INSTRUMENTATION FOR PRECISION MEASUREMENTS OF ANISOTROPY IN THE COSMIC MICROWAVE BACKGROUND

by

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Abstract

Instrumentation for Precision Measurements of Anisotropy in the Cosmic Microwave Background

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We describe a series of instruments designed to measure the anisotropy of the cosmic microwave background (CMB). The CMB is understood to be the relic radiation of the Big Bang; as the first radiation to propagate freely in the universe when it transitioned to an optically thin state, it is the oldest light in existence. As such, it represents a snapshot of the universe at a very young age, and encodes information vital to our understanding of the evolution of the cosmos into the complex system we observe today. This information is encoded in the statistical distribution of temperature differences (or anisotropy) in the microwave sky. These differences present a faint contrast of 1 part in $10^5$, or approximately $30\mu$K on a 3K background.

The MSAM/TopHat series of high altitude balloon-borne experiments were designed to measure CMB anisotropy with high precision. We report on the development of ultra-high sensitivity detectors for the MSAM2 instrument, and the results of the initial MSAM2 flight in 1997. For the MSAM2 1997 flight, we find significant excess variance in the 90 and 105 GHz sky maps. This excess variance is likely due to CMB temperature fluctuations, but the possibility of foreground contamination cannot be excluded. We also report on the development of the photometer for the next generation Tophat instrument, which flew on an Antarctic long duration circumpolar balloon flight and observed a 48° diameter section of sky centered on the South Celestial Pole.
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Chapter 1

A Brief Introduction to Modern Cosmology

Inquiry into origins is surely as old as human thought. Every civilization, in every recorded epoch, has had some construct to explain why things are the way they are, and how they came to be that way. If we were to ask our ancestors, far back into antiquity, some of the most fundamental questions one can ask: How old is the Universe? What is it made of? How long will it last?, they would have provided the answers with confidence. In our scientific age the questions remain the same, but answers that fit our post-Enlightenment sensibilities have, until relatively recently, proved elusive to the tools of science.

Cosmology is the study of these questions in a scientific context. It could be argued that Newton was the first cosmologist in a modern sense, when he extended his theory of gravitation from terrestrial phenomena to encompass the motion of heavenly bodies. This crucial hypothesis, that the rules of physics are everywhere the same, forms the bedrock of cosmology. Subsequent observations relevant to cosmology were slow in coming, however, and theories were slower yet; devoid of testable physical theories, the field remained a metaphysical backwater, with few serious practitioners. For example, we have long known the night sky is dark, a cosmologically interesting fact, but scientific explanations for this, the simplest of observations, inevitably led to paradox. It was only in the twentieth century that the pace quickened, and cosmology grew into the mature and active field it is today (although the former is still debated by some [1].) We now know why the sky is dark, and are uncovering clues that promise to lead us to answers to the fundamental questions.

It is remarkable that science should find itself in a position to address the properties of the universe on the largest scales, but as Peebles notes, it is "where the astronomy and physics has led
The beginning of the trail that has become modern cosmology can be traced to Einstein in 1915. The field equations of General Relativity provide a description of gravity, the dominant interaction on cosmological length scales. Scientists now had a framework within which questions about dynamics at cosmological distances could be posed.

This framework allows real, testable predictions to be made. Let us take the Copernican principle, which held that the earth is not the center of the solar system, and apply it to our expanded vista that encompasses the entire universe. We rename it the Cosmological Principle, or perhaps more aptly the Principle of Cosmic Modesty: That is, our point of observation, necessarily on or around earth, is unprivileged. We posit that our observations, in a statistical sense, would agree with those of any other observer anywhere. Clearly, this is in some sense a necessary assumption, for if our view of the sky were unique, any hope of discovering any universal property of the cosmos would be frustrated by our inability to observe most of it. It is also clearly the simplest assumption, so in the absence of evidence to the contrary we take Occam’s advice and start here. This introduces the concepts of translational and rotational invariance into cosmology: If we were to observe elsewhere, we would find similar statistical properties, as we would if we were to observe from the same place but in different directions. These properties impose homogeneity and isotropy on the geometric structure of the universe (Fig. 1.1).

Figure 1.1: 2D Illustration of Homogeneity and Isotropy. The figure on the left has a uniform linear gradient from top to bottom. This picks out a preferred direction but there is no preferred location. This object is homogeneous but anisotropic. The figure in the center has a uniform radial gradient. In the center there is no preferred direction, but this only holds in one location. This object is isotropic but inhomogeneous. The figure on the right has no preferred location or direction, and is both homogeneous and isotropic.

The Cosmological Principle forces the spacetime metric describing the universe on the largest
scales to take Friedman-Robertson-Walker form,

\[ ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \]  

where \( a \) is a time dependent scale factor, and \( k \) parameterizes the curvature of the space \[3\]. The rotational and translational invariance of this object may be easily demonstrated. Geometry has given us the ruler we need to measure spacetime intervals in our homogeneous and isotropic cosmology. Now Einstein’s field equations provide the physics that tells us how the geometry evolves in time. Applied to the Friedman-Robertson-Walker metric, the field equations become the Friedman equations (see e.g. Weinberg \[4\] for a complete derivation),

\[ \frac{\ddot{a}}{a} = \frac{4\pi}{3}(\rho + 3p), \]  
\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3}\rho - \frac{k}{a^2}, \]

relating the scale factor to the density \( \rho \) and pressure \( p \). Given an equation of state relating \( \rho \) and \( p \) for the field sources (e.g. radiation, relativistic matter, etc.), the dynamics of the universe are solved. Note that as written, the Friedman equations do not admit a static solution for the scale factor \( a \) (unless pressure \( p \) is negative). Hence, given General Relativity, one could have predicted in 1915 that the scale factor of the universe would be changing. Einstein derived exactly this, but preferred the rather bizarre hypothesis of the existence of negative pressure (an energy source with equation of state \( p \sim -\rho \)) to the idea of a nonstatic universe. This source, the so-called cosmological constant, facilitates a static, albeit unstable, solution to Equations 1.2 and 1.3. It was in this form that Einstein initially presented General Relativity.

It was with some regret, then, that Einstein greeted Hubble’s 1929 observation of the linear relation between distance and redshift \[5\], the ”expansion of the universe”. Surely the prediction of this phenomenon would have ranked as one of the greatest triumphs of theoretical physics. Einstein would later rate the cosmological constant as his biggest mistake\[^1\]. This observation, illustrating Einstein’s ”mistake”, represents the birth of modern observational cosmology. At last the field had an observation directly testing predictions on cosmological scales.

With the introduction of the idea of a dynamic spacetime metric into cosmology, the quantity \( \dot{a}/a \) arises in many contexts, and proves to be a convenient parameterization of the universe’s large scale dynamics. It is named the ”Hubble Constant” in honor of Hubble’s discovery of the time

\[^1\]It is ironic that Einstein’s greatest mistake may in fact have been abandoning the cosmological constant, as it again plays a central role in present day cosmological theory in the guise of inflation, and there is now compelling observational evidence that a cosmological constant plays a role in the current phase of the evolution of the universe.
varying nature of the scale factor. Note that the dimension of the constant is inverse time, so it implicitly sets a characteristic time and length scale, concepts incompatible with an infinite and ageless universe. With the expansion observation and General Relativity, the stage was set for the next advance in our understanding of the cosmos.

If objects in the universe are observed to be receding from each other, at earlier times they must have been closer together. Hence, at earlier times the universe must have been denser and hotter. If we trace this evolution far enough back in time, we deduce that the universe originated in an epoch of extremely high temperature. The hypothesis that the universe began at some time in the past introduces the revolutionary idea that the cosmos is of finite age. The model based on this simple hypothesis has been dubbed (by its detractors) the "Hot Big Bang". Its intimate relation to the Friedman-Robertson-Walker equations discussed earlier have also led to this title being used interchangeably with "Friedman-Robertson-Walker Cosmologies". Like any theory, it is remarkable not for what it was designed to model, but for what it predicts. The Big Bang Model makes several testable predictions.

The first depends on the nuclear physics developed soon after Hubble's observation. If the evolution of the universe is played back far enough in time (to ~ 1 minute after the Big Bang), the universe is dense and hot enough to disassociate atomic nuclei to free protons and neutrons. Hence, if we imagine the time arrow pointed forward again, around a minute after the Big Bang we see atomic element production in this primordial furnace, the temperature of which as a function time we know from our cosmological model. This coalition between the Big Bang and nuclear physics makes precise predictions about the cosmological abundances of the light elements. The observed deuterium and helium abundances are in striking agreement with the predicted Big Bang Nucleosynthesis (BBN) values (see e.g. Kolb and Turner [6]).

In the forties, Alpher and Herman [7] realized that a hot Big Bang carries another implication. As the universe continues to expand and cool, the temperature eventually (around 100,000 y after the big bang, or equivalently at a redshift $z \sim 1000$) drops below that necessary to keep hydrogen ionized. Hence, the Big Bang model predicts that at a very specific time, the universe transitioned from an optically thick plasma of tightly coupled photons and baryons (or more precisely, photons tightly coupled to electrons via Thomson scattering, and electrons coupled to protons via the Coulomb interaction), to a decoupled system of free streaming photons and neutral hydrogen. This transition occurs when the universe cools to around 3000 K, somewhat cooler than the surface of the Sun\(^2\). The tight coupling of the photons to the baryons provides a thermalization mechanism,

\(^2\)This occurs at a much lower temperature than the $kT = 1\text{Ry}$ that one might guess because the photon/baryon
so the photons released at this decoupling event\(^3\) should have a thermal, or blackbody, distribution of energies, and, in the absence of magnetic fields or any other additional property that would select some preferred orientation, they should be isotropically distributed.

These thermally distributed photons free-stream after decoupling, but expansion of the universe redshifts them. The effect of the redshift on the spectrum may be understood most simply by considering the effect of the redshift on a single mode. Since the spectrum is thermal, the occupancy number per mode is given by the Planck function,

\[
\langle n \rangle = \frac{1}{e^{\frac{h \nu}{kT}} - 1},
\]

\[
= \frac{1}{e^{\frac{hc}{kT \lambda}} - 1}.
\]

The occupancy number is constant in the absence of interactions [2], and the wavelength scales with the expansion, \(\lambda(t) \sim a(t)\), so as the universe evolves in time

\[
\langle n \rangle = \text{constant}
\]

\[
= \frac{1}{e^{\frac{hc}{kT(t) a(t)}} - 1}.
\]

This is only possible if \(T(t) \sim 1/a(t)\). This is independent of \(\lambda\), so the temperature associated with each mode scales exactly inversely with the expansion, and the blackbody spectrum is preserved. We thus have the remarkable result that the big bang model not only predicts an isotropic background of photons from the epoch of decoupling, but that the energies should be thermally distributed, and the temperature of the radiation should be that of the temperature at which decoupling occurred, redshifted by the expansion since decoupling. Redshifting a 3000 K blackbody by 1000, we would expect to find this radiation at 3 K today. A 3 K blackbody peaks at around 200 GHz, in the microwave portion of the spectrum, so the Big Bang model predicts that we should see an isotropic, thermal Cosmic Microwave Background (CMB) today.

The story of the interplay between theory and observation in the prediction of and search for the CMB is a splendid illustration of the nonlinear way in which scientific advances usually occur. The prescient work of Herman, Alpher, and later Gamow, was largely ignored, forgotten, or misinterpreted, and several pieces of observational evidence for the CMB were missed as a consequence (see Partridge [8] or Weinberg [4] for a thorough review). In the 1960s, Dicke’s group at

\(^3\)This is also referred to as recombination, a misleading term since neutral hydrogen had never existed until this time.

ratio is large; from BBN constraints this is around \(10^9\) [6]. Heuristically, a thermal distribution of photons at a temperature lower than that corresponding to 13.6 eV has enough photons in the Wien tail to keep the hydrogen completely ionized if the photon/baryon ratio is large.
Princeton, apparently independent of the previous work in the field, set out to build a receiver with the specific aim of detecting this cosmological radio signal. It is thus ironic that two researchers at Bell Labs, working on a problem unrelated to cosmology, using a technique for receiver gain stabilization developed by Dicke, were the first to make an unambiguous detection of the CMB that was recognized as such. The radiation temperature was not far from that predicted by Alpher and Herman some fifteen years earlier, and was smoothly distributed across the sky, completely uncorrelated with galactic latitude. Given the impact on cosmology that this momentous result, once confirmed, would have, the findings were published under perhaps the most devastatingly understated title in the history of scientific writing [9].

Although we have focused on the Big Bang (the victors inevitably write history), until the confirmation of the existence of the CMB there were several competing models of cosmological evolution. It is a testament to the importance of this discovery that subsequent to the detection of the CMB, models not based on a hot Big Bang were largely discarded. In the face of the observational evidence in the form of the expansion, the light element abundances, and the existence of the CMB, other models simply became untenable. This falsification of competing models represented a real milestone in the development of cosmology into a mature physical science: At last, data with real discriminating power began to trickle in.

The scientific payoffs gleaned from observations of the CMB were only just beginning with its detection. The prediction of the thermal spectrum of the radiation was vindicated to high precision by the FIRAS instrument on the COBE satellite. The observed spectrum of the cosmological background (Fig. 1.2) is observed to match that of the instrument’s onboard reference blackbody well, in a frequency range from 2 to 21 cm\(^{-1}\), with a temperature of 2.725 K \(\pm\) 0.002 K [11]. No systematic deviation from the Planck spectrum is detected. Competing theories must account for this perfect thermal spectrum in the face of evidence that the universe is optically thin out to a redshift of at least 5.8, so local absorption and reemission of starlight by dust, for example, is ruled out as a potential local source. It is a marvel that the most economical explanation for the source of this background is that it is the remnant radiation of cosmological creation, originating

\[4\] For a fascinating snapshot of the state of cosmology immediately prior to the detection of the CMB, see Bondi et al. [10].

\[5\] The Big Bang model also provides the explanation for the puzzle mentioned earlier: Why is the sky dark at night? This paradox (the so-called Olbers Paradox) is as follows: If the universe is of infinite extent and infinitely old, any line of sight must eventually land on a star. Hence the night sky should be as bright as the surface of a star. The Big Bang solution is simple: The universe is not infinitely old! (other physical effects play a role here, but the finite age is the crucial point)

\[6\] From the fact that quasars have been observed at this redshift. This is the most distant object (other than the last scattering surface of the CMB) currently reported [12]. Another, unconfirmed candidate has been observed at \(z=6.68\) [13].
in an epoch when the universe was only 10 millionths of its present age.

Figure 1.2: FIRAS measurement of the CMB spectrum. The model is the Planck function, and hence has one free parameter, a temperature of 2.725 K. The data points and the error bars corresponding to the ± 2 mK measurement uncertainties are obscured by the thickness of the line representing the model. From Mather, et al. [11].
Chapter 2

Cosmic Microwave Background Anisotropy

We have argued that the preponderance of observational evidence supports the Big Bang model, and that the CMB is the relic radiation of a hot, dense universe, carrying information imprinted on it when the cosmos was only on order 100,000 years old. As such, careful studies of all of the CMB’s degrees of freedom promises to reveal precious cosmological clues. In general, a photon field can carry information in three distinct ways:

- In its distribution in frequency space (i.e. its spectrum),
- in its distribution in space,
- and in its polarization state.

Clearly, information encoded in any of these ways is of fundamental cosmological interest. As mentioned earlier, the spectrum in the vicinity of the peak of the Planck curve has been measured to extraordinary accuracy by the FIRAS instrument\(^1\) on COBE. However, the long wavelength portion of the spectrum may still have some secrets to reveal about the earliest epoch of star formation, and about very early particle decay processes that would perturb the chemical potential of the CMB (processes, that is, that would not conserve photon number). A NASA satellite mission that will probe the far Rayleigh-Jeans portion of the CMB spectrum in the 15-0.3 cm range in an effort to uncover some of the characteristic signatures of these processes is currently in the planning stages [14].

In regards to its spatial distribution, we have argued that the CMB should be isotropic, based on the cosmological principle. However, clearly the cosmological principle breaks down on small

\(^1\)See [http://space.gsfc.nasa.gov/astro/cobe/](http://space.gsfc.nasa.gov/astro/cobe/) for more information on FIRAS.
enough scales. At distances of 100Mpc and below, the existence of large structures and voids (see e.g. [15], [16]) shows that the homogeneity assumption is not accurate to arbitrary order, and clearly there is a rich spectrum of structures (clusters, galaxies, solar systems, etc) on smaller scales.

2.1 Sources of anisotropy in the CMB

As mentioned in the first chapter, gravitation is the dominant interaction on the largest scales\(^2\), so it is expected to govern structure formation processes. If the evolution of the large scale structure we see today occurred via the gravitational interaction, we might expect that seeds of this structure were present during the CMB decoupling epoch, and that they thus influenced the spatial distribution of the CMB at some level. To understand this departure from perfectly uniform density we require an understanding of the origin and growth of inhomogeneities in the universe.

Early fundamental work on this problem was done by Sunyaev and Zeldovich, Sachs and Wolf, Silk, and others [17], [18], [19]. This has been an area of intense research; for a modern treatment, see e.g. Hu et al. [20]. If we assume some primorial seeding mechanism, as provided in e.g. inflationary scenarios, we may ask how these seeds evolve in time until decoupling, at which time their distribution is stamped on the CMB. Since we may observe this resulting anisotropy, the CMB provides a window back to the earliest epoch of structure formation, and study of its distribution provides powerful observational tests of structure formation theories.

Prediction of the statistical properties of anisotropy in the CMB requires knowledge of the species that make up the seeds of large scale structure (e.g. baryons, dark matter, cosmological constant\(^3\), etc.), their relative amounts, and the total energy density. Since all of these inputs affect the predicted anisotropy spectrum, measurement of the CMB provides real discriminatory power between competing models based on these constituents. A detailed treatment of these species and their signatures on the CMB is beyond the scope of this introduction; however, a heuristic argument for how anisotropy may be generated in general is relatively straightforward.

The Sachs-Wolf Effect The presence of seeds (density perturbations) implies that the CMB must be anisotropic at some level, since

\(^2\)In the absence of a significant aggregate charge asymmetry.

\(^3\)This is usually parameterized in terms of \(\Omega_i = \rho_i / \rho_c\), the ratio of the density of the \(i^{th}\) constituent to the critical density; the critical density \(\rho_c\) is the density that would make the large scale geometry of the universe flat. Note that flat models (as favored within many theoretical frameworks) thus require \(\sum \Omega_i = 1\).
Chapter 2: Cosmic Microwave Background Anisotropy

- photons from higher density regions must climb out of potential wells, and are redshifted to cooler temperatures.

- The overdensities cause a time-dilation effect, effectively making the overdense regions appear younger, i.e. hotter [8].

Note that these are competing effects. The net result is a temperature perturbation

\[
\frac{\delta T}{T} = \frac{\delta \Phi}{3e^2}.
\]

On superhorizon sized scales\(^4\), where plasma dynamics can play no role due to causal disconnection, this effect plays a dominant role.

**Acoustical oscillation modes in the photon/baryon fluid** Recall that prior to recombination, all the matter in the universe was tightly coupled to the radiation, forming a fluid of photons and baryons. We previously assumed this fluid was homogeneous in density on all scales. If we now introduce density perturbations\(^5\) on this smooth background, several new phenomena occur. A sub-horizon sized region of baryon overdensity would tend to grow in time, since gravitation is the dominant interaction, and a rarefaction would tend toward lower density. The baryons however, are tightly coupled to the photons, that, as a Bose gas, resist compression. These competing forces set up an oscillatory system\(^6\). These acoustic oscillations\(^7\) depend closely on the parameters that go into determining the oscillator.

- The photon/baryon ratio, for example, is clearly something like the ratio of the spring constant to the mass, so a smaller ratio implies a lower resonant frequency, and more baryons imply more compression in the regions of overdensity, altering the oscillation amplitude. The compressions and rarefactions of this fluid correspond, respectively, to photon temperatures slightly above and below the mean.

- The compressed, higher density regions redshift the hotter photons via the Sachs-Wolfe effect mentioned earlier. However, since the density distribution is dynamic on sub-horizon scales,

\(^4\)For the universe we observe today (\(z \sim 1000\)), the horizon scale is around \(10^6\) for most cosmological models.

\(^5\)Take these over/underdensities to be small enough that linear perturbation theory applies. In addition to greatly simplifying the analysis, it turns out that this approximation is adequate to describe evolution to the structure we observe today.

\(^6\)An early mentor pointed out to me that in 350 years of modern physics we’ve come to understand two things: The inverse square law and the harmonic oscillator. These two models nicely account for physics of the early universe as well.

\(^7\)“Optical reverberations”, due to Weiskopf [21], provides a more accurate mental picture.
Chapter 2: Cosmic Microwave Background Anisotropy

the potential wells evolve in time. The redshift effect in the time-varying case is called the integrated Sachs-Wolfe effect.

- The horizon size at the last scattering surface sets a characteristic scale for oscillations, since modes larger than the horizon size cannot evolve.\(^8\)

- The velocity of the fluid Doppler shifts the photons.

All of these effects (and others) conspire to produce a characteristic imprint of features on the CMB before decoupling.

The important point to take from all this from an observational viewpoint is that, given a cosmological model, the characteristics of the resulting oscillations can be derived. These oscillations then predict the statistical distribution of hot and cold spots in the photons, via the relatively straightforward processes described above. The era of decoupling essentially takes a snapshot of the photons,\(^9\) hence the matter distribution in this era, and these photons free-stream to us along the background geometry of the universe, redshifted but unscattered, and are seen as a radiation background. The characteristics of the oscillations and the geometry of the universe set a characteristic angular scale for the oscillations on the microwave sky today,\(^10\) that manifest themselves as temperature fluctuations.

The most common way in current use to represent the predicted temperature fluctuations from a given model is to plot its predicted angular power spectrum (Fig. 2.1); this is the fluctuation power on a given angular scale, plotted as a function of the spherical harmonic multipole moment $\ell$ since it is natural to describe the temperature distribution we observe now as a power series expansion on the curved sky.

2.2 The correlation function and the angular power spectrum

We now develop the language for describing how these processes at the last scattering surface appear on the microwave sky today. Cosmological models predict a fluctuation power as a function of correlation length, which maps to an angular scale on the sky today through the background geometry of the universe and the distance to the last scattering surface. The two-point correlation

---

\(^8\)Simply because superhorizon scales are causally disconnected, so there is no way to build up power in these modes.

\(^9\)The duration of the decoupling event is small relative to cosmological time scales, but not instantaneous. Hence some processing occurs as the photon mean free path transitions from small to essentially infinite, washing out anisotropy on the smallest scales.

\(^10\)Note that there are both dynamical and geometric effects that determine the fluctuation scale.
Figure 2.1: An illustration of the dependence of the angular power spectrum of the CMB on the underlying cosmological parameters. Note the characteristic "acoustic" peak around $\ell = 200$. Here, we have taken the Hubble constant $h = 0.72$, while the baryon density $\Omega_B$ has been varied in the range $0.046 \pm 0.020$; $\Omega_\Lambda$ has been simultaneously varied s.t. the total density $\Omega = 1$. Calculations were performed with the CMBFAST code [22].

\begin{equation}
C(\theta) = \left< \frac{\delta T}{T}(\theta_0) \frac{\delta T}{T}(\theta_0 + \theta) \right>, \tag{2.2}
\end{equation}

provides a complete description of the statistical properties of a Gaussian distribution\(^{11}\). If we can measure the temperature correlations on the sky today as a function of angle, we can directly test cosmological models. Since we observe these fluctuations on the curved sky, it is conventional to expand in spherical harmonics

\begin{equation}
\frac{\delta T}{T}(x) = \sum a_{\ell m} Y_{\ell m}(x). \tag{2.3}
\end{equation}

We measure the fluctuations $\delta T/T$, and would like to know the power at a given spatial frequency. Note that small $\ell$ corresponds to large angular scales; as a rule of thumb $\ell \sim \pi/\theta$. Exploiting the

\(^{11}\)Most, but not all, cosmologies predict Gaussian fluctuations. If Gaussianity does not hold, higher order correlation functions are needed to unambiguously characterize the temperature distribution.
orthonormality of this basis we can isolate the amplitude of a given mode

$$a_{\ell m} = \int \frac{\delta T}{T} Y_{\ell m}^* d^2 x.$$  (2.4)

The curved space analog of the Wiener-Khinchin theorem connects the correlation function to the power spectrum,

$$C(\theta) = \frac{1}{4\pi} \sum_\ell \sum_{m=-\ell}^{m=\ell} |a_{\ell m}|^2 P_\ell(\cos \theta)$$  (2.5)

$$|a_{\ell m}|^2 = 2\pi \int_{-1}^{+1} C(\theta) P_\ell(\cos \theta) d(\cos \theta),$$  (2.6)

where the $P_\ell$ are the Legendre polynomials. This provides a description of any distribution on the sky to arbitrary precision as $\ell \to \infty$. Note however, that since temperature differences arise from random processes, the models do not predict sky temperature but their statistical properties. Hence, we are not interested in the actual coefficients $a_{\ell m}$ measured but rather their ensemble average, i.e. the average power on a given angular scale

$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle.$$  (2.7)

This quantity, or as it is more frequently presented, the power per logarithmic $\ell$ interval,

$$\frac{\ell(\ell + 1)}{2\pi} C_\ell,$$  (2.8)

is the quantity theory predicts (again, see figure 2.1).

### 2.3 Anisotropy measurements

The search for anisotropy has been another interesting interplay between theory and experiment. As experimental upper limits on the size of fluctuations in the CMB continued to tighten (with the exception of the detection of a kinematic dipole at the mK, or 1 part in $10^3$ level), theoretically expected fluctuation sizes diminished as well, until in the early nineties the Big Bang model itself was in a state of crisis, as there seemed little room left within it to accommodate for the observed structure in the universe.

#### 2.3.1 Large angular scale detections

It was not until the results of the Differential Microwave Radiometer (DMR) instrument on the COBE satellite were published in the early 1990s that a definitive detection of anisotropy of cosmological origin was made ([23], for a recent reanalysis see Tegmark and Hamilton [24]). With $7^\circ$
beams, the DMR was only sensitive to the lowest $\ell$ values, or largest angular scales. These scales correspond, as we have seen, to superhorizon scales at the surface of last scattering, and are thus thought to mirror the primordial density perturbations via the Sachs-Wolfe effect. The detected fluctuations were at the 20 $\mu$K level, an extremely faint temperature contrast $\delta T/T$ of one part in $10^5$. These results were confirmed by the balloon-borne FIRS instrument [26] in a companion letter.

2.3.2 Medium angular scale detections: Current state of the art

The subdegree scale anisotropy that is so rich in cosmological information has been the subject of intense scrutiny in the post-COBE era. The drive towards higher resolution imaging of the CMB has been facilitated by the experimental effort towards developing ever more sensitive detectors, allowing the scanning of comparable sized patches of the CMB, in a comparable amount of time, at ever finer resolution. This effort has imposed ever tighter constraints on both both models and parameters. The "standard" cold dark matter model, for example, favored in the mid- to late nineties when anisotropy had been detected but was poorly characterized, is now completely inconsistent with the observational dataset, and models with a substantial cosmological constant component are favored.

The sophistication and sensitivity of the latest generation of instruments designed to measure anisotropy is ushering in an era that some have called "precision cosmology". The goal of simply making statistically significant detections of anisotropy has been replaced by the goal of making high resolution images of the temperature fluctuations, allowing tests of cosmological models with unprecedented discriminating power. Tophat [27], one of the instruments that forms the basis of this work, is an instrument that has been developed by workers at the University of Chicago, Goddard Space Flight Center, the Danish Space Research Institute, the Bartol Research Institute, and the University of Wisconsin, to map the CMB at subdegree angular scales from a long duration balloon platform with unprecedented accuracy. The development of the photometer for this instrument is discussed in this work in §3.3.2. Further references on the scientific results of this mission may be found in §3.3.1. MAXIMA and BOOMERANG, two high-altitude balloon borne instruments contemporary with Tophat, have both reported significant anisotropy detections with a well defined peak in the power spectrum around $l \sim 200$ [28], [29].

WMAP\textsuperscript{12}, a high resolution successor to the orbital DMR instrument, has recently published 23-94 GHz CMB anisotropy maps with SNR per pixel $> 1$ out to $l=658$. These measurements

\textsuperscript{12}The Wilkinson Microwave Anisotropy Probe.
have clearly set a new standard in CMB anisotropy observations. The results tightly constrain the underlying cosmological model, determining to within a few percent a whole host of fundamental cosmological parameters (age of the universe, time of decoupling, matter density, etc.) \cite{30}, \cite{31}. The WMAP results lend real credence to the use of the term "precision cosmology". The third degree of freedom of the CMB, its polarization state, is just beginning to be explored. There are theoretical reasons to expect the CMB to be polarized on medium angular scales \cite{32}, \cite{33}, and recent results from the DASI group \cite{34} have just yielded the first definitive detection of polarization of the CMB. The detection of polarization adds information that breaks some of the degeneracies between cosmological parameters that exist for anisotropy data alone; in addition, gravitational waves leave a characteristic imprint on CMB polarization that will, if observed, provide a direct observational probe of inflation. This nascent field will certainly be the focus of intense efforts in the immediate future, and promises to be the the next great frontier in observational cosmology.

### 2.3.3 Planned observations

An additional satellite mission, PLANCK \cite{35}, is planned in 2007. PLANCK’s combination of high resolution optics and ultra-sensitive detectors, on a stable, systematics free\footnote{In an environmental sense.} orbital observing platform, will likely allow full sky maps with resolution and signal to noise high enough to completely characterize the angular power spectrum of the CMB due to primary anistropy sources\footnote{Primary sources are those that cause anisotropy on the last scattering surface. Secondary sources are those that distort the CMB after decoupling.}. If this instrument performs as advertised it is likely to be the last word in measurements of CMB anisotropy. The PLANCK instrument will also measure polarization, and if sensitivity is as planned, the measured angular power spectrum will have dramatic consequences for both the physics of recombination and inflation itself, extending the probative power of observational cosmology back to time scales as early as $t \sim 10^{-32}$ s \cite{36}.

### 2.4 The future of CMB studies

Although the end of anisotropy characterization appears to be in sight, this in no way implies cosmology is close to being solved. The current measurements can be accounted for with models that have been previously developed, but this may be more a function of the fact that it is quicker to generate a model than it is to make a clean measurement. It seems likely that with the rapid
aggregation of observational data that we are experiencing today, new, unanticipated features will arise that will challenge our current paradigm for understanding structure formation.

Observationally, polarization studies of the CMB are unlikely to be completed with the Planck mission. In many ways, observational work in this field is in a position similar to the one anisotropy observations were in twenty years ago, due to the small expected signal levels relative to the anisotropy. Systematic effects in polarization measurements at the $\mu$K level have yet to be identified and accounted for. There is also the question of secondary CMB anisotropy that arises on small (arcminute) angular scales due to interactions of the CMB with matter during the first generation of galaxy formation. These fluctuations may contain information from the vast cosmological “dark ages”, from decoupling until the first generation of stars turned on. For an interesting discussion of post-PLANCK CMB observations, see Peterson et al. [37].

The current generation of CMB anisotropy measurements has finally put us in a position to address the fundamental questions we asked earlier in a scientifically well-posed way: What is the universe made of? How old is it? How long will it last? But for every potential answer there are many questions. The recent influx of data has triggered a euphoria of sorts in cosmology, with grandiose claims of vindicated models and implications of a deep understanding of the fundamental constituents that the universe is comprised of, and their evolution into the structure we observe today. This seems overly optimistic; consider the following:

- The isotropy of the CMB in the Big Bang model needs a period of superluminal expansion (inflation) to account for the uniformity of the radiation across causally disconnected regions. The idea of inflation leads to some conclusions consistent with observation, but makes no claims that are easily falsifiable\footnote{The $\Omega = 1$ prediction is a reasonably strong one, but inflationary scenarios exist that allow deviations from this, so if the universe were found to be closed, for example, this would not falsify the theory.} with the current tools available to observational cosmologists. Inflation relies upon a scalar field, an object that is not known to exist. Admittedly it shares this trait with the Standard Model, but sharing an unconfirmed concept with a well established theory hardly seems a virtue.

- 30-90% of the closure density of the universe in current models is made up of dark matter, the existence of which is inferred by observation but has never been directly detected.

- The cosmological constant, a quantity that has no microphysical motivation at all, plays a central role in current models.

While it is clear that our understanding of the cosmos has increased dramatically during the current
"golden age" of cosmology, it is also clear that the field is still young and there is much work to be done. Only time (and observational effort) will tell which concepts we find useful today will persist, and which will be the twentieth century's equivalent of the aether.
Chapter 3

Instrumentation

The detection of anisotropy in the CMB has been a great technical challenge. The extremely faint contrast of the temperature fluctuations ($\delta T \sim 30\mu$K on a 3K mean), in the presence of potentially confusing foreground emission from terrestrial and local astrophysical sources, places stringent demands on instrument sensitivity and stability, with tight control on systematic errors. For this reason, the CMB is in general observed with instruments that have been custom built and optimized for this purpose. Indeed, the instrumentation requirements that need to be fulfilled for successful anisotropy detection have been a driving factor in the development of ultra-high sensitivity mm-wave radiometer technology, and rapid advances in this field have provided us the opportunity to do the "high-precision" cosmology we speak of today.

One such development and observation program is the MSAM (Medium Scale Anisotropy Measurement)/Tophat collaboration, a joint effort of the University of Chicago, Goddard Space Flight Center, the Danish Space Research Institute, the Bartol Research Institute, and the University of Wisconsin. The goals of the collaboration are

- the design and construction of instruments capable of measuring CMB anisotropy on medium angular scales,
- observation of CMB anisotropy,
- analysis of the observational data, constraining $\delta T/T$ as a function of angular scale.

In this work, we report on the observations with and results from the MSAM2 instrument, as well as the development of the next generation Tophat instrument.
3.1 Instrumental considerations and requirements

We briefly summarize the experimental challenges quantitatively in order to pin down a target instrument performance, and illustrate the tradeoffs involved. As we have seen, the temperature contrast that we are trying to detect is small\(^1\) \(\delta T/T \sim \delta P/P \sim 10^{-5}\). A receiver with a 10 GHz bandwidth observing a 3K sky presents a power \(P = kT \, d\nu \sim 0.5\) pW at its output terminals\(^2\) (see appendix A); the fluctuation power \(\delta P\) is thus a miniscule \(10^{-17}\)W. To detect a temperature fluctuation of this size in e.g. ten seconds, the radiometer equation (again, reference Appendix A) implies that, to achieve a signal to noise of one, we need a system temperature on order \(T_{\text{sys}} = 10\)K, or a noise equivalent temperature (NET) \(T_{\text{sys}}/\sqrt{\Delta \nu} = 100\mu\text{K}\sqrt{s}\).

The time scale for the detection is set by the total observing time and the need to minimize the sample variance of the sky pixels observed. Recall that the goal is to characterize the statistics of the temperature fluctuations on the sky as a function of correlation length. To do this with any statistical significance we must obviously observe many pixels. Note that since theory predicts statistical properties of the temperature correlation function, and we have only one sky to observe, there is an unavoidable cosmic variance limit on our measurement accuracy. Clearly this must be a function of angular scale, for at small scales we can measure many temperature differences, while at large scales there are few independent samples. This limit is given by \(\sigma^2_\ell = (2/2\ell + 1)C_\ell^2\). In addition, with the exception of instruments on orbital platforms, we are restricted to limited sky coverage\(^3\). This augments the cosmic variance limit by the inverse fractional sky coverage \([38]\),

\[
\sigma^2_{\ell,A} = \frac{4\pi}{A} \frac{2}{2\ell + 1} C_\ell^2
\]

where \(A\) is the area observed in steradians. Hence, the variance of our measurement of a \(C_\ell\) depends on the statistical error due to instrument noise and the sample variance due to the limited number of pixels available for observation. Approaching the intrinsic cosmic variance limit clearly requires maximizing sky coverage, which, given a typically fixed observing time, competes with the reduction in variance on a given pixel by integrating in time. Given a fixed amount of observing time and a fixed instrument sensitivity, observing a field size that gives a signal to noise of 1 per pixel is a nearly optimal observing strategy, striking a compromise between the significance of a detection

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\(^1\)We work in the Rayleigh Jeans approximation for simplicity. CMB observations near or past the peak of the 2.725K Planck function at 160 GHz are subject to thermodynamic corrections that further reduce the fluctuation power (but not the contrast).

\(^2\)Assuming 100% optical power transmission efficiency through the instrument. This is of course never realized in practice.

\(^3\)Even for instruments on spacecraft, the removal of observations in the galactic plane typically limits sky coverage.
per independent pixel (independent "pixels" are discussed below) and cosmic variance (Knox [39]).

Real telescopes do not yield measurements that can be pixelized to arbitrary precision. In practice, all instruments have finite resolution, and the radiometer output is proportional to the sky distribution weighted by a beam shape $B(\vec{x})$ (see appendix A),

$$ S(\vec{x}_0) \sim \int T(\vec{x}) B(\vec{x}_0 - \vec{x}) \, d\vec{x}, \quad (3.2) $$

where $T(\vec{x})$ is the sky temperature distribution. This convolution of the sky distribution means that structures smaller than the beamsize will not be detected; the finite beam size essentially imposes a low pass filter on the instrument’s sensitivity in $\ell$ space. If an instrument’s beam is well modeled by a Gaussian in angle with standard deviation\(^4\) $\sigma$, its $\ell$ space sensitivity rolls off at high spatial frequencies by a factor

$$ B_\ell = e^{-\sigma^2(\ell+1)/2} \quad (3.3) $$

Hence, as might be expected, detection of fluctuations on subdegree angular scales requires a beamsize smaller than a degree.

In summary, an instrument designed to observe anisotropy in the CMB must:

1. Have a high enough sensitivity to give a statistically significant detection in each pixel, while

2. covering an adequate amount of sky to reduce sample variance.

3. Have a beam small enough to resolve the cosmologically interesting angular scales.

The points above address statistical errors only and are hence an idealization of the design constraints; of course systematic errors must be tightly controlled to prevent them from dominating at these levels. Insulation against systematic error imposes a separate, largely independent set of constraints on CMB experiments, and is a central part of practical instrument design; indeed, the majority of the data analysis effort for even the most carefully designed experiments is usually directed towards understanding and removing residual systematic effects. While the sensitivity requirements for a CMB telescope can be neatly framed in terms of NET, integration time and beamsize, systematic effects are instrument specific and often quite subtle. A set of ”best practices” based on past experiences has arisen in the CMB field to provide a designer with some guidance to building a robust experiment, but at the levels of sensitivity needed for anisotropy detection unanticipated instrumental residuals invariably emerge and must be accounted for. A complex,

\(^4\)It is more common to refer to the beam size of an instrument in terms of its full width half maximum, $\theta_{FWHM} = 2\sqrt{2\ln 2}\sigma$. 

multiply modulated sky measurement is desirable to ensure that these systematic contaminants are as orthogonal to the sky signal as possible. These issues will be explored in depth in the subsequent instrument design and analysis sections.

Assuming systematic errors are kept in check to below the statistical limit, experiments that achieve the sensitivity target specified above may be expected to yield power spectrum results comparable to those shown in Fig. 3.1.

Figure 3.1: The angular power spectrum measurement resolution of two instruments with NET=150 µK√s and $\theta_{FWHM}=20'$ beam sizes. The first, plotted over an underlying standard CDM model, assumes a 4.5 hour observation of a 12 square degree patch of sky. These are typical numbers for an instrument with a single beam observing from an overnight flight on a high altitude balloon. The second, plotted over an underlying ΛCDM model, assumes a 10 day observation of an 1800 square degree patch of sky, half of which is usable due to editing. These numbers could be expected for a long duration high altitude balloon flight. On large angular scales (small $\ell$) the cosmic variance dominates the error. On medium angular scales the instrument noise dominates. On the smallest scales, the finite beam size fails to resolve features, causing the variance to diverge as $\ell > 1/\theta_{FWHM}$. 
3.2 MSAM2

The first generation of the Medium Scale Anisotropy Measurement (MSAM), designed and built at MIT, was the first instrument to detect fluctuations in the CMB on subdegree angular scales [40]. The radiometer was designed to interface with a balloon borne telescope, with sufficient sensitivity to measure anisotropy on an overnight flight. This instrument flew three times, reobserved the same sky fields to confirm results, and has been cross correlated with the Saskatoon dataset [41], placing strict limits on the systematic errors of both experiments. For the most recent analysis of the MSAM data, see Wilson et al. [42].

The MSAM2 instrument, designed and built at Brown University/Wisconsin, extends on the capabilities of the first generation MSAM in beam resolution and sensitivity. The balloon gondola [43],[44], and optics and cryostat [45] are documented in previous works. Here we summarize these results, and detail subsequent refinements of the instrument. We also describe the design and performance of the bolometric radiometer used for MSAM2. Some additional discussion of the gondola in the context of telescope pointing during CMB observations may be found in §4.3.

3.2.1 The balloon-borne observing platform

The extremely faint CMB temperature fluctuation contrast can be easily dominated by local contaminants. Atmospheric emission, in particular, can hinder CMB anisotropy measurements in two distinct ways

- The constant in-band emission presents a large radiometric load, effectively lowering sensitivity.

- Atmospheric instability mimics fluctuation power on the sky, confusing the sky signal.

As is evident in Fig. 3.2, the spectrum of the atmosphere is complex, with contributions from many molecular species. Above 40 GHz, the advantages of observing at high altitudes are substantial [47]. Although balloon borne observations are necessarily of shorter duration than ground based observations, and designing an instrument for the rigors and requirements of ballooning adds an order of magnitude to the complexity of the experiment, the net increase in sensitivity more than compensates for the reduced integration time and added effort.

Figure 3.3 illustrates the observing geometry typically available to balloon borne CMB instruments. The experiment is generally limited to a range of zenith angles between 29° and 45°, because of obstruction due to the balloon near the zenith and increasing airmass and atmospheric emission.
Figure 3.2: Atmospheric emission for ground and balloon based observations. The ground site spectrum illustrates a best case observing scenario from a stable mountaintop site. The bandpasses for the MSAM2 and Tophat instruments are overplotted (Tophat channel 5 at 600 Ghz is not shown). The drastic reduction in atmospheric emission when observing at 35 km clearly illustrates the advantage of observing from a high altitude balloon platform. Atmospheric modeling for Tophat was done with the Grossman AT code [48].

towards the horizon, along with the increasing risk of 300K earthshine coupling to the optics. Our next generation Tophat instrument, to be described later, takes a radical new approach by observing from the top of the balloon, eliminating obstruction from the balloon and the risk of earth limb contamination.

3.2.2 Optics

The mm-wave optical design of a CMB telescope must achieve a delicate balance between several competing figures of merit (resolution of angular scales of interest, adequate optical throughput, high directivity). In addition, the optics contribute to the loading on the radiometer by thermal
Figure 3.3: Observing range for a bottom hung balloon borne payload. In the low-pressure (~ 3 Torr) environment at an altitude of 35 km, the balloon expands to a volume of 40 million cubic feet.

Emission. A low emissivity (ε ~ 0.01) aluminum surface at 250 K5 contributes a brightness comparable to that of the CMB. Hence, cooled optics (where feasible) may play an important role in reaching target instrument sensitivities.

The small signal level from the CMB also sets stringent limits on the beam directivity. Even if the beam accepts off-axis power at -70 dB relative to the on-axis response, a signal comparable to the CMB fluctuation signal will result if this portion of the beam falls on the 300 K earth. In practice high directivity is achieved by underilluminating the primary mirror so that power near the mirror edges is small, and diffraction at the edges is minimized. In addition, the optical elements are surrounded with reflective ground screens to insure that any residual beam spillage is diverted away from the 300K earth onto the cold sky. Note that the goal of underilluminating the optical elements competes with the goal of minimizing the beam size, since the quantity $A_e \Omega_B$ is a constant (see §A.2).

5A typical temperature for an optical surface at balloon float altitudes.
Figure 3.4 depicts the MSAM2 optical system. The optical elements are arranged in an off-axis Cassegrain configuration. The off-axis secondary eliminates support structures that could emit or diffract power into the main beam. The sky signal is gathered by a 1.3m diameter primary mirror and reflected to a hyperbolic secondary mirror nutating at 2.5 Hz. The motion of the secondary modulates the sky signal up into a portion of the post-detection audio band where the detectors have optimal noise properties (see the following section on the MSAM2 bolometric detectors). The optical signal from the secondary then enters the vacuum space of the radiometer through a 0.020” polypropylene window, and is split into two beams by a wire-grid polarizer. The beams are each gathered by cooled tertiary mirrors at 77K, which direct the power into low and high frequency corrugated feed horns\(^6\) cooled to the temperature of the liquid helium pot. These horns couple a single electromagnetic spatial mode to rectangular waveguide. The reduction in étendue that single mode systems suffer relative to multimode systems\(^7\) is compensated by the simple, well-understood Gaussian beam patterns such systems yield. In addition, the development of cryogenic systems that are capable of cooling detectors to lower temperatures than previously possible in a balloon borne package compensates for the reduced throughput, allowing the MSAM2 detectors to achieve nearly background limited sensitivity\(^8\). High coupling efficiency is also an advantage of single mode designs.

The heat budget for the 100 mK cold stage for the MSAM2 detectors cannot tolerate the parasitic heat loads that would be imposed on the system by even the lowest conductivity stainless steel waveguides. Instead, the cold stage is supported by Kevlar thread tensioned between the stage and a support frame, and the optical signal is fed into the detector housing across a small (0.005”) waveguide gap. A choke groove around the rectangular waveguide joint improves the match across the discontinuity; insertion loss across the full receiver band was less than 1dB, and was found to be insensitive to changes in alignment that were mechanically possible given the support structure.

Astrophysical foregrounds from local galactic processes are potential contaminants that may mask the underlying cosmological signal. Hence, spectral resolution is essential for unambiguous CMB anisotropy detection. Fortunately, the spectral indices of the primary culprits (synchrotron radiation, bremsstrahlung, and dust) are all markedly different from the blackbody spectrum of the CMB, so data from a multi-spectral band instrument can be modeled as a linear sum of CMB signal and contaminants, and the contribution of the contaminants can then in principle be subtracted from each channel. MSAM2 is triply immunized against foreground contamination:

\(^{6}\)The polarizer and low and high frequency horn arrangement essentially acts as a first stage of frequency duplexing.

\(^{7}\)Also see §A.2.

\(^{8}\)See §3.2.4.1
• It incorporates a 5 channel radiometer, with channels spanning E, W, and D bands (65-170 GHz),

• it measures near the CMB’s spectral peak at 160 GHz, where dust emission is low and synchrotron and bremsstrahlung are small relative to the CMB signal, so the cosmological-to-foreground signal contrast is nearly optimal (Fig. 3.5), and

• the observing strategy focuses on fields with low dust emission (based on composite 3000 Ghz DIRBE-IRAS survey data [82]).

Hence, the CMB is expected to dominate the astrophysical signal measured by MSAM2.
Figure 3.5: MSAM2 spectral coverage relative to dominant astrophysical foregrounds.

Single mode optics yield a further benefit when designing an instrument’s spectral resolution: Relatively precise stripline designs for band defining filters may be used. In MSAM2 the low frequency optical signal is triplexed by a stripline filter into three sub-bands; the high frequency signal is duplexed to two. These signals are then fed to the single-mode optimized detectors. The stripline filters used in MSAM2 were sufficiently selective, with acceptable in-band transmission, but robustness over multiple thermal cycles was an issue. The nominal band centers for MSAM2 are presented in Table 3.1.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (GHz)</td>
<td>72</td>
<td>90</td>
<td>105</td>
<td>140</td>
<td>165</td>
</tr>
</tbody>
</table>

Higher order modes may in principle propagate through the optical system up to the bandpass filters. This poses a potential loading risk to the bolometers since the high frequency (Near IR) transmission of the bandpass filters is unknown, and also would represent an additional parasitic thermal load on the cold stage itself. We therefore cascade the optical signal with an additional low pass filter constructed from quartz beads embedded in polypropylene and inserted in each of the waveguides just before the bolometer box discontinuity. Wilson [45] and Farooqui [46] document
the "thick grill" tests performed on MSAM2 to measure the Near IR rejection of the MSAM2 telescope.

3.2.3 Cryogenics

It is necessary to cryogenically cool the radiometric elements of a receiver in order to achieve the sensitivity\(^9\) necessary to detect the CMB. The bolometric detectors we use in MSAM2/Tophat require an operating temperature around 100 mK in order to achieve near background limited performance\(^10\). The development of cryogenic cooling technology capable of achieving these temperatures, in a package compact enough to be flown on a high altitude balloon, has been a crucial step towards realizing the sensitivities necessary to make high precision CMB anisotropy measurements.

3.2.3.1 The cryostat

A cross section of the MSAM2 cryostat is also shown in Fig. 3.4. There are three cryogen tanks: A buffer liquid nitrogen (LN\(_2\)) tank that serves to reduce the optical loading on an internal liquid helium (L\(^4\)He) tank, and an extension LN\(_2\) tank that cools the tertiary optical elements and the beam splitter to 77K. The helium fill port is left open to atmosphere during flight, cooling the L\(^4\)He to 1.4K in the ~ 3 Torr environment at 35km (fig. 3.6) altitude. For ground operations the helium port is actively pumped to reach these temperatures. The L\(^4\)He pot at 1.4 K provides an acceptable bath temperature from which the next stage of refrigeration can operate. The nitrogen pressure is regulated to 4psi absolute during flight to prevent the liquid from freezing.

3.2.3.2 The adiabatic demagnetization refrigerator (ADR)

The sub-Kelvin temperatures required for high sensitivity bolometric detector technology are difficult to achieve with the robustness and weight constraints that high altitude ballooning impose. Small \(^3\)He refrigerators with internal adsorption pumps have been flown successfully, but are limited to temperatures above ~ 240 mK. For MSAM2, a compact magnetic refrigerator capable of regulating a cold stage at 100 mK for the duration of an overnight flight has been developed. This represents the coldest temperature achieved on a balloon borne platform.

\(^9\)Instrument sensitivity is typically characterized by the system Noise Equivalent Temperature (NET) in CMB work; see §A.4 for a definition.

\(^10\)The background limit is the limit set by the intrinsic fluctuations in the signal, i.e. photon noise. See the subsequent section on bolometric detectors for a quantitative discussion. The bath temperature needed to attain a given sensitivity is somewhat higher for Tophat due to the multi-mode optical design.
Figure 3.6: Temperature as a function of pressure for liquid helium. The Clausius-Clapeyron equation, \( dp/dT = l/T \Delta v \), relates the temperature of an equilibrated two-state gas/liquid system to the pressure of the gas; \( l \) is the latent heat of vaporization of the liquid and \( \Delta v \) is the difference in specific volume of the two states. Since \( v_{\text{gas}} \gg v_{\text{liquid}} \), \( \Delta v \approx v_{\text{gas}} \). From the ideal gas law \( v_{\text{gas}} = RT/p \), so \( dp/dT = lp/RT^2 \Rightarrow p = p_0 e^{-l/RT} \), where we approximate \( l \) as a constant \( 88 \text{ J mol}^{-1} \), true to about 4\% for \( ^4\text{He} \) for the temperature range shown. See Reif [89] for a more thorough treatment.

Magnetic refrigeration was first suggested by Debye in 1926 [49]. At the time, pumping on liquid helium was the only technique capable of achieving sub-Kelvin temperatures. Debye’s technique takes an entirely different approach, exploiting the entropy associated with the spin orientation of magnetic moments in a paramagnetic material (typically a salt). Schematically, the process is as follows (see Fig. 3.7)

- A paramagnetic salt is thermally connected to a bath at temperature \( T_0 \). The spin orientation is random (aside from a small, intrinsic internal field), and the lattice, in equilibrium with the bath, experiences vibrations consistent with temperature \( T_0 \). Note that there is an entropy associated with the disorder of each of the two systems, and that the two systems are independent.
- A magnetic field is applied to the salt while the lattice is maintained at temperature \( T_0 \), i.e. the salt is isothermally magnetized. The disorder in the lattice system is constant, but the
Figure 3.7: Illustration of paramagnetic salt lattice/spin system. The temperature of the lattice is related to the rms displacement of the ions from their zero-point lattice positions, represented by the grid. The spin orientation of the ions forms a separate system with an associated entropy. If the spins are aligned by applying a magnetic field and the salt is then thermally isolated from its surrounding, the temperature of the lattice is coupled to the entropy of the spins, i.e. the applied magnetic field.

- The thermal link to the bath is broken, and the applied magnetic field is reduced. Since the system is now isolated, no heat is exchanged between the salt and its surroundings; the process is isentropic (adiabatic). As the spins randomize, the entropy associated with them becomes higher. Since the process is adiabatic, this entropy must come from the lattice. Hence, the temperature of the lattice is reduced.

Essentially, since the total entropy is a function of spin alignment ($B$) and lattice vibrations ($T$), $S = S(B, T)$, when the system is adiabatically isolated, $\Delta S = 0$, and $B$ is coupled to $T$. It is shown in the appendix that the entropy depends on the ratio of the magnetic energy to the thermal energy, so we can further state $S(B, T) = S(B/T)$, so if $\Delta S$ is constant, $B/T$ is constant, i.e. $B_i/T_i = B_f/T_f$.

Magnetic refrigeration techniques were dominant in low temperature laboratories until the development of dilution refrigerators in the 1960s, with their lower temperature capabilities and continuous cooling operation. However, the miniaturizability of ADRs makes them ideal candidate refrigerators for ballooning, since the hold times of ADRs can easily be made to exceed the duration
of an overnight balloon flight, making continuous operation or in-flight recycling unnecessary.

Magnetic refrigeration clearly requires the entropy associated with the spins to be large compared to the lattice entropy \( S_{\text{spin}} \gg S_{\text{lattice}} \)^11, so magnetic cooling would not be expected to work at high temperatures. The 1.4 K bath temperature available from a pumped \(^4\)He bath, however, provides an environment where this condition is fulfilled for several materials. For MSAM2, we chose the paramagnetic salt Iron Ammonium Alum (FAA, see Table 3.2). The temperature entropy diagram for FAA is shown in Fig. 3.8. The cycle proceeds as indicated by the arrows. The field is applied to the salt by placing it in the bore of a 3T superconducting solenoidal magnet^12. Note that the field is not reduced to zero on the demagnetization leg of the cycle. Instead, the ADR is designed with an operating margin that allows regulation at a temperature setpoint.

Table 3.2: Characteristics of Iron Ammonium Alum (FAA).

<table>
<thead>
<tr>
<th></th>
<th>( Fe_2 (SO_4)_3 (NH_4)_2 SO_4 24H_2O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composition</td>
<td></td>
</tr>
<tr>
<td>Molar density</td>
<td>482 g/gmol</td>
</tr>
<tr>
<td>Mass density</td>
<td>1.71 g/cm(^3)</td>
</tr>
<tr>
<td>Total angular momentum ( J )</td>
<td>5/2</td>
</tr>
<tr>
<td>Magnetic ordering temperature ( T_c )</td>
<td>30 mK</td>
</tr>
</tbody>
</table>

The coupling of the lattice temperature to the spins when the salt is adiabatically isolated provides a convenient method for temperature regulation. In practice, all cold stages will have some parasitic heat load on them. If there is still an applied field on the salt, this can be reduced to compensate for the heat input and maintain the stage at a constant temperature. This is represented by the bottom leg of the cycle: The salt remains at constant temperature as the entropy increases due to the heat input, by the gradual reduction of the applied magnetic field. In practice, the field is controlled by servoing the magnet current off a voltage input from a germanium resistance thermometer (GRT) on the cold stage^13. From Fig. 3.8 it is clear that the salt is best understood as an entropy reservoir; while regulated, the salt can absorb an entropy \( \Delta S = \Delta Q/T \), e.g., heat \( Q \) at temperature \( T \), or heat \( Q/2 \) at temperature \( T/2 \). For this reason it is best to demagnetize slowly relative to the internal time constant of the cold stage/salt system, since finite thermal conductances will inevitably result in thermal gradients, causing the salt to absorb heat during the cooling process at a lower temperature than necessary. Conversely, the optimization of

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^11To quote Reif’s analogy: A golf ball, no matter how cold, cannot cool a swimming pool by an appreciable amount!

^12American Magnetics, Inc., Oak Ridge, TN

^13The long time constants involved can make this control difficult. We use a proportional/integral/differential (PID) control loop as described in Forgan [51].
Figure 3.8: Entropy vs. temperature for Iron Ammonium Alum (FAA). Lines of constant magnetic field are overplotted, and the demagnetization cycle is shown. Temperature regulation at 100 mK can be maintained until the applied field reaches zero, at which point the refrigerator must be recycled. The analytic expression for $S(B, T)$ is furnished in the appendices (Equation B.4).

The refrigerator requires careful thermal design, insuring maximum thermal contact between the salt and the stage that must be cooled. This will result in a refrigerator that operates at optimal efficiency with a maximum duty cycle\textsuperscript{14}.

**Salt Pill** Making contact to the salt itself is the primary thermal conductance limit. We optimize this by constructing a salt "pill" as shown in Fig. 3.9. The construction is based on a design developed for SIRTF \textsuperscript{50}. The salt crystal is grown in a stainless steel housing filled with gold wires\textsuperscript{15}. Gold is used since it is resistant to the corrosive salt but still has acceptable thermal conductivity. For optimal thermal contact the number of wires must obviously be maximized, but not at the expense of occupying too much of the internal volume of the salt housing. This calls for maximization of the surface area/volume ratio of the wires. We chose 0.010" diameter wire as the smallest feasible to work with for this purpose. For the housing size shown we use $\sim$375 7" wires, yielding an 82 in$^2$ surface area in contact with the salt, and a 5% filling ratio of the wires in the

\textsuperscript{14}In this case, the amount of time regulating at the target cold stage temperature relative to the total operation time. We have achieved 98-99\% for our balloon borne refrigerator.

\textsuperscript{15}Johnson and Matthey Co., Fairfield, NJ. Quotes for large amounts of gold can vary wildly - shop around.
Chapter 3: Instrumentation

can. The gold wires are silver soldered\textsuperscript{16} into a cup in a 99.999\% Cu post that serves as a thermal bus to the cold stage that houses the detectors. This joint is sealed with Stycast 2850 to isolate the joint from the salt.

The salt is sealed in the stainless housing to prevent water evaporation. Water loss destroys the pill, since it is the water that is responsible for the large separation between the magnetic ions that yields a low magnetic ordering temperature. Welding has proven the most reliable sealing method, but the process is risky since the salt is also temperature sensitive. We have had good production yield by constructing large clamping jigs to hold the housing and serve as a heat sink. The bottom cap is then TIG welded to the housing on a rotary stage.

**Heat Switch** The operation of the ADR requires a method of disconnecting the cold stage from the bath for the transition from the isothermal leg to the adiabatic leg of the cycle. Traditionally there has been some tradeoff between performance and reliability in this component. Gas gap and superconducting heat switches have no moving parts, but the "off" state thermal conductivity typically dominates the heat load on the cold stage [52]. For MSAM2, a solenoid activated mechanical heat switch was developed [45]. This switch physically disconnects the salt pill from the bath, resulting in zero loading contribution from the switch itself in the off state. We have extended the capabilities of this design by building a mechanical heat switch activated by a stepper motor. This allows great flexibility of operation, since the switch can be positioned precisely, and the driving current can be adjusted externally, improving reliability.

The stepper driven switch is shown in Fig. 3.10. Preparation for operation at low temperature

\textsuperscript{16}Using a cadmium free compound that remains a normal conductor at low T.
simply involves disassembling the motor armature and degreasing the rotation bearings. After multiple acetone/methanol rinses in an ultrasonic cleaner, the bearings are baked out to remove any residual moisture and coated with a dry lubricant\textsuperscript{17}. The stepper motor\textsuperscript{18} shaft is fitted with a cam that drives two clamping arms that interface with flats on the salt pill shaft. The motor mount doubles as the thermal bus from the clamps to the helium plate. The thermal connection from the moving arms to the mount is made with a thick braid of high-purity copper wire. All components in the thermal bus from the clamps to the bath are made of Au plated high-purity copper. The salt pill shaft is gold plated as well. The contact points between the shaft and the clamping screws are particularly important for the final conductivity performance of the switch, as this interface joint is the primary conductance bottleneck. Gold plating and careful cleaning of the contacts is crucial.

We have measured the performance of the stepper actuated heat switch using the arrangement shown in Fig. 3.11. A metal film resistor and a silicon diode temperature sensor are fastened to a gold plated copper screw that simulates the shaft of the salt pill. This assembly is isolated from the bath by mounting on a low conductivity G-10 tube. The switch clamps on the screw, connecting it to the the bath. With the cold plate at 4.2 K, a current $I$ is applied to the resistor. The power generated by Joule heating in the resistor is routed to the bath through the switch. By measuring the temperature difference between the bath and the screw, we obtain the conductivity of the switch $G = I^2R/(T - T_0)$. Opening and closing the switch and repeating the measurement yielded a consistent $G = 30 \text{mW/K}$. The conductivity is a function of the force applied, which is set by the positioning of the clamping screws. It is possible that the motor is capable of greater torque and hence higher conductivity, so this number is not claimed to represent the optimal performance of the switch. It is however, already an improvement over previous designs and represents a workable number\textsuperscript{19}.

The coils of the stepper motor are not well heat sunk due to the composition of the motor body. This results in the internal components reaching a temperature well above that of the cold plate. We ameliorate this to some degree by stripping the coil leads as close to the motor as possible, and gluing the bare leads to a copper plate with Stycast. This plate is then bolted to the cold plate. This heating could be greatly reduced by rewinding the coils with superconducting wire as suggested by Porter \textit{et al.} [53]. Note, however, that the switch is stable in the on or off state, so

\footnotesize
\textsuperscript{17}Molybdenum Disulfide (Moly) powder  
\textsuperscript{18}Model P532, Portescap US, Inc., Hauppauge, NY  
\textsuperscript{19}Large "on" conductivity minimizes the time required for the cold stage to cool to the bath temperature after the salt has been magnetized. It is largely this cold soak time that determines the refrigerator’s duty cycle.
the motor only needs to be energized during the switching operation. Superconducting wire would also preclude motor operation at 77K, which we find convenient in the everyday cooling operation of the dewar.

**High current, low temperature wiring**  The superconducting magnet requires 7 A to achieve its maximum field of 3 T. This requires high-current wiring, which conflicts with the goal of minimizing the load on the L\textsuperscript{4}He stage. For MSAM2 we route the magnet leads and the single heat switch lead through the helium tank with superconducting wire. This provides ample current carrying capacity and low thermal loading, but necessitates a leak tight bulkhead between the helium tank and the vacuum space. The demands on this joint are stringent, since the helium is a superfluid during operation.

We constructed a feed-thru for this purpose similar to a design in Richardson and Smith [52]
Superconducting wires are routed through a copper flange with a conical nipple with very thin walls. The wires are then glued into the flange with Stycast 2850, with the Stycast coating the outside of the cone. This takes advantage of the differential contraction between the epoxy and the copper upon cooling, leaving the epoxy in tension on the copper at 4 K. We have found that it is very important for joint reliability to degas the Stycast by pumping on it for several minutes prior to application. After adopting the practice of degassing of the epoxy, we have had no failures in this component. The bulkhead is bolted to the L\textsuperscript{4}He plate with stainless steel screws and Belleville washers, and sealed with indium wire.

The difficulty of sealing against a superfluid leak does introduce an additional cryostat failure mode. An alternate approach we developed for another dewar exploits high temperature superconductor technology. We run conventional high current leads to the buffer nitrogen tank, which can easily absorb the additional heat load. The high current connections from the LN\textsubscript{2} tank to the L\textsuperscript{4}He tank are then made using high $T_c$ superconducting tape. The tape consists of many high $T_c$ ceramic filaments embedded in a silver-alloy sheath. The sheath provides good room temperature conductivity, while yielding good thermal isolation when cold. The tape is quite fragile, and the ceramic filaments are prone to breaking if bent. Enclosing the superconductors in a capsule as shown in Fig. 3.13 provides a robust package capable of handling very high currents between 77 K and 4 K with good thermal isolation characteristics.

\footnote{Over 1.5 years of operation, and approximately 25 thermal cycles.}
\footnote{BICC General Superconductors, Wrexham, Wales, UK}
Refrigerator module  The assembled refrigerator is shown in Fig. 3.14 (the heat switch has been removed for clarity). The magnet is housed in a high permeability case to minimize stray fields in the dewar. The salt pill shaft can be seen protruding from the magnet bore. The pill is suspended by tensioned wound Kevlar\(^{22}\), providing a rigid, low thermal conductivity support structure; parasitic loading with this arrangement is around 0.5 \(\mu W\) (Table 3.3). The three point support incorporates sprung posts on knife edge pivots to maintain uniform tension at all temperatures and over time as the Kevlar stretches. The modest size and mass of this ADR make it simple to incorporate in most any dewar design. Since the refrigerator is entirely electrical, installation requires only wiring (no leak tight plumbing) to the outside of the dewar, using one of the high current lead techniques

\(^{22}\)E. I. du Pont de Nemours and Company

<table>
<thead>
<tr>
<th>(\frac{dT}{dt})</th>
<th>17.2 (\mu K) min(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C(100mK))</td>
<td>1.850 J K(^{-1})</td>
</tr>
<tr>
<td>(Q = CdT/dt)</td>
<td>530 nW</td>
</tr>
</tbody>
</table>
described above. The refrigerator pictured is now in use in one of our lab cryostats at the University of Wisconsin.

**Extending the technology** As stated earlier, the entropy of the lattice must be small compared to the spin entropy for magnetic refrigeration to work. To reach our target temperature of 100 mK with enough entropy margin to yield an adequate hold time, FAA must start the demagnetization cycle at \( \sim 2 \) K, which requires a pumped helium bath. This is an operational inconvenience that adds considerable complexity to everyday dewar operations. We have built and tested an ADR capable of cycling from 4.2 K and yielding very long hold times (\( \sim 6 \) days), by using a second pill of different composition (GGG - gadolinium gallium garnet) as a buffer for the 100 mK FAA cold stage. The design is similar to an ADR developed at NIST [54]. In this two stage ADR design, the GGG is demagnetized simultaneously with the FAA, reaching a temperature of approximately 1 K when the FAA has reached its target temperature. The buffer pill effectively lowers the temperature at which the FAA pill demagnetization begins, and serves as a low temperature base for the FAA pill suspension. This greatly reduces the parasitic load on the 100 mK cold stage relative to single stage designs; we measure 160 nW for the two stage refrigerator tested. This refrigerator design is ideally suited for a long duration ballooning mission where 100 mK temperatures are required. For a complete discussion of the cryostat and two stage ADR, see Gundersen et al. [55].

### 3.2.4 Bolometers

At frequencies upward of 100 GHz, bolometric detection of radiation offers the highest sensitivity in the current state of the art [56]. A basic, commonly used bolometric detection scheme is shown in
Fig. 3.15. The bolometer is comprised of a radiation-absorbing material, a thermistor, and a weak link to a thermal bath at temperature $T_s$. The thermistor is biased by a DC voltage and a large load resistor, essentially current biasing the device. If optical power is coupled to the absorber, the temperature of the device will rise above that of the quiescent (dark) state. Since the resistance of the thermistor changes along with this temperature change, the voltage at the "Signal Out" terminal will change. Note that since the bolometer thermally detects the incident radiation, this detection scheme is phase incoherent, and is intrinsically "square-law"\textsuperscript{23}.

Some of the parameters over which the bolometer design must be optimized are immediately clear from this model. For a given input optical power level, the temperature that the bolometer reaches, hence the output signal level, is inversely proportional to the conductance of the thermal link. It is therefore desirable to minimize the conductivity $G$ of the link. However, the components of the bolometer that are isolated from the bath have some heat capacity $C$ that must cool through the link. Hence, the rate at which independent measurements can be taken is constrained by a time constant $\tau = C/G$. To maximize sensitivity faced with the time constant constraint, it is therefore clear that it is desirable to minimize $C$, and then minimize $G$ until the maximum tolerable time constant is achieved. Thermal power is also dissipated in the bolometer by the current through

\textsuperscript{23}That is, it is proportional to the square of the incident field intensity, or the power.
the resistive thermistor, changing the detector’s operating point. Optimization of the detector’s sensitivity therefore requires careful adjustment of the bias voltage for a given optical power input. Sensitivity optimization over these design and setpoint parameters requires a full noise model for the detector.

3.2.4.1 Bolometer responsivity and equilibrium noise model

Models that permit detailed analysis and optimization of bolometers have been developed and validated [58], [59], [60], [56]. Here we state some important results to provide context for reporting on the detectors developed for the MSAM and Tophat projects.

The bolometer shown in 3.15 absorbs optical power at its input and presents a voltage signal at its output. Clearly, the voltage output per unit of power input is an important parameter for characterizing the device. This quantity, the responsivity, is given by

\[ S = \frac{IR\alpha}{G(1+i\omega\tau)}, \]

where \( I \) is the bias current, \( R \) is the bolometer resistance, the parameter \( \alpha = R^{-1}dR/dT \) characterizes the thermistor’s temperature dependence, \( G \) is the conductance of the weak thermal link, \( \omega \) is the frequency at which the input optical power is modulated, and \( \tau \) is the time constant.

Figure 3.15: Principle of bolometric signal detection.
described above. This expression can be derived by writing down the dynamic energy balance for the bolometer’s absorber (see appendix B.2). Viewed as the transfer function for the detector, it is seen that the bolometer functions as a single-pole low-pass filter with a 3 dB knee at $\omega = 1/\tau$. For the detectors developed by the MSAM/Tophat collaboration for CMB observations, responsivities are on the order of $10^9$ V/W.

The responsivity relates power at the bolometer input to the voltage signal at the device output. To complete the picture, we need a model of the noise processes intrinsic to the bolometer to evaluate its suitability for use for CMB measurements. A figure of merit for the sensitivity of the detector is required here; it is typical to use the Noise Equivalent Power (NEP) of the detector, the incident power necessary to equal the noise power in a 1 Hz bandwidth, for this purpose in bolometer work. Various physical processes, both internal to and independent of the detector, contribute to the total NEP of a bolometric radiometer:

- The resistive thermistor element generates Johnson Noise at the bolometer output with spectral density $v_n^2 = 4kTR$, ($V^2$ Hz$^{-1}$). The responsivity may be used to relate this voltage noise source to the detector input,

$$\text{NEP}_J^2 = \frac{4kTR}{|S|^2} \text{ (W}^2\text{Hz}^{-1}),$$

yielding the Johnson noise contribution to the total detector NEP.

- There is a thermal fluctuation noise associated with the heat flux $\dot{Q}$ through the weak thermal link between the absorber and the bath. The NEP of this noise source is

$$\text{NEP}_T^2 = 4kT^2G \text{ (W}^2\text{Hz}^{-1}),$$

- Subsequent noise in the detector readout electronics is a potential contributor to the total system NEP. This contribution can generally be made negligible by using a cooled amplifier with gain as the first readout stage, as was implemented for MSAM2.

- Non-ideal behavior, such as excess noise with a $1/f$ spectrum, inevitably contributes to the detector NEP at low frequencies.

- Photon noise intrinsic to the optical signal limits the ultimate sensitivity of the system.

The last contribution noted does not represent a technical limitation of the radiometer but rather a fundamental limit for measurement of an optical signal over a given bandwidth. It is useful to
determine the photon noise contribution to the NEP of a given radiometer, as this defines the best performance achievable for a given measurement. When the intrinsic noise terms dominate the total radiometer NEP, the radiometer is said to be "detector noise limited". When the detector noise terms have been minimized to the point that the photon noise term dominates, the radiometer is said to be "photon noise limited" (this lower limit on the NEP is also sometimes called the "BLIP" limit\(^{24}\)). It is important to note that all sources of radiative loading contribute to the BLIP noise; in practice, thermal emission from optical elements, the atmosphere, and astrophysical foregrounds all degrade the sensitivity of a BLIP limited bolometer to a level higher than that imposed by the signal (the CMB) itself. For a detailed discussion on estimating photon noise limits, see Hauser [61] or Mather [58]. Here, we will simply summarize these important papers with the following result: For an instrument with throughput \( A \Omega \) and optical efficiency \( \eta(\nu) \), observing a source with emissivity \( \varepsilon \), the photon noise NEP contribution is given by

\[
\text{NEP}_{\text{PHOT}}^2 = \int 2h\nu A\Omega \eta(\nu) \frac{2h\nu^3/\varepsilon^2}{e^{h\nu/kT} - 1} \left[ 1 + \frac{\eta(\nu)}{e^{h\nu/kT} - 1} \right] d\nu \tag{3.7}
\]

Since the individual noise sources detailed above are uncorrelated, the total NEP at a post-detection audio frequency \( f \) may be obtained by calculating the quadrature sum of the various noise contributions,

\[
\text{NEP}^2 = \text{NEP}_J^2 + \text{NEP}_T^2 + \text{NEP}_{AMP}^2 + \text{NEP}_{1/f}^2 + \text{NEP}_{PHOT}^2. \tag{3.8}
\]

Figure 3.16 compares the intrinsic detector noise terms\(^{25}\) for the Tophat radiometer, with bolometers heat sunk to a 220 mK bath, to the BLIP noise limit from the CMB alone for each radiometer channel. Assumed radiometric optical efficiency is 30\%. As stated above, additional optical power from the beam forming optics, as well as astrophysical foregrounds (dust emission) in the higher frequency channels, elevate the BLIP limit beyond that shown. It is apparent that the Tophat radiometer approaches the BLIP limit.

Given a model for estimating detector NEP as described above, the algorithm for designing a detector for a specific application [60] may be broadly described as follows:

- Estimate the optical loading on the detector, including signal, foregrounds, and instrument emission.

- Fix the operating temperature of the bolometer relative to the bath. A \( \Delta T/T_{\text{bath}} \sim 0.5 \) is a typical value to begin with for an optically loaded detector [59].

---

\(^{24}\)Various origins for the acronym are claimed - most common is "Background Limited Performance".

\(^{25}\)Calculated from Mather’s detector model using code written by D. Cottingham.
Chapter 3: Instrumentation

Figure 3.16: BLIP limit (CMB contribution alone) vs. frequency for the Tophat radiometer. The NEP due to noise processes in the detector alone approach the CMB BLIP limit.

- Compute the conductance \( G = P/\Delta T \) required for the target operating temperature.
- Check detector time constant \( \tau = C/G \) relative to required modulation frequency \( \omega \).
- Compute noise (or, w/ optical efficiency estimate, sensitivity). Optimize sensitivity by selecting optimum bias current.
- Evaluate relative to sensitivity required for measurement. Observing time and other measurement considerations may enter here.

Each design stage is of course subject to technical constraints (bath temperature achievable given a cryostat design, conductivity values possible given detector materials, etc.). Lowering temperature is generally always desirable, although detector speed may limit detector NEP above DC at low \( T \).

3.2.4.2 Monolithic silicon bolometers

Various techniques are used for bolometer construction. For both MSAM2 and Tophat, we utilize\(^{26}\) monolithic silicon bolometers as described in Downey \textit{et al.} \[57\]. In this approach, the thermistor element and electrical leads are ion-implanted into bulk silicon. Thin support legs, which form

\(^{26}\)The detectors for both experiments were built within the collaboration at Goddard Space Flight Center.
the weak thermal link to the bath, are formed by etching. The bolometer is then coated with a metal with surface resistance selected to match the detector to the feed for optimal radiation absorption efficiency. The resulting device is robust and free of complications (such as Kapitza resistance between internal components) that plague composite bolometer structures. Monolithic construction also facilitates the minimization of the bolometer heat capacity, since bonding agents, which may dominate composite bolometer heat capacity budgets, are not needed.

The temperature dependence of the implanted thermistor is well parameterized by the relation

\[ R = R_0 \exp \left( \frac{T_0}{T} \right) \quad (3.9) \]

where \( T_0 \) is a strong function of the doping density and \( R_0 \) is a function of the doping density and the thermistor geometry. Processing must be controlled so that workable resistances at the intended operating temperature are achieved. Resistances in the 1-10 MΩ range are used in combination with fixed, cryogenically compensated 80 MΩ load resistors\(^ {27} \); this presents an output impedance that is easily read out by a JFET for the first stage of amplification. The JFET\(^ {28} \) is cooled to reduce its NEP contribution to below that of the bolometer itself\(^ {29} \). The complete readout circuit for the MSAM2 bolometer is shown in 3.17.

The conductivity of the weak thermal link is primarily due to phonon conduction and therefore varies with temperature as \( T^3 \). The conductance of the link is characterized by a conductance

\(^ {27}\text{MSI chip resistors via Sunbelt Micro, Deltona, FL.}\)
\(^ {28}\text{InterFET Corporation, Garland TX. For MSAM2 we use 2N6451 n-channel JFETs, regulated at } T = 100K \text{ in a box mounted to the MSAM2 L4He tank.}\)
\(^ {29}\text{At 100 K, approximately 20% of the devices tested met the required } 5 \text{ nV/ } \sqrt{\text{Hz} } \text{ specification required for use with the MSAM2 detectors; hence all devices were hand selected.}\)
parameter $G_0$, 

$$G_0 = G / T^3 \quad \text{(W K}^{-4}) \text{).}$$  \hspace{2cm} (3.10)

The bolometer developed for MSAM2 is shown in Fig. 3.18. This detector is designed to couple to a single waveguide mode; the absorber of the bolometer is aligned with the $\vec{E}$ field of the dominant TE$_{10}$ mode of a rectangular waveguide. Bismuth is evaporated onto the bolometer absorber to provide the surface resistance necessary for terminating the feed, and adjustable quarter-wave backshorts are incorporated to optimize optical efficiency. Absorption efficiencies deduced from individual monochromatic reflection measurements are better than 90% across the full radiometer bandwidth (65 - 170 GHz) [62]. This single-mode design yields a radiometer with well controlled optics, allows the use of conventional stripline filters, and minimizes the bolometer size. However, as a consequence of the reduced optical throughput relative to multi-mode systems, the photon noise limit is lower, requiring colder operating temperatures to achieve BLIP limited performance. Development of the lightweight 100 mK adiabatic demagnetization refrigerator described earlier

![Figure 3.18: MSAM2 single-mode monolithic silicon bolometer.](image)
facilitated use of this detector design.

3.2.4.3 MSAM2 bolometers: Pre-flight device characterization

Each bolometer produced is characterized by in-lab testing before use. We perform dark tests to measure the conductance \( G \) of the weak thermal link, and determine the \( R_0 \) and \( T_0 \) parameters in the thermistor resistance model (eq. 3.9). For these tests, a sequence of measured bias voltages is applied to the series combination of the load resistor and the bolometer, and the DC voltages at the bolometer terminal are read out (reference Fig. 3.17). The bias voltage sequence is repeated over several different bath temperatures. The absolute DC level at the FET output is of course device dependent and somewhat arbitrary. In addition, non-ohmic contacts at the bolometer pads can lead to polarity dependent voltage drops across the detector; this phenomenon must be eliminated for a useful detector. For these reasons, for each DC voltage \( V_{BIAS} \) we apply to the bolometer, we apply an inverted voltage \( -V_{BIAS} \). The bolometer resistance is then given by the total change in voltage at the bolometer and the total change in the bias voltage,

\[
R_{BOLO} = \frac{R_{LOAD}}{(\Delta V_{BIAS}/\Delta V_{BOLO}) - 1}. \tag{3.11}
\]

An idealized example of time-ordered data from a DC measurement of a bolometer, and the \( R(T) \) curve and load curve derived from the data, are shown in Fig. 3.19. By measuring load curves at different base temperatures, as illustrated in Fig. 3.20, the device parameters \( R_0, T_0, \) and \( G_0 \) are determined; this establishes the DC responsivity of the detector and provides the data necessary to predict noise characteristics for a given background. A typical set of measured and derived parameters for an MSAM2 bolometer is shown in Table 3.4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_0 )</td>
<td>380 ( \Omega )</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>13 K</td>
</tr>
<tr>
<td>( G_0 )</td>
<td>20 nWK(^{-1})</td>
</tr>
<tr>
<td>( S ) (DC)</td>
<td>( 1 \times 10^9 ) ( \text{VW}^{-1} )</td>
</tr>
<tr>
<td>NEP</td>
<td>( 8 \times 10^{-15} ) ( \text{WHz}^{-1/2} )</td>
</tr>
<tr>
<td>NET, Rayleigh-Jeans, 5 K loading, ( \eta=0.2 )</td>
<td>320 ( \mu K ) ( \text{s}^{1/2} )</td>
</tr>
</tbody>
</table>

Prior to the 1997 MSAM2 flight, we performed a series of measurements with a variable temperature cold termination inserted into each of the microwave feedhorns as shown in Fig. 3.21. The cold load was constructed in a manner similar to the calibrator used for the FIRAS instru-
Figure 3.19: An idealized example of DC characterization of a bolometer with no incident optical power. The top left panel shows the time-ordered data from a simulated load curve measurement. A sequence of voltages $V_{BIAS}$, of positive and negative polarity, are applied to the series combination of the 80 M$\Omega$ load resistor and the bolometer. Joule heating in the thermistor raises the temperature of the bolometer, lowering the resistance as the bias voltage increases. This causes the bolometer voltage $V_{BOLO}$ to flatten out vs. $V_{BIAS}$ as $V_{BIAS}$ increases. Solving for the bolometer current vs. the bias voltage yields the load curve shown in the lower right. The device shown has parameters $R_0 = 100\Omega$, $T_0 = 15K$. 
Figure 3.20: Example bolometer load curves at varying bath temperatures. For the bolometer shown, $R_0 = 380 \Omega$, $T_0 = 13K$, and $G_0 = 20 \text{nW K}^4$.

Figure 3.21: Block diagram of MSAM2 optical efficiency measurement.
Table 3.5: MSAM2 pre-flight optical efficiencies, as measured with cold terminations at the feed horns.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>efficiency</td>
<td>9%</td>
<td>18%</td>
<td>44%</td>
<td>10%</td>
<td>7%</td>
</tr>
</tbody>
</table>

ment [11]. Using the configuration shown, the load curve procedure described above was repeated with the cold termination at temperatures between 5 and 20 K. Since the load completely fills the horn aperture, the power incident on the optical system is determined. By including an incident optical power term in the bolometer energy balance, the absorbed optical power is determined; this provides a measure of the system optical coupling efficiency $\eta$. Load curves with varying optical loading also allow direct determination of the optimal bias point for a given background.

The optical efficiencies measured via this method prior to flight were lower than desired. The stripline multiplexers had given problems previously [45], and had been repaired, but new problems were suspected. Unfortunately, there was at this point no prospect of reopening the radiometer to effect fixes and still having the instrument ready for the summer of 1997 flight window. The measured efficiencies (Table 3.5) were sufficient for achieving our science objectives, but degrade the instrument’s sensitivity somewhat relative to the design goal.

3.2.4.4 MSAM2 bolometers: Post-flight performance analysis

MSAM2 was launched on 1 June 1997. It acquired approximately 270 minutes of CMB observations in the course of its overnight flight, along with several planet observations for calibration purposes, and other diagnostic data such as in-flight bolometer load curves. The data from the load curves indicates that the in-flight loading was substantially higher than that anticipated from pre-flight estimates (Table 3.6). This excess loading compromised sensitivity relative to the estimate in Table 3.4, as shown in Fig. 3.22. An account of the investigation into the source of the excess loading in MSAM2 is provided in §3.2.4.5; a full discussion of the observation is provided in chapter 4.

Periodic "glitches" are evident throughout the MSAM2 time-ordered flight data. These events, caused by high energy cosmic rays interacting with the detectors, are endemic to observing from a high-altitude platform. Although they require data editing to prevent contamination of the CMB data, they are very brief in duration and, given the small cross-section of the MSAM2 detectors, quite infrequent$^{30}$. Glitch events can also be exploited to measure detector characteristics; they are

$^{30}$Glitch editing necessitated the removal of approximately 5% of the MSAM2 CMB scan data. Details are provided in chapter 5.
Table 3.6: MSAM2 in-flight operating point and loading estimates. The optical efficiencies of Table 3.5 are used to infer the effective temperature of the optical load on the detectors, based on the in-flight bolometer operating points. $C_0$ is the dark channel.

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{bias}}$</td>
<td>61.9</td>
<td>38.7</td>
<td>101.9</td>
<td>38.5</td>
<td>63.2</td>
<td>38.9</td>
</tr>
<tr>
<td>$T$</td>
<td>126</td>
<td>133</td>
<td>161</td>
<td>129</td>
<td>113</td>
<td>102</td>
</tr>
<tr>
<td>$R$</td>
<td>3.38</td>
<td>8.77</td>
<td>1.24</td>
<td>12.56</td>
<td>4.28</td>
<td>-</td>
</tr>
<tr>
<td>$G(100 \text{ mK})$</td>
<td>21</td>
<td>37</td>
<td>21</td>
<td>18</td>
<td>37</td>
<td>-</td>
</tr>
<tr>
<td>$P_i$</td>
<td>0.8</td>
<td>1.9</td>
<td>2.9</td>
<td>0.8</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td>$T_{\text{LOAD}}$</td>
<td>46</td>
<td>51</td>
<td>32</td>
<td>29</td>
<td>28</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 3.22: Calibrated MSAM2 in-flight power spectral densities (Rayleigh-Jeans sensitivity, transfer function deconvolved). The data shown is a short segment of the west CMB scans. The power spectrum is calibrated using the data from an observation of Jupiter during the flight. Note that to obtain a sensitivity estimate in mK s$^{1/2}$, the vertical axis should be divided by $\sqrt{2}$. The narrow-band features at harmonics of 2.5 Hz are chop-synchronous offsets.
\[\delta\text{-function depositions of energy into the detector, and hence the resulting signal in the time-ordered data is the time-domain representation of the transfer function of the entire signal chain.}\]

In principle, the full transfer function of the instrument (from optical input to electrical output) in the time domain can be described by the impulse response of the detector, which we have shown to be that of a single-pole low-pass filter parameterized by time constant \(\tau\), convolved with the time-domain representation of the electronics transfer function. The preamp schematic shown in Fig. 3.17 was used to calculate the exact expressions for the readout electronics’ transfer function. Spectrum analyzer measurements, both pre- and post-flight, agree with the calculated transfer function to high precision (Fig. 3.23). All components are military spec and rated to 5% or better over a much wider temperature range than that actually seen in flight (the temperature of the readout electronics is actively regulated, Fig. 3.24.) Thermo-vac testing at the NSBF facility in Palestine under flight-like pressure and temperature conditions uncovered no gain stability issues. We therefore have a high degree of confidence in the in-flight stability of the readout electronics, and would expect that the glitches could be fit to a model with the bolometer time constant the single free parameter.

The gain settings for the west CMB scans were high enough that glitches typically railed at the ADC input. For the north scans and the planet scans, however, the gains were set lower and most glitches were within the dynamic range of the readout electronics. We nevertheless find marginal agreement between the transfer function inferred from the particle hits and that calculated and measured on the ground (Fig. 3.25). Given the confidence we have in the stability of the rest of the signal chain, we consider the possibility that the bolometer response to the particle hits is not well modeled by a single-pole low-pass filter.
Figure 3.24: Temperature of the MSAM2 readout electronics box over the duration of the 1997 flight. Active temperature regulation maintained the preamps at 285 K for the duration of the time at float.

Figure 3.25: Comparison of in-flight glitches (black) and the transfer function model (red). Residuals (blue) to a fit with a single free parameter (the bolometer time constant) are poor.
Figure 3.26: Bolometer equivalent circuit, with FET input capacitance. For \( R_L >> R_b \), the circuit on the left may be replaced by the Thevenin equivalent circuit on the right, with the bolometer represented by an AC voltage source \( v_b \) with output impedance \( R_b \). This output impedance combines with the FET input capacitance to form an RC lowpass at the JFET gate.

The simplest refinement to the model of the frequency response of the bolometer is the inclusion of the effect of the input capacitance at the cold JFET (Fig. 3.26). This adds an additional term to the signal transfer function

\[
s_{FET} = \frac{1}{1 + i\omega\tau'},
\]

where \( \tau' = R_bC_{FET} \), resulting in an effective responsivity (c.f. Equation 3.4)

\[
\tilde{S} = \frac{IR\alpha}{G(1 + i\omega\tau)} \frac{1}{1 + i\omega\tau'}.
\]

This shunt capacitance effectively changes the responsivity to that of a two pole low-pass filter, rolling off signal faster than predicted from the bolometer model alone. However, the input and stray capacitance on the bolometer gate lead is small (~ 50 pF), and incorporating this element in the transfer function model failed to replicate the observed behavior for any reasonable input capacitance values. A phenomenological model that replaces the single detector time constant with two time constants, and couples the incident energy equally between them, comes closer to describing the observed response. The general idea of a multiple time constant model was motivated in part by information provided by the McCammon group at the University of Wisconsin, based on experience developing monolithic Si detectors for X-Ray work. The hypothesis is that the internal mechanisms for thermalizing incident energy and power signals\(^{31}\) may differ; all analytical power

\( ^{31} \)These terms are borrowed from communications engineering parlance. Consider an arbitrary signal \( g(t) \).

The energy of the signal is defined as \( E_g = \lim_{T \to \infty} \int_{-T/2}^{T/2} |g(t)|^2 \, dt \). The power of the signal is defined as \( P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 \, dt \). Signals for which \( E_g \) is finite, such as a particle hit well localized in time, are termed energy signals. Signals for which \( P_g \) is nonzero and finite are termed power signals.
is removed from the glitch events in this case, since the system transfer function for particle hits differs from the transfer function for the CMB scans.

We determine which transfer function to use for the CMB analysis by comparing the $\chi^2$ of fits to the Jupiter raster for the phenomenological glitch model with the $\chi^2$ for the calculated model. We find that the calculated model, with the bolometer time constant as the only free parameter, is preferred. The bolometer time constants determined by these fits are provided in Table 3.7. The time constants quoted are consistent with those measured in earlier lab tests [45]. Detailed discussion of the transfer function model and the fits to the planet data is provided in chapter 5.

Table 3.7: MSAM2 in-flight bolometer time constants, as determined by fits to the Jupiter raster.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.41 ± 0.11</td>
<td>2.10 ± 0.06</td>
<td>3.89 ± 0.11</td>
<td>4.53 ± 0.07</td>
</tr>
</tbody>
</table>

Figure 3.27: In-flight voltage noise, MSAM2 west scan 4, channel 2. The signal at the input to the ADC (top trace) is referenced back to the bolometer gate lead (bottom trace) by deconvolving the instrument transfer function, including the cold JFET ($G = 18$), AC preamps ($G \sim 1000$ mid-band), and post-amps ($G = 204$). Signal band voltage noise is 20-25 nVHz$^{-1/2}$. 
3.2.4.5 Excess in-flight optical loading of the MSAM2 detectors.

We performed a series of tests on the MSAM2 cryostat after the 1997 flight to determine if the source of the excess optical loading was internal to the instrument. To simulate the loading conditions at float in a lab environment, an external cold load was constructed. This load is similar in principle to the one described above that was designed for measuring optical efficiency at the horn inputs; here the beam dump is moved to the cryostat vacuum window instead. Since the load is external to the cryostat, it requires its own cooling. We converted a small IR labs LN$_2$/L$^4$He dewar (the "Gold Dewar") for this purpose. A cold load, composed of deep concentric metal rings with a triangular cross section and covered with mm-wave black epoxy$^{32}$, was mounted to the Gold Dewar L$^4$He cold plate. Embedded thermometry was incorporated in the design to provide monitoring of the temperature distribution on the cold load. The entire dewar was then bolted to the vacuum window (or "snout") of the MSAM2 dewar, as shown in Fig. 3.28.

![Figure 3.28: MSAM2 cryostat with cold load attached. Collaborators happily pondering the source of the excess loading in the MSAM2 cryostat are, from left to right, Josh Gundersen, Lucio Piccirillo, and the author.](image)

Recall that from DC characterization of the detectors with no optical power absorbed, we establish the resistance vs. temperature dependence of the detectors. Then, using the internal cold load we establish the optical efficiency of the system up to the feed horn input. During these tests on MSAM2, we measured detector temperatures consistent with the cold load, i.e. no excess was

---

$^{32}$Eccosorb CR-114
observed. This directed our attention to the optics beyond the feed horns - the dewar extension that houses the tertiary mirrors and beam-splitting polarizer (refer to Fig. 3.4 for the extension position).

With the cold load at approximately 8K on the dewar snout, we ran load curves on the detectors and measured temperatures consistent with those in flight, i.e. the excess loading scenario was replicated. The source of the excess loading was thus confirmed within the cryostat.

MSAM2 relies on a polarizer to align the low and high frequency beams on the sky. This beam combining technique exploits the single polarization, single mode acceptance of the feed horns: The horns are rotated such that the accepted polarization aligns with the polarization that is fed, via the polarizer, out the vacuum window to the external optics. For the low frequency horn, this is the power transmitted through the polarizer, for the high frequency horn, this is the power reflected off the polarizer, as is shown schematically in Fig. 3.29. Note that for both horns, the cross-polarized beam is directed at the internal wall of the 77K cavity housing the optics. This "beam" does not fall on an optical surface and is free to scatter in the cavity; it essentially views a 77K blackbody. However, the measured cross-polarization acceptance of the feeds is attenuated 20 dB relative to the polarization-aligned beam, implying a negligible optical power contribution to
the detector loading from cross-polarized components. This suggested in the original design phase that this beam-splitting approach was feasible.

We therefore began by investigating other potential excess loading sources in the dewar extension. An initial cause of concern was the flatness of the tertiary mirrors. The surface after the final manufacturing cut exhibited significant structure with a period of 0.100" and peak-to-peak amplitude of .003". The depth is small compared to a wavelength, and the calculated diffracted power using a simple model of the surface is small, but due to the vagaries introduced into the more realistic calculation due to cavity effects in the extension we proceeded to polish the mirrors. As expected, the excess loading was unchanged.

Scattering and absorption at the polarizer could cause excess loading: If the beam is scattered from the polarizer into the 77K cavity, or if the 77K polarizer is highly emissive, this would directly increase the background power incident on the detectors. We performed a series of warm measurements on the polarizer itself, and found that in all cases the mm-wave insertion loss was less that 1.5%. This mechanism for generating excess loading was ruled out.

The cross-polarization attenuation of greater than 20 dB quoted earlier was measured with the horn and polarizer mounted on a lab bench test jig. In principle, in situ misalignment of the horn relative to the polarizer could lead to a larger cross-polarization acceptance. The complex, non-planar geometry of the horn/tertiary/polarizer system makes this scenario quite plausible. To test this, we remachined the horn mounts such that they were able to rotate approximately \( \pm 15^\circ \) about the optical axis. We then repeatedly cooled the system with the horns rotated throughout their range of motion, and found no variation in loading (and confirmed that the initial horn position was optimally aligned with the polarizer.)

Still suspicious of the polarizer, we simply removed it from the system. Note that without the polarizer serving as a beam splitter, the low frequency beam is transmitted directly out of the dewar (see Fig. 3.29), while the high frequency beam views the interior of the extension. We cooled the cryostat in this configuration, and found bolometer temperatures consistent with those expected from the cold load in the low frequency channels, while the high frequency channels were essentially unchanged from previous measurements.

With the upcoming MSAM2 1998 campaign hanging in the balance, and no time to implement a fundamental optical design change, we considered the following work-arounds:

- Fill the extension tank for the flight with liquid Neon (LNe) rather than LN\(_2\). This would uniformly decrease the excess loading in the extension from any internal source, due to LNe's lower boiling point (27 K at 760 Torr). LNe has a latent heat of vaporization somewhat less
than half that of LN$_2$; this would have necessitated extensive modifications to the extension cryogen tank to maintain the hold time needed for an overnight flight.

- Fly MSAM2 in the test configuration shown to yield low optical loading; that is, fly with the three low frequency channels only, omitting the polarizer from the experiment. The loss in total instrument sensitivity suffered by removing the two high frequency channels (channels 4 and 5) would have been more than compensated for by the increase in sensitivity in the three low frequency channels\textsuperscript{33}; recall that the variance on a quantity measured $i$ independent times is related to the variances of the $i$ individual measurements by

$$\frac{1}{\sigma^2} = \sum_i \frac{1}{\sigma_i^2}. \quad (3.14)$$

The MSAM2 radiometer is capable of achieving an NET of 150 $\mu$K$_{RJS}^{1/2}$ in a low background environment. Using Equation 3.14 for a rough estimate, running with three channels degrades this sensitivity by a factor of $\sqrt{3/5}$ to 190 $\mu$K$_{RJS}^{1/2}$ - still competitive with its experimental contemporaries in mid-1998 and much better than that achieved in the 1997 flight. However, spectral resolution would obviously be impaired.

In the end, we elected not to refly MSAM2, primarily so the science team could devote its full attention to our next-generation balloon-borne instrument, Tophat. The MSAM2 cryostat was put into continual use the following two years as a test and validation platform for Tophat bolometers.

### 3.3 Tophat

The advantages of performing CMB observations from a balloon-borne platform are numerous: atmospheric contamination is (nearly) eliminated, the platform is extremely flexible in terms of the instrument sizes and configurations that are technically feasible, and the vehicle cost is relatively low - facilitating rapid instrument development and the use of cutting edge detector technology. A major disadvantage for surveying instruments, however, is the viewing position. As illustrated previously in Fig. 3.3, a gondola suspended from the bottom of a balloon cannot observe above a 61° elevation, due to obstruction from the balloon. With Tophat, we have developed an instrument that observes from a platform mounted on top of a balloon, allowing an unobstructed view of the zenith, hence minimizing atmospheric loading and earthshine signal contamination.

\textsuperscript{33}Particularly since the sensitivity of channel 5 was markedly worse than the others in the 1997 flight.
3.3.1 A new observation platform

Tophat represents the current state of the art for a balloon-borne instrument. It combines the advances related to moving a science package to the top of a balloon with the latest in long-duration ballooning (LDB) technology. The National Scientific Ballooning Facility’s LDB program now offers vehicles capable of float times extending up to several weeks, and supports launches from Antarctica - Tophat exploits both of these capabilities to radically extend the surveying scan strategy pioneered with the FIRS instrument.

The logistics of launching a top-mounted package are somewhat more involved than those required for a conventional bottom-hung gondola, as might be expected. The top mounted science package must first be lifted into position by a tow balloon (Fig. 3.30). The primary balloon is then inflated, and the tow balloon is released. The primary balloon is then launched using a release vehicle in the usual way. Weather conditions (surface wind speeds in particular) must be ideal. In
addition, weight limits for top-mounted packages are stringent. The entire Tophat science package weighs 270 lbs; weight reduction to this level while maintaining cryogenic temperatures for the duration of an LDB flight necessitated development of an extremely efficient cryostat. Due to the stringent constraints on the total allowable mass at the top of the balloon, all components of the package that obtain no benefit from the balloon-top position (communications, data recording, package houskeeping, power, etc.) are housed in a much larger gondola that is suspended from the bottom of the balloon in the conventional way. Communications and power transfer between the packages is accomodated by integral wiring in the balloon.

The configuration of the Tophat "Spinner" telescope is shown in Fig. 3.31. The 1 meter, on-axis Cassegrain telescope is mounted on a rotating platform, with its optical axis fixed at a 12° zenith angle. The secondary is supported by taut Kevlar thread to avoid scattering and diffractive effects that can arise from reflective support structures. This arrangement is designed specifically for observing from an Antarctic latitude; the instrument spins at 1/16 Hz on its rotation stage at a latitude of $-78^\circ$, so each rotation sweeps the beam on the sky through a 24° diameter circle tangent to the south celestial pole (SCP). As the sky rotates, the center of the scan rotates. In this way, Tophat uses the rotation of the earth itself as a source of signal modulation. This scan strategy results in a highly repetitive, thoroughly interconnected survey of a 48° diameter portion of sky centered on the SCP, as shown in Fig. 3.32. Since each scan circle passes through the SCP, each point observed is referenced to the SCP in an interval that is at most 1/2 the rotation period, or 8 seconds. Pixels are observed many times, with the beam in many different orientations as the scan passes through each pixel, resulting in a multiply modulated, thoroughly interconnected dataset.

Each pixel is simultaneously observed by a five channel bolometric radiometer. A sixth "dark" channel is used to identify non-optical signals that may couple to the detectors and result in systematic effects. The entire radiometer/telescope assembly is surrounded by a reflective ground/sun shield that minimizes optical pick-up due to emission from the earth limb or the sun.

Tophat was launched on 4 January 2001, and collected approximately 100 hours of CMB survey data. The instrument remained aloft until 31 January 2001, setting a record for the longest duration zero-pressure balloon flight. The flight path is shown in Fig. 3.33. All telescope and detector systems performed as designed. An anomaly arose in the balloon vehicle itself; an altitude dependent tilt of the top platform persisted throughout the flight. This caused each circular scan to miss the SCP by an amount equal to the platform tilt relative to local $g$, resulting in an increase in the total amount of sky coverage, and a decrease in the total integration time per pixel. Interconnectedness at the SCP was therefore somewhat compromised as well.
Figure 3.31: The observation geometry of the Tophat telescope. The top-of-the-balloon vantage point and the inclined optical axis yield an unobstructed view of an annular region about the local zenith, providing an observation scheme unrivalled among suborbital platforms.
Figure 3.32: The Tophat scan strategy. The optical axis is inclined 12° from the local zenith and the instrument is rotated in the local azimuth plane. The instrument observes from 78°S latitude, so rotation of the telescope results in a scan tangent to the south celestial pole (SCP), as shown in the top left panel. Sky rotation precesses the scan about the SCP, but the scan circles remain tangent to the SCP, as shown in the top right panel (the path of the local zenith on the sky is shown in blue). The sky coverage after eight sidereal hours is shown in the lower left. At 24 sidereal hours, the telescope rotation combined with sky rotation combines to yield complete coverage of a 48° diameter cap centered at the SCP. The scan density has been thinned for clarity of illustration; the actual instrument completes 5400 rotations in one day.
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Figure 3.33: The flight path of Tophat 2001. Although the instrument gradually spiralled in towards the south pole, the CMB data was gathered at the beginning of the flight at roughly constant latitude.

Analysis of the Tophat 2001 flight data has been completed at the time of this writing [64]. The miniature Tophat “Indigo” dewar is documented in a recent paper by Fixsen et al. [63]. The science package, including the dewar, external optics and telescope rotation mechanism, has been documented in a series of papers and theses by Bezaire [65], Crawford [66], and Aguirre [67]. The subsequent sections of this work will describe the development of the bolometers for Tophat, the production of its band-defining filters, and the design and execution of an experiment to measure Tophat’s optical efficiency.

3.3.2 Tophat ”Indigo” bolometers

Monolithic silicon bolometers, as described in §3.2.4.2, were built for the Tophat radiometer as well. Tophat observes further into the far infrared (150 - 600 GHz) than MSAM2; at these frequencies single-mode techniques are infeasible. Instead, Tophat uses a “light pipe” optical feed, in which multiple spatial modes propagate through the optical system. Winston concentrators are then used to couple power from the optical feed to the bolometers. This approach requires use of quasi-optical filters to define the frequency bands; the inductive-capacitive mesh filters built for Tophat
for this purpose were constructed at Wisconsin and are described later in this work (§3.3.3). The bolometers, concentrators, and filters are all incorporated in the compact photometer shown in Fig. 3.34.

![Figure 3.34: Tophat five channel photometer mounted to the cold plate of the Tophat Indigo dewar. The broadband optical power incident at the optics block input passes through a series of quasi-optical dichroic filters that, through repeated duplexing, multiplex the signal into five channels. Winston cones then concentrate the optical power onto the bolometers.](image)

Since the Tophat bolometers (dubbed "Indigo", to match the Indigo dewar) are fed by multi-mode Winston cones, the physical shape of the absorber is fundamentally different from the MSAM2 waveguide-coupled, single mode absorber - the power density of the signal occupies a larger solid angle at the output of the cone relative to the waveguide, requiring a larger, radially symmetric absorbing element. The support legs must still meet the dual requirements of providing a high mechanical resonant frequency while hitting the thermal conductivity design target derived from the sensitivity requirements. It was anticipated that these dual requirements might necessitate
extra measures to reduce the thermal conductivity of the link beyond simply varying the cross-section to length ratio of the legs\textsuperscript{34}, so various leg geometries were explored (masks yielding right angle support legs, "zig-zag" legs, edge roughened legs, etc. \cite{68}) The final detector design used in flight, however, achieved adequate thermal isolation between the absorber and the frame by using a simple, straight leg geometry, as shown in Fig. 3.35.

As with MSAM2, the Indigo detectors are sputtered with a 50 nm thick film of bismuth that serves as a mmm-wave termination on the absorber. A final SiO passivation layer is deposited to prevent degradation of the bismuth coating when exposed to the atmosphere. Post-processing, detectors are immediately integrated into mounts designed to interface with the Winston cone feeds and provide $\lambda/4$ backshorts. Because of the broad spectral coverage of the Tophat optical system, two backshort depths were produced, with the "shallow" backshort serving for channels 3 and 4, and the "deep" backshort serving for channels 1, 2, and 5 (the deep backshort is actually $\sim \lambda/2$ for channel 5). A mounted Tophat bolometer is pictured in Fig. 3.36.

Further Indigo detector fabrication details will be provided in an upcoming paper \cite{69}.

3.3.2.1 DC characterization and heat capacity measurements

The Tophat bolometers were designed and built through an iterative process of construction at Goddard Space Flight Center and testing at the University of Wisconsin. Initial geometric design parameters were determined using bolometer modeling software developed within the group. Actual device parameters were measured on prototype devices at Wisconsin; this data was then used to refine the detector design. All Tophat bolometer tests were performed using the ADR in the MSAM2 blue dewar. The ease with which arbitrary temperatures in the range of 100 - 600 mK can be selected makes the MSAM2 ADR ideally suited for bolometer characterization.

The readout circuit shown in Fig. 3.37 was found to be convenient for purposes of DC characterization of the Tophat bolometers. By setting switches on the external preamp cards appropriately, different gain levels at the cold JFET can be selected, and the read-out gain relative to the bolometer voltage can be measured by feeding the bias voltage directly to the JFET gate. The various operation modes of the readout circuit are summarized in Table 3.8.

The device layout of the Indigo bolometer is shown in Fig. 3.38. The absorber contains two ion-implanted thermistors; incorporating two thermistors in the same absorber affords some flexibility in selecting device characteristics at expected operating temperatures, since the thermistors can be read out singly, in parallel, or in series. This makes optimal device parameters easier to achieve.

\textsuperscript{34}Due to waveguide-type "phonon mode" effects in the silicon at low temperatures.
Figure 3.35: Tophat multi-mode monolithic silicon bolometer.

Figure 3.36: Electron microscope image of a mounted Tophat bolometer. The 2.4 mm absorber disk is centered over a tuned backshort integral to the mount. The detector pictured has 5 µm wide legs; flight detectors had \( \sim 35 \) µm legs. Image by Rainer Fettig of GSFC.
Figure 3.37: A flexible circuit for DC characterization of bolometers. The load resistor (80 MΩ) is mounted to the bolometer box and held at the cold stage temperature (for Tophat bolometer characterization, 200-500 mK). The JFET is mounted to the L⁴He stage with a weak thermal link and regulated at 100K.

Table 3.8: Readout circuit switch positions for detector measurement modes. Reference schematic in Fig. 3.37.

<table>
<thead>
<tr>
<th></th>
<th>SW1</th>
<th>SW2</th>
<th>SW3</th>
<th>SW4</th>
<th>SW5</th>
<th>SW6</th>
</tr>
</thead>
<tbody>
<tr>
<td>cold FET as source follower</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>cold FET w/ gain ~ 20</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>readout DC gain measurement</td>
<td>1</td>
<td>0</td>
<td>0 (1)</td>
<td>1(0)</td>
<td>1(0)</td>
<td>0 (1)</td>
</tr>
</tbody>
</table>

Figure 3.38: Tophat Indigo bolometer geometry and pinout. Integral load resistors (R1-4) are incorporated in the bolometer frame.
in the face of relatively coarse control of intrinsic \((R_0, T_0)\) thermistor parameters. In addition, four thermistors are implanted in the bolometer frame. Since these thermistors are tightly coupled thermally to the bath, they are essentially fixed resistors, and combinations of them can be used as the load resistor (as shown in Fig. 3.15) for the device. This approach again affords flexibility in selecting values, while simplifying wiring and reducing volume.

In the course of performing the device characterization (as described in §3.2.4.3) on the first Indigo bolometer prototypes, it became apparent that the heat capacity of the detectors was substantially larger than predicted by the device models. We therefore developed a routine characterization process that yielded direct data on each detector’s heat capacity in addition to the other bolometer parameters of interest, and tracked heat capacity along with \(R_0\), \(T_0\), and \(G_0\) as the designs were iterated.

An additional benefit of implantation of thermistor pairs in the absorber is that it facilitates a particularly direct method of determining the heat capacity of the detector, since one of the thermistors can be used as a heater while the other is monitored with a steady bias current as in normal operation (e.g. in Fig. 3.38, T1 (T2) is monitored while current is fed to T2 (T1) to provide a known heat input to the absorber disk). This approach avoids some of the complications that can arise in the purely electrical characterization of detectors, since the energy deposition is coupled to the read-out thermistor purely by thermalization in the absorber disk. Nonideal effects (e.g. electrical-field effects) that may affect the heater thermistor’s resistance when relatively large pulses are applied are irrelevant to the measurement, since although the heater’s resistance may vary in a complex way throughout the duration of the pulse, the voltage across and current through the heater are monitored at all times.

We apply an electrical pulse \(v_p\) to the heater’s bias line and simultaneously monitor the heater voltage \(v_h\). Some small shunt capacitance\(^{35}\) across the detector was apparent in \(v_h\); this capacitance affects the total dissipated energy (particularly at low \(T\) where high heater resistance leads to long RC time constants) and must be accounted for. Including the contribution of the capacitance, the dissipated energy may be calculated from the monitored signals \(v_p, v_h\) by

\[
q = \int dt \ v_h \left( \frac{v_p - v_h}{R_L} - C_{FET} \frac{dv_h}{dt} \right). \tag{3.15}
\]

Additionally, some capacitive coupling between the heater bias line and that of the monitored thermistor was evident in the test cryostat, so a Gaussian pulse generator\(^{36}\) was synthesized to

\(^{35}\) \(\sim 50\) pF. Approximately 25 pF input capacitance is expected at the gate of the JFET used for read-out. The balance may be attributed to stray capacitance in the wiring.

\(^{36}\) The author acknowledges the assistance of Mark Supanich, who performed the Labview programming for the
minimize $dv_p/dt$ while keeping the pulse width short relative to the time constant of the detector. The signals $v_p$ and $v_h$ were sampled at 10 kHz by a National Instruments DAQPad to provide adequate resolution in determining $q$. The equivalent circuit for the heater, as well as examples of a bias line pulse and the measured heater voltage response to the pulse, are shown in Fig. 3.39.

The deposition of heat $q$ in the detector via the heater results in a signal $v(t)$ in the monitored thermistor. Using the $R(T)$ characteristics for that thermistor, the resulting voltage pulse height $\Delta v$ can be related to a temperature excursion $\Delta T$, thus directly yielding the heat capacity at disk temperature $T_d$, $C(T_d) = q/\Delta T$. Alternately, $v(t)$ can be fit to a detector model to obtain the same information.

Operationally, detectors were characterized by cooling a batch of three simultaneously to a bath temperature of interest $T_b$. Load curves were then taken at temperature $T_b$ by running OS9 scripts\(^{37}\) that commanded the bolometer bias circuit through a predetermined sequence of bias voltages. Bias was next set to a fixed value, and a heat pulse was injected into the detector as

\(^{37}\)Running on the "BSB", the flight DAQ system for MSAM2 developed by D. Cottingham at Goddard.
described above. $T_b$ was then incremented upward and the measurement process was repeated. In this way, thermistor parameters as well as heat capacity as a function of temperature are fully determined for each device.

3.3.2.2 Initial Tophat Indigo bolometer performance

As previously mentioned, tests on the initial Tophat detectors indicated heat capacities far in excess of the design estimate, based on the component budget for monolithic Si bolometers of Table 3.9.

The first sign of trouble was an obvious settling time visible upon changes of bias current magnitude. Subsequent investigation revealed time constants well in excess of one second (Fig. 3.40) at 300 mK, corresponding to heat capacities approaching a nJ/K, several hundred times the expected values (Fig. 3.41). Excess heat capacity was present in all the prototypes, although it was somewhat nonuniform, with values at a given temperature varying by up to a factor of two between devices. Fits to a power law $C(T) = aT^\beta$ yielded indices $0.8 < \beta < 1.2$.

Since the measured values were so large, considerable effort was dedicated to validating that the results were indeed intrinsic to the detectors and not some artifact of the measurement instrument or analysis. Some concern about adsorption of helium onto the detector was raised, but steps taken to evaporate any condensate on the detector resulted in no change in the measured $C(T)$, so this hypothesis was discarded. In addition, repeated heat capacity measurements at Chicago and GSFC in different dewars, using different measurement methods, resulted in consistent values of $C(T)$ [70].

38 After cooling to operating temperature, we ground the n-channel JFET source and apply a positive voltage to the bolometer return, resulting in $v_{gs} > 0$. In this mode, the JFET is essentially a diode and a large current flows through the bolometer, heating it to well above 4K.

Table 3.9: Estimated heat capacity budget for the Tophat Indigo bolometer.

<table>
<thead>
<tr>
<th>Component</th>
<th>$C_1$ (J K$^{-2}$)</th>
<th>$C_3$ (J K$^{-4}$)</th>
<th>$V$ (µm$^3$)</th>
<th>$\rho$ (g µm$^{-3}$)</th>
<th>$C$(270 mK) (pJ K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si disk</td>
<td>-</td>
<td>2.6 × 10$^{-7}$</td>
<td>2.3 × 10$^{4}$</td>
<td>2.33 × 10$^{-12}$</td>
<td>0.27</td>
</tr>
<tr>
<td>Thermistors</td>
<td>8.5 × 10$^{-18}$µm$^{-3}$</td>
<td>-</td>
<td>3.6 × 10$^{5}$</td>
<td>-</td>
<td>0.83</td>
</tr>
<tr>
<td>Leads</td>
<td>1.76 × 10$^{-17}$µm$^{-3}$</td>
<td>-</td>
<td>2.1 × 10$^{4}$</td>
<td>-</td>
<td>0.10</td>
</tr>
<tr>
<td>bismuth</td>
<td>3.2 × 10$^{-7}$ g$^{-1}$</td>
<td>5.66 × 10$^{-6}$g$^{-1}$</td>
<td>2.2 × 10$^{5}$</td>
<td>9.79 × 10$^{-12}$</td>
<td>0.42</td>
</tr>
<tr>
<td>SiO$^a$</td>
<td>2.0 × 10$^{-6}$(T/1K)$^{0.3}$g$^{-1}$</td>
<td>-</td>
<td>4.5 × 10$^{5}$</td>
<td>2.1 × 10$^{-12}$</td>
<td>0.34</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
</tr>
</tbody>
</table>

$^a$Heat capacity dependence assumed similar to silica, for which $C(T)$ scales like $T^{1.3}$ at low T [71].
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Figure 3.40: Excess time constants exhibited by the initial Indigo detector batch. The trace is proportional to the bolometer voltage. At 1260 records, an electrical pulse is coupled to the detector (1 record = 1.6s). At 1315 records, the bias polarity is flipped. At 1340 records, the detector is again pulsed. Note the several second time constant (operating conditions: $V_{\text{bias}} = \pm 12.5$ mV, $T_{\text{bath}} = 237$ mK.)

Figure 3.41: Heat capacity vs. absorber temperature for an initial Tophat Indigo prototype. $C(T)$ is fit to a power law, $C(T) = \alpha T^\beta$. For the device shown, $\beta = 0.86$. For all prototypes produced in the first run, $0.8 < \beta < 1.2$. The error bars shown are estimates obtained by forcing $\chi^2_{\nu} \rightarrow 1$. 
With the effect confirmed to be intrinsic to the detectors, we began searching for possible surface contamination. Scanning electron microscope (SEM) images of one sample from the initial prototype batch indicated a ubiquitous, unidentified filamentary substance (Fig. 3.42) coating the detector surface. However, another detector from the batch had somewhat less of the substance on the surface, but still had large heat capacity, so a simple correlation between the visual contaminant and the excess heat capacity was not clear. Thorough SEM inspection of detectors before testing was implemented, but excess heat capacity was observed even on relatively "clean" samples.

Auger electron spectroscopy (AES) on a wafer from the batch used to make the prototypes revealed nothing unexpected in the unprocessed wafers, so attention turned to the processing itself. An Auger spectrograph obtained from the surface of a completed detector revealed several unexpected species, including Fe and Cr. The Fe was of particular interest, as a magnetic system was one contaminant suspected at the outset, since such a system would be capable of causing the large heat capacities observed. It was subsequently discovered that an anodized aluminum plate in the reactive ion etch (RIE) chamber in which the detectors were processed was replaced with a stainless steel plate, resulting in deposition of components of the steel on the detectors during the etch process. The steel plate was removed from the RIE chamber, and another batch of detectors was processed. The time constants of the detectors produced after identification of the contamination,
while substantially reduced, still exceeded design estimates by a factor of approximately 20 at 270 mK (Fig. 3.43).

We attempted to further isolate the source of the excess heat capacity by producing a set of three detectors with variations in their coating schedules: one coated with SiO and Bi as usual, one coated with SiO only, and one uncoated. Again, all devices exhibited excess heat capacity. Furthermore, the time constants measured showed no sensible correlation with the coating schedules (SiO + Bi faster than uncoated; SiO only slowest), suggesting that the source of excess capacity was unrelated to the coating steps, that the heat capacity contribution from the unknown source was large compared to that from any of the coatings, and that its variation from device to device was also large compared to the contribution from any of the coatings (Fig. 3.44).

At this point, to obtain an independent measurement of the time constants of the Tophat detectors, we prepared an experiment in which known energy deposition to the absorber disk was provided by a radioactive source. A sample of $^{210}$Po, a monoenergetic (5.3 MeV) $\alpha$ emitter, was situated in a mount in the MSAM2 bolometer box, separated from the bolometer absorber by a Cu aperture tuned to set the event rate to a time scale slow compared to the expected bolometer time constant (note that the rate should be slow enough that the detector will relax back to its equilibrium state for many time constants with high probability, if concurrent load curves are to be taken for device characterization.) The detector under test was then processed as usual, with the $\alpha$ source providing periodic probes of the detector’s dynamic response.

To obtain a satisfactory event rate, we calculate a stop aperture size based on the intrinsic event rate of the source into $2\pi$ steradians, $n$, and the geometry of the source relative to the detector. Let the source be a disk of radius $r_d$. The disk is covered by a stop of the same radius, of thickness $t$, with a hole of radius $r_h$ in the center. If the detector is close enough to the stop that it completely fills the $\alpha$-particle beam exiting the stop, the event rate from the stopped down source at the detector is approximated by

$$n' = \left( \frac{r_h^2}{r_d^2} \right) \left( \frac{r_h^2}{2t^2} \right) n. \quad (3.16)$$

The first term is just the aperture effect of the stop, while the second term approximates the ratio of the solid angle subtended by the aperture $r_h$ at a distance $t$ from the source, to the full hemisphere over which rate $n$ is quoted. Solving for $d = 2r_h$, the diameter of the aperture required to obtain rate $n'$ given intrinsic rate $n$, we find

$$d = \sqrt{32t^2r_d^2 \left( \frac{n'}{n} \right)}. \quad (3.17)$$

---

39Kindly provided by Professor Lynn Knutson, University of Wisconsin Experimental Nuclear Physics.
Figure 3.43: Heat capacity vs. absorber temperature, post-RIE fix Tophat Indigo prototype. $C(T)$ is fit to a power law, $C(T) = \alpha T^\beta$. For the device shown, $\beta = 0.78$. Heat capacity is reduced by a factor of approximately 20 relative to the initial prototypes (c.f. Fig. 3.41). Functional dependence on temperature is similar. For reference, the estimated $C$ at 270 mK is 0.002 nJ K$^{-1}$. The error bars shown are estimates obtained by forcing $\chi^2 \rightarrow 1$.

Figure 3.44: Time constants vs. absorber temperature, post-RIE fix, with varied coating schedules. No sensible trend in time constants vs. coating schedule is evident, suggesting that the coatings are not the dominant heat capacity contributor. $V_{\text{bias}} = \pm 12.5$ mV for all data shown.
Our source had a diameter of 5 mm and was estimated to have \( n \sim 1000 \text{ s}^{-1} \) into the hemisphere above the disk; the Cu stop thickness was 1/16". We selected an aperture \( d = 0.023" \) to yield a rate of \( 1/4 \text{ s}^{-1} \) (Fig. 3.45). Note that the diameter of the aperture is not small compared to the thickness of the stop so the effective solid angle is somewhat larger, and the expected rate may therefore be larger than the target rate.

![Figure 3.45: Geometry of the α-particle stop used in bolometer heat capacity measurements.](image)

The emitted α-particles are monoenergetic, but the energy deposited in the bolometer absorber depends on the stopping power of the bismuth and silicon. The Bethe-Bloch equation is used to calculate the fraction of the kinetic energy that the α-particle deposits in the absorber due to ionization losses as it passes through the disk [72]. Typically, one integrates the equation from the initial energy to 0 to determine the range of a particle in a material. Here, however, we integrate across a distance to determine the energy deposited as the particle completely traverses the material. Results for the bismuth and silicon in the Tophat bolometer absorber, as well as data sufficient for performing the full calculation, are provided in Table 3.10.

After calculating the stopping power of the materials in the absorber, we approximate the energy deposited by the α-particles as follows: Let the distance traversed by the α-particle in the substance be \( x \); define the mass length of the bulk substance as \( \xi = \rho x \). The stopping power of
the substance, $dE/d\xi$, is generally a function of the kinetic energy per unit mass of the energetic particle. However, if the loss is small, we can approximate the stopping power as a constant and estimate the energy deposited by the particle as

$$
\frac{1}{z^2} \frac{dE}{d\xi} \approx \kappa,
$$

(3.18)

\[
\downarrow
\]

$$
\Delta E \approx \kappa z^2 \rho \Delta x,
$$

(3.19)

where $z$ is the particle’s charge. The stopping power $dE/d\xi$ increases with decreasing kinetic energy, so this approximation places a lower limit on the energy deposited as the $\alpha$-particle transits the bolometer disk. Hence, heat capacities derived in this approximation are lower limit estimates as well.

The material width traversed by the $\alpha$-particle, and hence the deposited energy, can vary by $1/\cos(20^\circ)$, or 6% because of the finite thickness of the stop aperture. This uncertainty is not large enough to dominate the error budget in the heat capacity measurement. For future experiments we could work with lower event rates, hence a smaller aperture and a smaller uncertainty in the energy deposited. As the measurement is refined to higher precision, the constant stopping power assumption will have to be reexamined. A segment of the detector signal timesream with $\alpha$-particles transiting the bolometer disk is shown in Fig. 3.46.

The 0.11 pJ deposited per $\alpha$-particle by the $^{210}$Po source is nearly ideal for characterizing bolometers with heat capacity similar to our design estimate. With the excess capacity we observe, however, we obtain marginal signal to noise on the pulses. The detector we characterized in this way had heat capacity consistent with the other detectors in the batch, but the small temperature excursions resulted in poor measurement resolution, so the electrical pulse method was favored for subsequent measurements. For lower heat capacity devices the $^{210}$Po method would be preferred. Results are shown in Table 3.11.

Table 3.10: $\alpha$-particle stopping power of Tophat bolometer absorber. Stopping power is calculated for the energy to mass ratio of the incident particle (in MeV/amu) $\sim 1$. For $^{210}$Po, $E_\alpha = 5.3$ MeV.

<table>
<thead>
<tr>
<th>Absorber</th>
<th>Atomic Number</th>
<th>$\rho$ (g cm$^{-3}$)</th>
<th>$x$ (µm)</th>
<th>$\xi = \rho x$ (g cm$^{-2}$)</th>
<th>$(dE/d\xi)/z^2$ (MeV m$^2$ kg$^{-1}$)</th>
<th>$\Delta E$ (MeV)</th>
<th>$\Delta E$ (pJ)</th>
<th>$\Delta E/E_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>14</td>
<td>2.3</td>
<td>5.0</td>
<td>$1.1 \times 10^{-3}$</td>
<td>16</td>
<td>0.74</td>
<td>0.12</td>
<td>14 %</td>
</tr>
<tr>
<td>Bi</td>
<td>83</td>
<td>9.8</td>
<td>0.039</td>
<td>$3.8 \times 10^{-5}$</td>
<td>5.0</td>
<td>0.0076</td>
<td>0.0012</td>
<td>0.14 %</td>
</tr>
</tbody>
</table>
Figure 3.46: Time stream of biased Indigo detector with incident $\alpha$-particles. Device 8X is shown. Operating conditions are $T_{bath} = 234$ mK, $V_{bias} = -25$ mV. Major ticks on the $x$-axis are 2s.

Table 3.11: Detector heat capacity derived from $\alpha$-particle pulses (device 8X). Results are similar to those measured on other devices using the electrical pulse method (c.f. Fig. 3.43).

<table>
<thead>
<tr>
<th>T (mK)</th>
<th>C (pJ K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>343</td>
<td>34</td>
</tr>
<tr>
<td>290</td>
<td>31</td>
</tr>
</tbody>
</table>
Chapter 3: Instrumentation

With a persistent heat capacity excess and flight scheduling pressures looming, we reevaluated the thermal conductance of the Indigo detector legs. Scaling up the leg cross section to increase conductance $G$ of course reduces the detector time constant $\tau = C/G$, but at a cost of an increased contribution to the detector DC NEP, $NEP_T = \sqrt{4kT^2G}$.

The performance trade-off involved is best analyzed by considering the impact of the detector time constant on the NEP as a function of audio frequency. The detector time constant has no effect on DC NEP but determines the increase in NEP with frequency. For a given heat capacity, increasing $G$ increases the DC NEP but may reduce the NEP at some audio frequency of interest $f$ (see §B.2 on bolometer responsivity). Based on the Tophat beam size and modulation scheme, we expect an instrumental half power point due to the optics alone around $f = 5\text{Hz}$. Constraining the Tophat detectors to have NEP at 5 Hz no worse than $\sqrt{2}$ times the DC NEP, we arrived at a new detector design with $G_0$ scaled up a factor of 6 relative to the previous version. The tradeoff in NEP between the two designs is shown in Fig. 3.47. The resulting NEP, while somewhat higher than the initial design target, is entirely sufficient for achieving the Tophat science goals.

Time constant measurements performed on the new devices confirmed that our speed target had been reached (Fig. 3.48). Additionally, it was discovered that the heat capacity of the new devices

![Figure 3.47: Change in Indigo detector NEP with scaled-up leg geometry. Some low frequency sensitivity is traded for sensitivity at 5 Hz by increasing the thermal conductance $G_0$. All other detector physical parameters, as well as loading conditions, are fixed for the comparison. Optimized biasing and 80 MΩ load resistors are assumed in both cases. Simulation performed using D. Cottingham’s Boloweb.](image-url)
Figure 3.48: Time constants of Indigo detectors with rescaled $G_0$. Top panel shows detector-to-detector variation at fixed $T$; bottom panel shows variation for each detector with temperature.
was about 30\% of that measured on the previous devices, or roughly 3-4 times the initial design estimate of Table 3.9. Combined with the decrease in the heat capacity achieved in the previous production run, the presence of an uncontrolled but diminishing concentration of a contaminant in the processing might be inferred (Fig. 3.49).

Figure 3.49: Change in heat capacity of Indigo detectors over several processing runs. The reduction suggests an exponential decline in contamination over time, consistent with gradual purging of Fe contamination in the RIE chamber. The heat capacity estimated from the sum of the contributions of the individual bolometer constituents is indicated by the dotted line.

3.3.2.3 Detector performance

With all of the performance criteria satisfied, bolometers from the batch based on the increased $G_0$ design were identified as flight candidates. The detectors selected for the Tophat flight photometer, along with their measured parameters, are presented in Table 3.12 ($^{40}$). The temperature dependance of the detector heat capacities were fit to a physically motivated linear + cubic model. The range of bath temperatures at which the heat capacity was measured varied from device to device due to scheduling and hold time issues with the cryostat, so error bounds vary as well. Negative coefficients for the linear terms reflect a poor lever arm on that parameter due to limited measurement range. These coefficients should thus be thought of simply as useful for interpolating

$^{40}$The definition of parameter $\alpha_{270}$ of Table 3.12 differs slightly from the $\alpha$ introduced in Equation 3.4. Detector resistance vs. temperature is here fit to the model $R(T) = R_{270}e^{-2\alpha_{270}(\sqrt{270mK/T} - 1)}$ (i.e. fit is referenced near the expected operating temperature instead of using the $R_0,T_0$ parameterization of Equation 3.9), where $\alpha_{270}$ is dimensionless, and $\alpha_{270} = \sqrt{270mK/T} (d\ln R/d\ln T) = \sqrt{270mK/T} \alpha$. 
Table 3.12: Measured Tophat flight detector parameters. All $R_{270}$ values are for two thermistors in parallel. Resistance vs. temperature is fit to $R(T) = R_{270}e^{-2\alpha_{270}(\sqrt{270}mK/T - 1)}$, heat capacity vs $T$ is fit to $C_1T + C_3T^3$, thermal conductance vs. $T$ is fit to $G = G_0T^3$

<table>
<thead>
<tr>
<th>Channel</th>
<th>SN</th>
<th>Leg Width $(\mu m)$</th>
<th>$G_0$ (nW K$^{-4}$)</th>
<th>$C_1$ (pJ K$^{-2}$)</th>
<th>$C_3$ (pJ K$^{-4}$)</th>
<th>$R_{270}$ (MΩ)</th>
<th>$\alpha_{270}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21x</td>
<td>36.6</td>
<td>29.2 ± 1.2</td>
<td>-16 ± 6</td>
<td>730 ± 60</td>
<td>110±?</td>
<td>-6.55±?</td>
</tr>
<tr>
<td>2</td>
<td>14x</td>
<td>43.0</td>
<td>28.06 ± 0.19</td>
<td>15 ± 2</td>
<td>300 ± 20</td>
<td>99.6 ± 0.8</td>
<td>-6.602 ± 0.019</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>35.9</td>
<td>22.1 ± 0.18</td>
<td>15 ± 10</td>
<td>240 ± 140</td>
<td>128.2 ± 1.1</td>
<td>-6.60 ± 0.02</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>35.9</td>
<td>21.64 ± 0.14</td>
<td>9 ± 9</td>
<td>280 ± 100</td>
<td>184.5 ± 1.3</td>
<td>-6.80 ± 0.02</td>
</tr>
<tr>
<td>5</td>
<td>13x</td>
<td>40.7</td>
<td>18.8 ± 0.2</td>
<td>4 ± 6</td>
<td>280 ± 60</td>
<td>164 ± 2</td>
<td>-6.60 ± 0.03</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>36.2</td>
<td>18.1 ± 0.9</td>
<td>-11.8 ± 1.7</td>
<td>410 ± 20</td>
<td>129 ± 2</td>
<td>-6.36 ± 0.07</td>
</tr>
</tbody>
</table>

heat capacity within the range at which the detector was originally characterized. In all cases, the cubic term dominates the contribution to the $C(T)$ fit.

The detector resistances (and, of course, the resistances of the frame thermistors intended for use as load resistors as well) were somewhat higher than anticipated based on the doping schedules used for the device implantation. This will be discussed in detail in an upcoming paper [69]. High impedances were mitigated somewhat by running all detectors with the dual thermistors paralleled as described earlier, but values were still large. Out of concern for microphonic pickup with such high output impedances, as well as to keep the detectors out of a voltage biased state, we investigated an approach in which detectors were run at optimum bias (as determined by the noise model alone) if the output impedance $R_b||R_L$, at optimum bias, was less than 20 MΩ; otherwise we selected a bias large enough to limit $R_b||R_L$ to 20 MΩ.

Imposing a ceiling on the detector output impedance is equivalent to imposing a floor on the amount of total input power to the detector. The input optical power, by channel, for the Tophat photometer is summarized in Table 3.13. The optical loading is much higher in the high frequency channels, primarily due to emission from the ambient temperature optics. Hence, the amount of excess bias above optimal required to limit the detector to the impedance ceiling, if any, will be smaller in the high frequency channels. Indeed, as shown in Table 3.14, only channel 5 settles under 20 MΩ under simulated operating conditions at optimal bias; the biases of all other channels are determined solely by the maximum output impedance condition. The penalty in NET paid to limit output impedances is summarized in the bottom row. In addition, the bias voltages required to attain these impedances are quite high due to the high resistance of the frame thermistors.
Table 3.13: Optical loading budget for the Tophat 5 channel photometer. Dust model assumes a temperature of 20 K with a spectral index of 1.5, and optical depth $2 \times 10^{-5}$ at 20 cm$^{-1}$. Atmospheric loading is modeled at 35 km using the AT code described earlier in this work. Optics are assumed to be at 250 K, with an emissivity of 0.03 at 5 cm$^{-1}$. Optical efficiency is assumed to be 0.1 in all channels, with spectral coverage simulated by idealized models of the actual Tophat bandpasses and bandwidth 1.5 cm$^{-1}$. Results calculated with Boloweb.

<table>
<thead>
<tr>
<th>$\nu_{\text{center}}$</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMBR</td>
<td>241.08</td>
<td>219.18</td>
<td>65.03</td>
<td>38.19</td>
<td>3.27</td>
<td>fW</td>
</tr>
<tr>
<td>Dust</td>
<td>0.02</td>
<td>0.05</td>
<td>0.34</td>
<td>0.50</td>
<td>1.45</td>
<td>fW</td>
</tr>
<tr>
<td>Atmosphere</td>
<td>0.01</td>
<td>0.01</td>
<td>0.14</td>
<td>0.37</td>
<td>3.31</td>
<td>pW</td>
</tr>
<tr>
<td>Optics</td>
<td>3.58</td>
<td>7.27</td>
<td>33.04</td>
<td>45.69</td>
<td>120.55</td>
<td>pW</td>
</tr>
<tr>
<td>Total</td>
<td>3.83</td>
<td>7.50</td>
<td>33.24</td>
<td>46.11</td>
<td>123.87</td>
<td>pW</td>
</tr>
</tbody>
</table>

Table 3.14: Simulation of Indigo detector operating parameters under the loading conditions of Table 3.13, with output impedance limited to 20 M$\Omega$ and frame thermistors used as load resistors. For purposes of illustration thermistors in all channels, as well as load resistors, are assumed identical with fiducial parameters $R_{270}=155$ M$\Omega$, $\alpha_{270}=6.25$, $G_0=20$ nW K$^{-1}$, $C_1=20$ pJ K$^{-2}$, $C_3=320$ pJ K$^{-4}$. $T_{\text{bath}}$ is taken to be 240 mK. Results calculated with Boloweb.

<table>
<thead>
<tr>
<th>$V_{\text{bias}}$</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7061</td>
<td>0.6908</td>
<td>0.5721</td>
<td>0.5023</td>
<td>0.8317</td>
<td>V</td>
</tr>
<tr>
<td>Bolo temp</td>
<td>381.6</td>
<td>381.6</td>
<td>381.6</td>
<td>381.6</td>
<td>446.1</td>
</tr>
<tr>
<td>$G$</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
<td>1.8</td>
</tr>
<tr>
<td>$C$</td>
<td>40.7</td>
<td>40.7</td>
<td>40.7</td>
<td>40.7</td>
<td>55.2</td>
</tr>
<tr>
<td>$R_b$</td>
<td>21.3</td>
<td>21.3</td>
<td>21.3</td>
<td>21.3</td>
<td>9.7</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-5.26</td>
<td>-5.26</td>
<td>-5.26</td>
<td>-5.26</td>
<td>-4.86</td>
</tr>
<tr>
<td>$R_{\text{bias}}$</td>
<td>330.9</td>
<td>330.9</td>
<td>330.9</td>
<td>330.9</td>
<td>330.9</td>
</tr>
<tr>
<td>Output impedance</td>
<td>20.0</td>
<td>20.0</td>
<td>20.0</td>
<td>20.0</td>
<td>9.4</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>18.9</td>
<td>19.3</td>
<td>22.7</td>
<td>24.9</td>
<td>23.3</td>
</tr>
<tr>
<td>NET$_{RJ}$</td>
<td>128</td>
<td>79</td>
<td>40</td>
<td>37</td>
<td>33</td>
</tr>
<tr>
<td>NET$<em>{RJ}$ (at optimum $V</em>{\text{bias}}$)</td>
<td>81</td>
<td>58</td>
<td>39</td>
<td>37</td>
<td>33</td>
</tr>
</tbody>
</table>
Non-Ohmic Behavior  The high bias voltages required for this overbiasing scheme exposed a weakness in using heat sunk ion-implanted thermistors as integral load resistors for bolometers. For small electric fields, thermistor resistance is purely Ohmic and is well described by Equation 3.9. As fields increase however, thermistor resistance at fixed $T$ decreases. This behavior may be modeled by an exponential field-effect at low temperature $T$ that modifies the low $E$-field resistance by a factor (Mather [60], Rosenbaum et al. [73])

$$R(T, E) = R(T, 0) e^{-C e^a \sqrt{T_0}{2kT^3}}$$

(3.20)

where $R(T, 0)$ is given by the hopping-conduction model, $C$ is a constant of order unity, $e$ is the electron charge, $a$ characterizes the electron localization, and $k$ is the Boltzman constant.

Zhang et al. [74] note that an electric field effect is indistinguishable from a power density effect by scaling cuboidal thermistor dimensions (Fig. 3.50), and they present their results in terms of the power density $P/V$ in the device. For comparison with their results on thermistors similar to those in the Indigo bolometers, we note the power density in the Tophat bolometers, with a 15% margin above the highest bias stated in Table 3.14: The Indigo thermistor dimensions are $600 \mu m$ \times \text{600 $\mu m \times 0.75\ \mu m$, which with a 1 V bias into 300 M$\Omega$ yields a power density on order of 6 \times 10^3 Wm}^{-3}, assuming two thermistors in parallel. At 281 mK, these power densities caused a resistance decrease of $\sim$30% in their work, on thermistors with similar $T_0$ to the Indigo devices. Additionally, since the thermistor has a negative $\alpha$ and is provided with a voltage bias, a situation in which thermal runaway can occur exists if the heat sinking to the bath is finite\textsuperscript{41}. Indeed, an example of

\textsuperscript{41}Power dissipated in the thermistor under voltage biasing is $V_b^2/R$. Dissipated power increases the detector temperature by an amount $\Delta T = P/G$. Poor heat sinking implies a small $G$, resulting in a significant positive $\Delta T$. For thermistors with $\alpha < 0$, $R$ decreases, which thereby increases $P$, and thermal runaway occurs.
this occurred when load curves incorporating large biases were first attempted after integrating the Tophat detectors into the Indigo flight dewar, while using the frame thermistors as load resistors. Bias voltages around a volt drove enough current through the load resistor/detector circuit to drive the $^3\text{He}$ refrigerator well above its quiescent operating point, implying an large anomalous internal load due to Joule heating in the thermisters, i.e. a total collapse in thermistor resistivity. Some suggestion of this effect was found upon review of the $R(T)$ characterizations of the Indigo frame thermisters in the MSAM2 dewar, but since the conductance of the frame to the bath was unknown we were unable to unambiguously distinguish between an $E$-field effect and thermistor heating as the source\footnote{Estimates of the conductance between the frame and the bath, based on published conductivity values for the bonding agent used, suggest the resistance drop was primarily an $E$-field effect.} of the non-Ohmic behavior (Fig. 3.51). Simultaneous monitoring of the companion frame thermistor would allow this degeneracy to be broken in future measurements.

![Figure 3.51: Non-ohmic behavior in a Tophat Indigo frame thermistor. The ratio of bolometer voltage change $\Delta V_{\text{bol}}$ to bias voltage change $\Delta V_{\text{bias}}$ is plotted against $\Delta V_{\text{bias}}$ (the bias voltage change is from a bias voltage polarity flip, so $\Delta V_{\text{bias}} = 2|V_{\text{bias}}|$). The dotted line is a linear fit to the $\Delta V_{\text{bias}} < 0.4\text{V}$ data. For $\Delta V_{\text{bias}} < 0.4\text{V}$, the data are well described by a constant. At higher biases the thermistor exhibits non-Ohmic behavior.](image)

Due to these difficulties, we elected not to use the Indigo frame thermisters as load resistors, and opted instead to use fixed chip resistors similar to those used in MSAM2. For each detector, 60 M$\Omega$ load resistors\footnote{The Tophat collaboration thanks Dan McCammon for kindly loaning resistor chips from his inventory for this project.} were built using two $0.060'' \times 0.060'' \times 30$ M$\Omega$ NiCr on Si chip resistors, wirebonded in series and mounted in an MSI 586C case. Wirebonding and sealing of the cases was performed by Sunbelt Micro. Although space constraints in the Indigo dewar were tight, we were able to integrate these small packages quite easily by epoxying them with Stycast 2850FT to the
bottom of the feed horns for each channel. This configuration provided a secure mount and a short run to the bolometer gate leads, as shown in Fig. 3.52. To prevent voltage biasing \( (R_{\text{bolo}} >> R_L) \)

![Figure 3.52: Load resistor mounting location (bottom view of Indigo photometer).](image)

it was still necessary to provide biases in some channels in excess of optimum, so we continued to impose the 20 MΩ output impedance condition when determining bias voltages.

The expected voltage noise at 2 Hz for each of the bolometers in the Tophat flight photometer is presented in Table 3.15. Because of the delay in the schedule caused by the excess heat capacity problems, noise characterization of the Tophat flight detectors was done \textit{in situ} in the Indigo dewar during integration in Palestine, TX, and during flight preparations at McMurdo. Noise measurements tended to coincide with noise debugging of the top gondola package in general, so measurements are mostly a composite of the intrinsic detector noise combined with the microphonics
Table 3.15: Predicted 2 Hz voltage noise spectral density for Indigo flight detectors, based on the measured parameters of Table 3.12. A bath temperature of 270 mK is assumed, with biases optimized and no optical load.

<table>
<thead>
<tr>
<th></th>
<th>NEP (\times 10^{-18}) W Hz(^{-1/2})</th>
<th>S (\times 10^8) V W(^{-1})</th>
<th>(v_n) (nV Hz(^{-1/2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 (21x)</td>
<td>67</td>
<td>5.23</td>
<td>35</td>
</tr>
<tr>
<td>C2 (14x)</td>
<td>65</td>
<td>5.24</td>
<td>34</td>
</tr>
<tr>
<td>C3 (21)</td>
<td>59</td>
<td>6.28</td>
<td>37</td>
</tr>
<tr>
<td>C4 (19)</td>
<td>58</td>
<td>7.10</td>
<td>41</td>
</tr>
<tr>
<td>C5 (13x)</td>
<td>54</td>
<td>6.71</td>
<td>36</td>
</tr>
</tbody>
</table>

that were being tracked down during pre-flight preparations. The microphonics-free region of the channel 3 spectrum, obtained a week prior to flight and shown in Fig. 3.53, is consistent with the value predicted in Table 3.15.

1/f noise Han et al. [75] have characterized the 1/f noise in ion implanted thermistors such as those used in Tophat. They find that their data can be well described as resistance fluctuations modeled by the relation

\[
\frac{<\Delta R^2>}{R^2} = \frac{\alpha_H}{N f}
\]

(3.21)

where \(N\) is the number of carriers in the thermistor volume and \(\alpha_H\) parameterizes the dependence of the fluctuations on thermistor temperature and \(T_0\) \(^{44}\). They also find an empirical relation for \(\alpha_H\) that is found to be valid for a very wide range of \(T_0\) values.

The Indigo bolometers were characterized over a temperature range corresponding to the range of operating temperatures expected in flight (typically 220 - 500 mK), which provides a poor lever for extrapolating out to temperatures comparable to \(T_0\). Direct conversion of our \((R_{270}, \alpha_{270})\) parameterization typically yields values of \(T_0\) greater than 40 K and small (\(\sim 200\Omega\)) \(R_0\) values\(^{45}\). Since \(R_0\) is typically a weak function of doping density, we instead obtain an estimate of \(T_0\) for the Indigo bolometers by forcing \(R_0\) to a range \(1000\Omega < R_0 < 3000\Omega\), which is motivated by prior knowledge of the range of \(R_0\) possible given the doping schedule and geometry of the Indigo thermistors.

For detector 14x, which served as the channel 2 detector in the flight photometer, this \(R_0\) range confines \(T_0\) to the interval \(27.2\) K \(< T_0 < 34.4\) K, given the measured in-flight resistance of 22.2 M\(\Omega\)

\(^{44}\)\(T_0\) as defined in Equation 3.9.

\(^{45}\)This may indicate a departure from resistance based on the \(R_0 e^{\sqrt{T_0/T}}\) parameterization, which has been documented by Zhang [76] for values of \(T_0/T > 25\).
Figure 3.53: Noise spectral density of a Tophat Indigo detector *in situ*. Channel 3 is shown in a ground test shortly before the January 2001 launch. The dewar snout was covered for the measurement. Microphonics at harmonics of $1/16$ Hz are prevalent across the spectrum, but the intrinsic voltage noise at 2 Hz is consistent with Table 3.15 above.
Table 3.16: In-flight channel 2 parameters used as input for the $1/f$ noise model. Spectral density is calculated for two thermistors of volume $V_{therm}$ in parallel.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>0.343</td>
<td>K</td>
</tr>
<tr>
<td>$T_0$ (estimated)</td>
<td>27.2 - 34.4</td>
<td>K</td>
</tr>
<tr>
<td>$V_{therm}$</td>
<td>$2.7 \times 10^{-7}$</td>
<td>cm$^3$</td>
</tr>
<tr>
<td>$V_{bias}$</td>
<td>0.103</td>
<td>V</td>
</tr>
<tr>
<td>$R_{bias}$</td>
<td>60</td>
<td>M$\Omega$</td>
</tr>
<tr>
<td>$R_{bolo}$</td>
<td>22.2</td>
<td>M$\Omega$</td>
</tr>
</tbody>
</table>

at 343 mK. Based on these extremal values, the parameters for detector 14x shown in Table 3.12, and the parameters shown in Table 3.16, we calculate the expected voltage noise spectral density $v_n$ by combining the detector noise model with the empirical $1/f$ noise model. Carrier density $N$ is taken to be $3.5 \times 10^{18}$ cm$^{-3}$. The modeled locations of $f_{knee}$ (46) given the $T_0$ range above are shown in Fig. 3.54. The derived range of $0.4$ Hz $< f_{knee} < 0.6$ Hz may be compared with the 0.35-0.5 Hz observed for this channel in flight (Fig. 3.55). A definitive comparison with the Han et al. results, however, requires a more explicit determination of $T_0$ for the Indigo detectors by directly measuring resistances at higher $T$.

A complementary discussion of the noise performance of the Indigo bolometers after integration into the Tophat radiometer, as well as other details on the in-flight performance, can be found in Crawford [66] and Cottingham et al. [69].

### 3.3.3 Inductive-Capacitive Mesh Filters

Band defining filters for the Tophat photometer were constructed by patterning periodic metallic structures on a plastic substrate in a manner similar to that described in Page et al. [79]. Application of modern lithographic techniques to the construction of these filters results in a robust final product with desirable sub-mm wave transmission and rejection characteristics.

### 3.3.3.1 Filter description

The function of the filters may be heuristically understood by considering the periodic structures illustrated in Fig. 3.56. The conductive mesh in the top left of the figure, which we call an "inductive mesh" following Ulrich [78], has a high-pass transmission characteristic - a phenomenon

---

46The frequency at which the spectral density of the $1/f$ component equals the spectral density of the other noise processes, or equivalently the frequency at which the composite spectrum reaches $\sqrt{2}$ times the high frequency spectral density.
Figure 3.54: Simulation of channel 2 voltage noise spectral density $v_n$, including a $1/f$ component as characterized by Han et al., under in-flight loading and biasing conditions. The measured $1/f$ knee varied between 0.35 and 0.5 Hz in flight.

Figure 3.55: Measured in-flight voltage noise spectral density, with best fit sky and systematics model subtracted. The spectrum has been normalized to the mean in the interval (2 Hz < $f$ < 6 Hz).
that may be familiar in the context of metallic screen that is often used to isolate measurement equipment from environmental RF interference. The circuit equivalent of the inductive mesh to an inductor in shunt across a transmission line is clear.

The characteristics of the structure complementary to the inductive mesh (in the top right of Fig. 3.56) may be inferred from Babinet’s principle. Let the scattering parameter $\tau_L(\omega)$ represent the transmission of the inductive mesh. The transmission of its complement is related to $\tau_L$ by

$$\tau_C(\omega) + \tau_L(\omega) = 1, \quad (3.22)$$

indicating a low-pass transmission characteristic for $\tau_C$. In analogy to a capacitive shunt across a transmission line we term this structure a capacitive grid. Viewing the grid as a generic two-port network, we have the further constraint on the scattering parameters (for either structure)

$$|\tau(\omega)|^2 + |\Gamma(\omega)|^2 = 1, \quad (3.23)$$

where $\Gamma(\omega)$ is the reflection coefficient for the mesh, and conductive losses in the structure are neglected. Hence, in reflection the inductive (capacitive) mesh functions as a low-pass (high-pass) filter. This suggests application of these meshes as spatial duplexers (or dichroics in optical parlance) for incident signals.

If inductive and capacitive meshes (with the same unit cell size but differing fill ratios) are superposed in the manner illustrated in Fig. 3.57, a symmetric, cross-shaped aperture pattern emerges. This inductive-capacitive mesh (ICM) structure has a resonant transmission characteristic, as might be expected by extending the circuit analogy to that of an $LC$ circuit in shunt. Since the respective $L$ or $C$ values are clearly a function of the individual mesh geometries, varying the mesh sizes provides a method for tuning the resonance of the structure. This tunability makes these structures useful for constructing bandpass filters. A common parameterization of the ICM
mesh geometry, along with some example values used for the Tophat photometer, are illustrated in Fig. 3.58.

Modeling of ICM filter transmission by equivalent circuit analysis neglects diffractive effects. For a more precise initial design of the Tophat photometer we used a finite element analysis (FEA) tool (fss.i) written in Yorick by Alexei Goldin. The performance of filters constructed based on the simulation was then evaluated, and the final design was achieved by applying small scaling corrections to the initial design to compensate for second order effects not included in the model.

A full wave FEA simulation of an ICM filter using one of the commercially available packages (HFSS, eeSOF), including the boundary conditions at the surface of the light pipe feed, a varying angle of incidence on the filter plane, and the effects of the supporting dielectric material would be informative.

We construct a single pixel, multi-band photometer out of these discrete elements by successively duplexing and then filtering the incoming light using the arrangement shown in Fig. 3.59. The incoming radiation is incident at 45° on dichroic A, a capacitive mesh. A passes the low frequency component of the signal, which is then further duplexed by dichroic B. Each component split at B is then individually filtered before being fed into the Winston concentrator for its respective channel. The high-frequency signal reflected at A is likewise further duplexed and filtered to define
the higher frequency channels. Since there is some loss associated with each dichroic and filter, the critical CMB channels ($C_1, C_2$) are situated in the photometer such that their signal is processed by a minimum number of filters. The grid parameters for each filter are provided in Table 3.17.

3.3.3.2 ICM filter construction

ICM filters were constructed for Tophat in the Wisconsin Center for Applied Microelectronics (WCAM). We have developed a process for producing ICMs that results in high definition \( \mu m \) scale patterns on plastic substrates with very high yields.

We begin by tensioning the plastic to be used as the ICM substrate material, using the fixture illustrated in Fig. 3.60. In this scheme, a set of clamping rings is used to capture the plastic, which is then smoothed into a drumhead using a concentric tensioning ring (the tensioning ring is held in place by adjustable dogs on the underside of the clamp). Under tension, the plastic is cleaned and degreased by rinsing liberally with acetone, wiping with a dust-free cloth, then rinsing with methanol. Any residual dust is purged with compressed nitrogen. To avoid imperfections in the final pattern it is imperative that all surface dust be carefully removed at this stage.

Four 3” silicon wafers, which serve as substrates for the patterning process, are then positioned on a disk that fits concentric with the plastic tensioning ring and coated with a light film of 3M
Figure 3.59: Frequency band definition scheme in the Tophat photometer.

Table 3.17: Grid parameters for the Tophat photometer ICM filters of Fig. 3.59 (filter A is a capacitive mesh from MSAM1).

<table>
<thead>
<tr>
<th>Designation</th>
<th>Function</th>
<th>( w ) (( \mu m ))</th>
<th>( L ) (( \mu m ))</th>
<th>( g ) (( \mu m ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>dichroic</td>
<td>158</td>
<td>777</td>
<td>882</td>
</tr>
<tr>
<td>C</td>
<td>dichroic</td>
<td>71</td>
<td>348</td>
<td>395</td>
</tr>
<tr>
<td>D</td>
<td>dichroic</td>
<td>67</td>
<td>251</td>
<td>318</td>
</tr>
<tr>
<td>E</td>
<td>bandpass</td>
<td>124</td>
<td>916</td>
<td>992</td>
</tr>
<tr>
<td>F</td>
<td>bandpass</td>
<td>91</td>
<td>591</td>
<td>728</td>
</tr>
<tr>
<td>G</td>
<td>bandpass</td>
<td>63</td>
<td>304</td>
<td>500</td>
</tr>
<tr>
<td>H</td>
<td>bandpass</td>
<td>57</td>
<td>265</td>
<td>455</td>
</tr>
<tr>
<td>I</td>
<td>bandpass</td>
<td>41</td>
<td>173</td>
<td>332</td>
</tr>
</tbody>
</table>
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Figure 3.60: Tensioning fixture for mounting plastic on silicon wafers.

75 repositionable adhesive\(^\text{47}\). This glue serves as a blocking adhesive for the photolithographic and deposition processes, so it must be able to stand up to immersion in developer and water (Norland Blocking Adhesive 106 was also found suitable for this purpose; it offers slightly better adhesion, but is more difficult to deblock and requires UV illumination to cure). Uniform coverage on the wafer edges is crucial for maintaining adhesion throughout the processing. After allowing the adhesive to set for \(\sim 10\) s, the tensioned plastic is placed over the wafers on the disk. Adhesion of the wafers to the plastic is nearly instantaneous. The tension on the plastic is then relieved, and the individual disks are cut apart with a razor. Excess plastic around the wafers can be trimmed with scissors\(^\text{48}\). The repositionable adhesive is granular, but the plastic surface that results should be nearly optically flat. This can be roughly confirmed by a cursory viewing of reflections off the plastic surface. Wafers with bonded plastic are transferred to the class 1000 clean room at WCAM for a final stage cleaning, and then brought into the class 100 wet lab for lithographic processing\(^\text{49}\).

\(^{47}\)The adhesive used for manufacturing Post It\(^\text{©}\) notes.

\(^{48}\)We have mounted both \(2.5\ \mu\text{m}\) (10HDS, 10 gauge) and \(23\ \mu\text{m}\) (Type C, 92 gauge) bi-axially oriented Mylar\(^\text{©}\) in this way. The \(2.5\ \mu\text{m}\) film may be preferred in general since the thinner dielectric perturbs the actual filter performance less relative to the model, but we found the \(23\ \mu\text{m}\) film easier to work with and mount in the photometer; the frequency shift due to the dielectric was nulled by scaling the filter grid parameters in the design to compensate.

\(^{49}\)Pre-treating of the plastic surface with a plasma etch, which was done to promote metal/plastic adhesion for the capacitive mesh/polypropylene Tophat dewar window, was found to be unnecessary for metallizing Mylar.
We utilized a "lift-off" patterning process, in which metallization of the plastic occurs after all of the photolithographic steps have been performed. In addition to yielding higher resolution patterns than an etching based procedure, lift-off is substantially more convenient since all of the wet lab processing is done in a single block. Lift-off can be performed with either positive or negative photoresist, provided the photomask is designed accordingly; if a given mask provides a given pattern using e.g. positive photoresist and etching, creating the same pattern with lift-off can be accomplished by using the same mask with negative photoresist, or making a complementary mask for use with positive photoresist. Since we initially designed our filter masks for an etching process using positive photoresist, use of the same masks with a lift-off process required identification of a suitable negative photoresist, which is a much less common product. We found the negative resist offered by Futurrex (NR5-1000Y photoresist, RD5 developer, and RR1 stripper) to be easy to use and well suited to lift-off processing. Since it uses a water-based developer, it does not introduce any difficult compatibility issues with positive-resist oriented processing facilities.

For lift-off to be effective, the photoresist thickness must exceed the thickness of the metal deposited on the plastic. The minimum thickness of the deposition is determined by the skin depth $\delta_s$ of the deposited metal at the lowest frequency of interest for the photometer. We use 99.999% Al, for which $\delta_s = 2000\,\text{Å}$ at 160 GHz. Thicker coatings provide better attenuation of evanescent modes but tend to yield poorer quality (hence lossier) patterns. We chose an Al layer thickness of 3300 Å as a compromise between these two competing criteria. For this metal layer thickness, a 1 µm photoresist layer works well.

The lift-off process flow used is illustrated in cross-section in Fig. 3.61. Room temperature NR-5 negative photoresist is applied to the center of a Mylar loaded silicon wafer (1) and immediately spun at 3500 rpm for 40s, yielding a uniform 1 µm coating across the plastic surface (2). The photoresist coating must be striation-free; irregularities indicate an insufficient initial dose of photoresist. The wafer is then "soft-baked" in a 100°C oven for 60s to cure the photoresist and allowed to cool.

The photoresist is developed by placing the ICM photomask in a vacuum chuck in a Süss aligner, positioning the sample underneath, and illuminating with a UV light source (3). Futurrex quotes a 200 mJ cm$^{-2}$ exposure sensitivity at 366 nm, which suggests a ~16 s exposure given the 12.5 mW

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50 Given a pattern, changing any of the following flips the parity of the pattern: Mask transparency, photoresist type (+/-), lift-off/etching process. Hence swapping any two yields the initial pattern.
51 Produced by Adtek Photomask, Montreal, Quebec, www.adtekphotomask.com
52 Of Franklin, NJ, www.futurrex.com
53 A convenient thickness check can be performed by viewing a nearby incandescent bulb through the metal layer. At this thickness the filament of a 100 W bulb will be dimly visible through the deposition.
54 Due to poor thermal contact between the silicon and the plastic, a hot plate bake is not an acceptable substitution.
Figure 3.61: "Lift-off" patterning process flow for ICM filter fabrication using negative photoresist.
cm$^{-2}$ source in the aligner. To aid lift-off, we overexpose by 25% to encourage sidewall undercutting in the developing process. Post-exposure, the sample is oven-baked at 100°C for 120 s.

The undercut ICM pattern appears after a coarse develop in RD5 resist developer for 15s followed by a fine develop for 15 additional s (4). The developing process is stopped by rinsing in a deionized water bath, and the sample is dried with compressed dry nitrogen. At this point, wet lab processing is complete and the sample is ready for metallization. Typically, batches of 10-20 wafers were processed in parallel in the wet lab, since many wafers can be processed simultaneously in the subsequent steps.

After developing is complete, the sample batch is loaded into a rotation stage in a CVC RF sputterer loaded with a 6" 99.999% Al target. A large target radius is recommended to ensure uniform coating thickness across all of the samples in the batch. A rapid rotation rate (15 rpm) is used to minimize the amplitude of heat excursions in the plastic as they transit the target. With the shutter above the target closed, the RF power is ramped up to 300 W with a 3 mTorr Ar background (yielding an aluminum deposition rate of 110 Å min$^{-1}$) and held for 3 min to purge the target. After purging, the shutter is opened and the samples are exposed to the target for 30 min (5).

After metallization, the sample batch is placed in an ultrasonic acetone bath to strip the exposed photoresist and lift off the desired metallization. Five to ten minutes is typically sufficient for complete lift-off, but timing is not crucial. Bubbles in the acetone should quickly begin nucleating around features in the pattern after immersion; failure to do so may indicate an overly thick metal layer or a thin photoresist layer. A clean room environment is not required for the final lift off step. Although Futurrex recommends use of their RR1 photoresist stripper, we found acetone just as effective and easier to work with. A finished ICM pattern results (6).

If 3M 75 adhesive was used to bond the plastic to the silicon, the filters are deblocked from the wafers by lifting them gently and spraying acetone between the plastic and the wafer. Separation is immediate. Norland blocking adhesive requires soaking in a dish detergent solution at an elevated temperature for ~20 min. The deblocked filters can be cleaned by a succession of acetone and methanol rinses. The final product is ready for mounting (7). A completed, mounted filter is shown in Fig. 3.62.

### 3.3.3.3 ICM filter performance

Transmission and reflection sweeps from 6 to 30 cm$^{-1}$ (180 to 900 GHz) of the completed filter patterns were measured on a Fourier Transform Spectrometer (FTS) at the University of Chicago.
Figure 3.62: Photograph of a mounted ICM filter. The metallized, patterned Mylar is glued to an aluminum frame with an elliptical aperture; the frame serves as a carrier for positioning the filter in the optics block.

This FTS data was initially used to refine the mesh parameters relative to the simulation derived values to fine tune the filter frequencies, and finally used for selection of the best (lowest loss) patterns for final integration in the flight photometers. Typical peak transmission for an individual ICM was better than -1 dB in the CMB channels (Fig. 3.63). The resulting frequency resolution for the assembled Tophat photometer is shown in Fig. 3.64.

3.3.4 Optical Loading and Optical Efficiency

To simulate in-flight loading conditions, we constructed a calibrator for Tophat that interfaced with the dewar snout in a manner similar to that described for MSAM2 in §3.2.4.5.

Our experience with MSAM2 highlighted the importance of pre-flight characterization of radiometer performance under optical loading conditions as close to those in flight as possible. In addition to providing an environment in which spurious sources of loading on the detectors can be detected, variation of the temperature of an external load provides a natural method for measuring the optical efficiency of the optics chain internal to the cryostat. Since the Indigo dewar is designed for optimal efficiency, given its small total mass and extended hold time requirement, the external cold load designed for Tophat provided the tertiary benefit of presenting a variable optical load to
Figure 3.63: Transmission of individual ICM filters vs. frequency. Filter designations correspond to those shown in Fig. 3.59.

Figure 3.64: Normalized transmission vs. frequency of assembled Tophat photometer.
the cooled optics internal to the cryostat. This allowed testing of the efficacy of the IR blocking window (and the intricate internal IR absorbing structures) used to minimize loading on the L$_4^4$He pot and the $^3$He refrigerator [63].

3.3.4.1 External calibrator design and construction

To succeed in its basic function as a calibration source a cold load must be black over the radiometer frequencies of interest, be isothermal across its absorbing surfaces, and be temperature controllable. To allow ourselves the option of fast modulation of the temperature of the load (to provide an AC optical input to the radiometer), we set the additional goal of minimizing the heat capacity of the absorbing structure.

The physical design of the Indigo cold load is shown in Fig. 3.65. It is constructed of a central cone with vertex angle 18.5°, surrounded by two thin-walled truncated conic surfaces ("fins") with the same vertex angle but progressively larger base radii. The internal surfaces of the fins are chamfered at the bottom to maximize the interaction of an entrant beam with the walls as determined by a simple ray trace analysis. Steps are taken in the machining process to ensure the edges of the fins and the tip of the cone are sharp to minimize diffractive effects. The central cone is hollowed and the fins are as thin as mechanically feasible to minimize heat capacity. This three piece assembly bolts with 4/40 screws and Belleville washers to a common base plate, which then interfaces with the cold plate of an IR labs L$_4^4$He dewar via cylindrical standoffs (Fig. 3.66). All components are copper, and all mechanical interfaces are polished prior to assembly to provide optimum internal thermal conductivity.

The external surface of the central cone, both surfaces of the inner fin, and the internal surface of the outer fin are coated prior to assembly with ~ 1 mm of Eccosorb CR-114, a mm-wave absorbing,
iron-loaded epoxy. The epoxy is mixed in a ratio of 100:1.5 by mass with Cab-O-Sil, an extremely fine silica (SiO$_2$) powder that serves as a thixotropic agent for the uncured resin. The silica has no effect on the mm-wave properties of the coating, but the increase in viscosity it provides makes uniform coating of complex surfaces much easier to achieve. Adhesion of the epoxy to the bare copper is promoted by a preliminary surface roughening with 60x sandpaper and a thorough degreasing with acetone and methanol (Fig. 3.67, top left). A silicon diode temperature sensor (SDTS) was embedded in the epoxy on the outer fin near the tip, to compare with thermometry in the base for detecting thermal gradients within the calibrator. Three coats of epoxy were required to build up the desired thickness. The external surface of the outer fin was covered with a layer of aluminum tape and ten layers of MLI to minimize optical loading on the calibrator not originating at the Indigo dewar itself (Fig. 3.67, top right).

The coating thickness represents a compromise between achieving an isothermal calibrator, which tends the design towards a thin coat on the copper substrate, and a mm-wave black calibrator which tends the design towards a thicker coating. Because the thermal conductivity of the copper substrate is much higher than that of the Eccosorb, if the temperature of the base plate is modulated the dominant gradient in the structure is normal to the surface. The Eccosorb has an inverse

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55 Emerson and Cumming Microwave Products, Randolph, MA. http://www.eccosorb.com/
57 Due to its extremely low density, the epoxy/Cab-O-Sil mix ratio is approximately 1:1 by volume.
Figure 3.67: Cold load construction sequence. Top left: the central copper cone and surrounding copper rings are cleaned and roughened with coarse (60x) sandpaper prior to coating. Top Right: The individual components are coated with Stycast CR-114 and mounted to the copper base. The exterior surface is uniformly covered with aluminum tape and 10 sheets of MLI. Diode temperature sensors are embedded in the top of the outermost fin and the base. Bottom left: 12 75Ω metal film resistors are attached to the base with Stycast 2850. The resistors are evenly distributed to provide uniform heating. Bottom right: The completed cold load mounted on the 4K plate of the dewar.
diffusion constant $1/\kappa = \rho C_p/k = 0.2 \text{ s mm}^{-2}$ at around 4 K [11], but the thermal impedance between the copper and the epoxy coating is not known. With an eye towards modulating the temperature of the load at around 0.5 Hz, we chose 1 mm as a conservative coating thickness to start with based on thermal properties.

CR-114 provides an attenuation of $>1.3 \text{ Np mm}^{-1}$ in the range $250 < f < 670 \text{ GHz}$ [80]. Assuming a uniform 1 mm coating of all of the optically relevant surfaces, a naive ray trace analysis in which a skew ray enters the calibrator at the least favorable angle (that which minimizes the number of specular reflections it experiences before exiting; five in this specific case) indicates an attenuation of greater than 50 dB over this frequency range. A ray incident parallel to the calibrator’s symmetry axis reflects 15 times off Eccosorb coated surfaces and hence experiences far greater return loss. To further ensure the blackness of the load presented to the Indigo dewar, the radius of the outermost fin matches the radius of the Indigo dewar window, and the assembly comes within 1/16” of it, completely filling the beam.

Once the coated calibrator parts are mounted to the base plate, the joints between the conic sections are filled with CR-114 using a syringe. Cab-O-Sil is omitted for this batch of CR-114, since wicking into the joints is desired. This is encouraged by placing the assembly in a Bell jar and pumping on the uncured Eccosorb; the resultant foaming as the epoxy is degassed ensures a thorough coating of the voids around the joints.

A means to modulate the temperature of the load is provided by the parallel combination of twelve 75 $\Omega$ resistors glued to the bottom of the base plate with Stycast 2850 epoxy (Fig. 3.67, bottom left.) The resistors are evenly distributed azimuthally around the base plate to ensure uniform heating. A silicon diode is mounted to the bottom of the base as well (not shown). The entire assembly is mounted to the cold plate (Fig. 3.67, bottom right) using between three and eight ETP copper or 6061-T6 aluminum standoffs. The number and composition of the standoffs are variable so that the conductance between the load and the bath can be adjusted. Given the vagaries of calculating thermal conductivities in the face of varying material purities and imprecise joint conductance estimates, some degree of tunability in the aggregate thermal link conductance is desirable.

To test the effects of the various Indigo dewar windows that were under investigation, and to eliminate any transmission or absorption uncertainties that a plastic dewar window on the calibrator would introduce, the cold load was designed to interface directly with the Indigo dewar.

\footnote{Note that a reflected ray traverses a minimum of \textit{twice} the coating coating thickness per reflection off the copper substrate.}
via a hermetic seal at the snout\(^59\). This allowed the calibrator cryostat to share a vacuum space with the Indigo dewar if so desired - completely removing the effects of the Indigo dewar vacuum window from the results.

### 3.3.4.2 Calibrator thermal performance

To provide an AC measurement of optical sensitivity, the calibrator must be capable of being modulated in temperature at a signal frequency for the Tophat detector electronics, which have a high-pass component with a knee at approximately 0.1 Hz. We set a maximum time constant constraint for the calibrator at \(\sim 1\) s \(\left(f_{knee} = 1/2\pi \text{ Hz}\right)\); a comparison of the transfer function of the calibrator with the Tophat readout electronics transfer function given a 1 s time constant is shown in Fig. 3.68. The maximum radiometer output signal for a given \(\Delta T\) at the calibrator occurs in this case when the calibrator is modulated at approximately 0.2 Hz.

The heat capacity of the calibrator is fixed by the design, but the time constant can be adjusted by varying the thermal conductance to the bath as described above. In a preliminary cooldown with three aluminum standoffs (the weakest thermal link out of the possible configurations), a 20 s time constant was measured. The conductance from the calibrator to the bath was 40 mW K\(^{-1}\) in this case. The 8K temperature at which the load equilibrated is consistent with 150 mW of

\(^{59}\)The calibrator dewar bottom plate accepts the Indigo dewar snout in a recess with an O-ring groove. The snout is then held in place by dogs in the calibrator bottom plate.
optical loading from the 1.34” diameter calibrator aperture, which was capped-off with a 300 K aluminum disk. The calibrator was remounted using five copper standoffs and recooled. In this configuration the measured conductance was 140 mW K$^{-1}$; the tighter coupling to the bath yielded an equilibration temperature of 6.3 K on the fin and 4.3 K on the base. This configuration yielded the desired $\sim$1 s time constant (Fig. 3.69).

![Figure 3.69: Impulse response of Tophat Indigo dewar calibrator (fastest configuration).](image)

The temperature signal on the calibrator fin with an $f = 0.14$ Hz, $A = 2.35$ V square-wave driving signal is shown in Fig. 3.70. The calibrator temperature is modulated by driving the base plate resistors with an HP3614A power supply operated as a voltage-controlled voltage source, with the input control voltage provided a function generator. The HP functions well in this mode for frequencies up to 1 Hz. The thermal low pass of the calibrator strongly attenuates the higher harmonics of the driving signal $v_s$, but there is still sufficient power in modes above the fundamental for use in the optical efficiency analysis$^{60}$. Here, with $0 < v_s < 2.35$ V, we achieve a 1.2 K modulation amplitude with a minimum temperature of 4.6 K. The ratio of the AC to DC components of the optical loading from the calibrator can be easily adjusted by varying the DC offset and amplitude of the driving signal provided by the function generator. The power spectrum of the optical signal relative to the electronics transfer function is shown in Fig. 3.71.

$^{60}$In one analysis method we used, optical efficiency is calculated by comparing the amplitude of the individual harmonics of the calibrator signal to the detector signal.
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Figure 3.70: Temperature response of the Tophat Indigo dewar calibrator fin to a square wave driving signal.

Figure 3.71: Power spectrum of Indigo calibrator optical signal (with a 0.14 Hz square wave driving signal) relative to the Tophat electronics transfer function.
3.3.4.3 Indigo dewar cold load tests and data analysis

The Indigo dewar is cycled and cooled in preparation for cold load tests, and then placed on its side with the snout vertical and allowed to equilibrate. When the boiloff rate from the $^4\text{He}$ tank has stabilized, the calibrator dewar is interfaced with the Indigo snout and evacuated. The pump-down is gradual to avoid rapid changes in mechanical loading on the Indigo dewar vacuum window. The $\text{LN}_2$ tank of the calibrator dewar is filled, but the mechanical constraints of the docked dewars (Fig. 3.72) preclude precooling the $^4\text{He}$ tank with $\text{LN}_2$, then dumping as is conventional.

As the calibrator $^4\text{He}$ tank is cooled, the loading on the Indigo radiometer input horn decreases as illustrated in Fig. 3.73. We obtain an estimate of the emissivity of the horn and the conductivity from the horn to the bath by relating the change in horn equilibrium temperature to the change in loading on the bath as measured by the change in cryogen boil-off (Table 3.18).

We obtain measurements of the optical efficiency of the radiometer by varying the temperature of the load and measuring the change in measured power at the radiometer output. As a consistency
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Figure 3.73: The temperature of the Indigo dewar input horn as the external calibrator is cooled. The top panel shows the uncalibrated voltage signal from the thermometers on the cold load (base: dotted line, fin: solid line). The bottom panel shows calibrated signals from thermometers on the Indigo dewar input horn.

Table 3.18: Thermal conductivity to the bath, and effective emissivity with a capacitive mesh dewar vacuum window, for the Indigo dewar input horn as inferred from the temperature change of the horn as the cold load is cooled to ~6 K.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial horn temperature</td>
<td>4.295 K</td>
</tr>
<tr>
<td>Final horn temperature</td>
<td>4.230 K</td>
</tr>
<tr>
<td>Initial L$^4$He boil-off rate</td>
<td>22 l hr$^{-1}$</td>
</tr>
<tr>
<td>Final L$^4$He boil-off rate</td>
<td>19 l hr$^{-1}$</td>
</tr>
<tr>
<td>Change in thermal load on bath</td>
<td>3.1 mW</td>
</tr>
<tr>
<td>Conductivity, horn to bath</td>
<td>48 mW K$^{-1}$</td>
</tr>
<tr>
<td>Change in optical load into snout</td>
<td>424 mW</td>
</tr>
<tr>
<td>Inferred average emissivity of horn</td>
<td>0.007</td>
</tr>
</tbody>
</table>
check, we perform both DC and AC measurements.

**DC analysis** We set the DC power dissipated in the cold load to a fixed value and run load curves on each bolometer channel. Using boloweb, we solve for the optical loading component $P_d$ for each channel. We then change the temperature of the load and repeat the process.

The input power to the radiometer from the calibrator at a given temperature is calculated from first principles by

$$ P_i = \int d\nu A\Omega \tau_n(\nu) B(\nu, T), \tag{3.24} $$

where $A\Omega$ is the system étendue, $\tau_n(\nu)$ is the normalized frequency response of the filter for channel $n$ as given in Fig. 3.63, and $B(\nu, T)$ is the Planck function. The calibrator brightness $B(\nu, T)$ at various temperatures is shown relative to the Tophat photometer spectral coverage in Fig. 3.74.

The dependence of the detected optical power $P_d$ on the input optical power $P_i$ is fit to a linear

![Figure 3.74: Brightness of Tophat calibrator at selected temperatures relative to Tophat's spectral resolution.](image)
model,
\[ P_d = \xi + \eta P_i \]  \hspace{1cm} (3.25)

where the gain term \( \eta \) represents the optical efficiency of the channel and the offset term \( \xi \) represents spurious loading. For the initial tests on the Tophat flight photometer\(^{61}\), data were taken with the calibrator at temperatures [5K, 10K, 17.5K, 77K]. The data by channel were all well fit by straight lines with \( 0 < \xi < 35 \) pW, and \( \eta \) as given in Table 3.19.

**AC analysis** We modulate the temperature of the cold load at 0.14 Hz as described in §3.3.4.2. The change in optical power detected \( \delta p \) is then related to the change in input power, as measured by thermometry on the cold load, to obtain the optical efficiency. We choose a large electrical bias, so that the electrical power dissipated in the detector is large compared to the optical power input. This removes the dependence of the detector’s responsivity on the optical loading (and hence optical efficiency), at the cost of some sensitivity.

Differentiating Equation 3.24, the change in power at the detector in channel \( n \) from a change in temperature \( \delta T \) at the cold load is given by
\[
\delta p = \eta \delta T \int d\nu A\Omega_n(\nu) \frac{\partial B(\nu,T)}{\partial T}. \hspace{1cm} (3.26)
\]

In the Rayleigh-Jeans portion of the spectrum, \( \partial B/\partial T \) is independent of \( T \), but for the higher frequency channels (4 and 5) thermodynamic corrections become important as illustrated in Fig. 3.75. For a mean calibrator \( T \) of 8.8 K with a peak-to-peak amplitude of 630 mK, however, simply evaluating \( \partial B/\partial T \) at the mean load temperature results in at most a 5% error in estimated efficiency, so we adopt this approach.

A fluctuation in optical power detected at the bolometer for a given channel is related to a change in the voltage measured at the ADCs by
\[
\delta p = \frac{\delta v_n}{\beta(\omega)S_n(\omega)}. \hspace{1cm} (3.27)
\]

\(^{61}\)In Palestine, TX, 5 August 2000.

Table 3.19: Initial Tophat radiometer optical efficiencies as determined by DC optical loading measurements.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.4%</td>
<td>5.0%</td>
<td>3.9%</td>
<td>1.6%</td>
<td>0.8%</td>
</tr>
</tbody>
</table>
where $\beta(\omega)$ is the magnitude of the readout electronics transfer function and $S_n(\omega)$ is the responsivity of the detector in channel $n$ under the bias and loading conditions of the measurement. Equating Equations 3.26 and 3.27, we solve for $\eta$

$$\eta = \frac{\delta v_n/\beta(\omega)S_n(\omega)}{\delta T \int d\nu A!\tau_n(\nu)\partial B(\nu, T)/\partial T};$$

(3.28)

here, $\eta$ is expressed entirely in terms of the measured quantities $v_n, T$.

We evaluate Equation 3.28 in practice by using Equation 3.26 (3.27) to scale the calibrator (detector) time-ordered data in Watts (Fig. 3.76). We then phase-align a demodulation vector to the time-ordered data and lock-in to each in software. The ratio expressed in Equation 3.28 is then calculated for the demodulated quantities. The initial optical efficiencies for the Tophat flight photometer measured in this way are provided in Table 3.20.

The optical efficiencies measured by the AC and DC methods agreed to within $\sim 30\%$, and were much lower than anticipated. The values are somewhat rough, but a problem was clearly indicated.
Figure 3.76: Comparison of time-ordered calibrator (blue) and detector (black) signals for the Tophat flight photometer AC optical efficiency measurements. Signals are calibrated in Watts.
The detection of this poor performance highlights the utility of having a cryogenic, controllable temperature load external to as much of a radiometer’s optics chain as possible. Subsequent to the initial system level optical efficiency measurements, follow-on tests of individual components in the optics block at the University of Chicago uncovered several misalignments and poorly performing elements [66], the sum of which yielded the initially poor efficiencies. Modifications to the photometer yielded the much improved values in Table 3.21.

Table 3.20: Initial Tophat radiometer optical efficiencies as determined by AC optical loading measurements. Results are similar to the DC analysis presented in Table 3.19.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<td>3.6%</td>
<td>2.2%</td>
<td>0.7%</td>
<td>0.7%</td>
</tr>
</tbody>
</table>

Table 3.21: Final Tophat flight radiometer optical efficiencies as determined by AC optical loading measurements.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>12%</td>
<td>6%</td>
<td>3%</td>
<td>2%</td>
</tr>
</tbody>
</table>
Chapter 4

MSAM2 Observations

For the remainder of this work we focus on the MSAM2 1997 CMB observations and the analysis of the data from that flight. Tophat flight operations and data analysis are discussed by Bezaire [65], Crawford [66], and Aguirre [67].

4.1 Pre-flight ground operations

The MSAM2 radiometer, primary mirror, and nutating secondary mirror, along with a new DAQ and commanding system, were integrated with the gondola previously used for MSAM1 at the NASA National Scientific Ballooning Facility (NSBF) in Palestine, TX in the spring of 1997 prior to the launch. This period of time was used to test integrated system function and ensure compatibility of the science package with the NSBF provided equipment (rigging, telemetry, etc.). Tests were also performed on individual flight subsystems during this time in NSBF environmental chambers to confirm operation in the low temperature, low pressure environment at 40 km [45]. Since many gondola components from the FIRS and MSAM1 experiments were utilized, MSAM2 had a strong platform of flight validated hardware to build upon.

4.2 The launch and the flight

Launch opportunities are defined by an intersection of science team criteria (science package readiness, ephemeris considerations, time of day) with NSBF weather criteria. A long term weather constraint on the flight window is due to the direction of the prevailing stratospheric winds, which must carry the package away from densely populated areas for safety reasons. For launches from Palestine, this means a flight westward, which is typically possible from mid-May to early September. Short term weather is of course variable and subject to day-to-day review. Surface winds at
launch must be very calm to keep launch dynamics manageable. Once the science team asserts flight readiness, daily weather briefings from the NSBF assess the likelihood of a launch opportunity. An intersection of all sets of criteria required for launch is not guaranteed; the MSAM2 1996 campaign never resulted in a flight.

The 1996 campaign did however facilitate bringing MSAM2 to flight readiness quickly in 1997. MSAM2 was launched relatively early in the season, at 01:24:25 UTC on 2 June 1997, under nearly ideal launch conditions. The sequence of events on the launch pad is shown in Fig. 4.1.

Figure 4.1: MSAM2 1997 launch sequence. *Top left:* Cryogens are topped off and telemetry commanding is checked on the launch pad. The launch vehicle ("Tiny Tim") suspends the science package by a pin in its release mechanism. *Top right:* As the sun sets, dewar servicing is completed and the ground shields are buttoned up. *Bottom left:* The payload is cleared for launch and the balloon is inflated. The balloon and the helium inflation hoses are visible in the background (the balloon is anchored to a spool vehicle until inflation is complete). The flightline extends back over the launch vehicle upwind, so that upon release of the spool the balloon (hence the package lift) will travel to a point directly above the package. *Bottom right:* The MSAM2 pin is released. Note the flightline is nearly normal to the ground.

The package ascended at a rate of 210-330 m min$^{-1}$, initially on a south-southeast heading
(directly towards Houston, TX), before abruptly turning to the west at an altitude of 20 km, 50 miles downrange and 1.5 hours into the flight. At 1.6 hours, the ADR, which was held at a magnet current of 1A for the launch sequence, was successfully ramped down to 100 mK. At 27 km altitude, 2 hours after launch, gondola attitude control was obtained, and execution of the flight plan commenced. The balloon reached a maximum altitude of 40 km 4 hours into the flight, then gradually descended to 38 km, while drifting on a predominantly westward course at a speed of 30 - 50 knots. The flight was terminated 10.5 hours after launch, 30 minutes after sunrise on the package. The complete flight trajectory is shown in Fig. 4.2.

In addition to a very broad ascent/descent profile, the balloon platform exhibits oscillations in altitude on approximately 5 minute time scales. These oscillations result in modulation of the ambient pressure as measured at the gondola. At a given altitude, small perturbations at the 100 mTorr level are also observed (Fig. 4.3). Since the atmosphere is potentially a significant contributor to the optical loading on the detectors, correlation between the ambient pressure and drift in the detector signals may be expected (as was the case for MSAM1).

Float time is not entirely devoted to CMB scans. Observations of the known position of celestial bodies with the on-board star camera are required to establish absolute pointing of the telescope, and planet observations are used to measure the mm-wave beam in flight, establish the position of the beam relative to the on-board star camera, and calibrate the radiometer. A summary of the MSAM2 1997 flight plan is provided in §4.4.

Real-time data is telemetered to the science team at the NSBF ground station via an on-board CIP (Consolidated Instrument Package) provided by NSBF. A complete data archive is also written to on-board disk drives which are recovered from the package after flight. Since MSAM2 is an actively pointed (as opposed to a survey) instrument, a relatively large amount of data must also be transmitted on the uplink to the telescope. While the telemetry on the downlink from the telescope is robust, the uplink is prone to dropouts. For this reason, instructions may be sent to the package in batches, which are then buffered and executed in the sequence in which they were sent [44]. This lessens the impact of the weak uplink on the relatively time critical flight plan.

4.3 Pointing and pointing reconstruction

The MSAM2 balloon gondola is described in detail by Fixsen et al. [43]. Here we briefly review the telescope pointing and attitude measurement subsystems to provide context for the subsequent analysis section on in-flight pointing reconstruction.
Figure 4.2: MSAM2 1997 flight path and 3D flight trajectory.
Figure 4.3: Gondola altitude and pressure vs. time. Balloon "porpoising", or altitude oscillations, occur with 300-500 m amplitude and a period of ~5 min. In addition to the variation of pressure with altitude (top panel, middle panel detail), atmospheric perturbations at constant altitude at the 100 mTorr level are also evident (bottom panel). The blocks of time devoted to each observation field (described in full in §4.4) are included for reference.
4.3.1 Beam pointing

The gondola provides an altitude-azimuth mount for the radiometer, as illustrated in Fig. 4.4. The elevation of the beam is controlled by twin servo motors that pivot the entire optical system and radiometer. Azimuth is controlled by a momentum wheel that spins on an axis coaxial with the balloon flightline; as the momentum wheel spins up, conservation of angular momentum rotates the package in the opposite sense. The package normally rotates freely, but angular momentum can be dumped to the balloon via a "jitter" mechanism at the payload/flightline interface point to prevent the momentum wheel from spinning up to arbitrarily large angular velocities.

The radiometer beam position is modulated relative to the instantaneous attitude of the telescope at 2.5 Hz by a nutating secondary mirror (the "chopper"). The resulting beam throw on the sky is ±70' at approximately constant elevation. The deflection of the chopper relative to its center position is monitored, but the resulting path of the beam on the sky is determined by fits to in-flight observations of Jupiter. A detailed discussion of the reconstruction of the position of the radiometer beam including the chop throw is deferred until §5.1.2.

4.3.2 Attitude sensors

On-board position sensors allow telescope attitude reconstruction in both local altitude-azimuth and absolute coordinate systems. Azimuth relative to north is determined by an on-board magnetometer, while elevation is determined by encoders in the elevation drive motors, and an inclinometer that measures the angle of the elevation truss relative to local $g$.

Gyroscopes on the elevation truss define a second "elevation/cross-elevation" (el, xl) coordinate system, with a rail-to-rail extent of approximately $10^\circ \times 10^\circ$. The gyro coordinate system is approximately inertial and hence fixed with respect to the sky. The origin of the (el, xl) system is reset before each particular observation begins, so the system is inherently differential. Pointing may be servoed off the local attitude sensors ("acquisition" mode) or off the gyros ("inertial" mode). An additional braking mode ignores absolute position, but seeks to zero the position signal derivatives with respect to time.

The relation between the inertial gyro system and equatorial ($\alpha, \delta$) coordinates is determined in flight by observations of celestial objects at known coordinates with an on-board star camera. In the final analysis, the star camera observations periodically fix the orientation of the telescope relative to the sky, and the gyro signals are used to interpolate the absolute pointing between the star camera observations. Planet observations, which yield a signal in both the optical camera and the radiometer, fix the position of the mm-wave beam in the star camera, and hence the position
Figure 4.4: MSAM2 1997 beam pointing and attitude measurement components. The elevation drive combined with the rotation of the gondola below the balloon (driven by the azimuth momentum wheel) form an alt-az mount for the radiometer. The nutating secondary mirror chops the beam $\pm 150^\circ$ at 2.5 Hz, at approximately constant cross-elevation, relative to the elevation/cross-elevation position of the telescope.
of the beam in right ascension and declination.

4.3.3 Pointing noise and pointing accuracy

A two minute segment of gyro data from a portion of the flight when the gondola was idling is shown in Fig. 4.5. The pointing noise in the radiometer signal bandwidth provides a means to estimate the amount of beam smearing due to pointing instabilities in the telescope. This noise is typically less than 12" Hz^{-1/2} above 0.5 Hz, and is completely negligible relative to the ~25' FWHM mm-wave beams.

Below 0.1 Hz, the pointing noise spectrum increases due to a combination of long time scale pendulations of the platform and intrinsic drifts in the gyros. The gyro drifts are monotonic, and are subtracted out by periodically re-referencing the pointing to the sky using the star camera observations. The camera itself provides 50" resolution. Drift offsets accumulated in the ~20 minutes between camera observations are typically <2".
Chapter 4: MSAM2 Observations

4.4 Observation fields

The goal of maximizing the amount of CMB observation time must be balanced with the need to adequately calibrate the radiometer, obtain a high-fidelity pointing solution, and perform systematics checks. The time budget for the MSAM2 1997 flight is shown in Table 4.1. The gondola boresight pointing for the full flight, in right ascension/declination vs. time and in right ascension vs. declination, is shown in Fig. 4.6.

4.4.1 Planet Observations

Scans and rasters of Jupiter occupied 80 minutes in the middle of the flight, and the data from these observations served as the primary calibration for the instrument. The 40° ⌀ disk is unresolved by the 25' mm-wave beams, and hence serves as a point source probe of the optical system. However, the planet is sufficiently bright that it provides a flux $T_J \Omega_J$ sufficient for a high signal to noise measurement, and indeed fits nearly ideally within the dynamic range of the receiver. A full raster of the telescope over the planet provides a probe of the full two-dimensional mm-wave beam profile, a precise measurement of the amplitude of the chopping secondary’s beam throw on the sky, a known flux input to the radiometer that determines the Volts/Kelvin scaling of the detector data, and a host of other second order detector and optical parameters. The telescope pointing in gyro coordinates for the Jupiter raster is illustrated in Fig. 4.7. The analysis of the Jupiter calibration data is treated in detail in § 5.1.

Although Jupiter’s disk size and temperature make it ideal for calibrating an instrument of MSAM2’s sensitivity and resolution, Mars is also useful as a secondary calibration target\(^1\). Mars

\(^1\)MSAM1, which observed Jupiter, Mars, and Saturn in its 1995 flight, provided the first measurement of the relative fluxes of these planets in the mm- and sub-mm bands [81].

<table>
<thead>
<tr>
<th>Event</th>
<th>Record(s)</th>
<th>Duration (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ascent to 27 km</td>
<td>653-5206</td>
<td>121</td>
</tr>
<tr>
<td>Mars scans</td>
<td>5577-6666</td>
<td>29</td>
</tr>
<tr>
<td>North CMB observations</td>
<td>7079-12331</td>
<td>140</td>
</tr>
<tr>
<td>Jupiter scans and rasters</td>
<td>12485-15294, 20610-20806</td>
<td>80</td>
</tr>
<tr>
<td>West CMB observations</td>
<td>15852-20432</td>
<td>122</td>
</tr>
<tr>
<td>Saturn scans</td>
<td>21215-21600</td>
<td>10</td>
</tr>
<tr>
<td>Moon raster</td>
<td>21930-22373</td>
<td>12</td>
</tr>
<tr>
<td>Star camera exposures</td>
<td>various</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 4.1: MSAM2 1997 flight chronology. One record corresponds to 1.6s.
Figure 4.6: MSAM2 gondola boresight pointing in equatorial coordinates $(\alpha, \delta)$ and equatorial coordinates $(\alpha, \delta)$ vs. time for CMB observations and planet calibrations.
was the first planet observed in the 1997 flight (Jupiter had not yet risen early in the flight); these initial observations provided the first indication of the loading problem in the MSAM2 radiometer. Saturn and the Moon were also observed as tertiary calibration sources.

The planet raster data also provides some insight into the dynamics of the balloon platform. Figure 4.8 compares the inclination of the telescope to the elevation gyro for the duration of the Jupiter raster\(^2\). The raster is a stepped sequence of constant elevation scans in inertial coordinates. Since Jupiter is rising as the raster is being performed, in local coordinates the raster would appear as a stepped sequence of elevation scans, with some elevation drift at each step due to the rising of the planet, if the attitude of the gondola was fixed. The data, however, shows some wander and several excursions to lower elevation during the raster. This indicates pendulation of the package on the flightline. Note, however, that the elevation gyro shows no corresponding perturbation - a testament to the tight servoing of the gondola attitude in inertial mode.

### 4.4.2 CMB observations

The MSAM2 1997 CMB observations comprised two distinct scan strategies. The first was oriented roughly due north and closely matched the constant declination scans of MSAM1. In this approach, the scan is centered on a point slightly east of north, and this center point is tracked in inertial coordinates until, due to sky rotation, the point reaches an azimuth symmetrically west of north from the east of north start point. Since the scans are at constant declination and are close to due north, the telescope stays at roughly constant elevation throughout the observation and minimizes

\(^2\)This is hence a comparison of the inertial and local coordinate systems.
Figure 4.8: Gondola pendulation. The top panel shows the elevation of the telescope in inertial coordinates vs. local coordinates during the Jupiter raster. The telescope is fixed in elevation in inertial coordinates while each cross-elevation scan in the raster is performed; if the gondola flightline remains parallel to local $\hat{g}$, the resulting change in the locally measured elevation will vary smoothly as the planet rises. Variations in elevation while the telescope attitude remains fixed with respect to the sky indicate perturbations of the flightline away from local $\hat{g}$. The bottom panel presents the same two signals vs. time.

The second strategy observes a horizontal patch of sky as it sets in the west. The telescope elevation throughout the measurement therefore necessarily varies more than it does for the north scans, but the resulting sky coverage is considerably more uniform; i.e. the sensitivity/pixel has less scatter. In addition, the westward sky available at the time of observation for this scan strategy had relatively low and uniform dust emission, minimizing the foreground contamination for the measurement.

The sky coverage of the MSAM2 north scans is shown superposed on a dust map of the observed
region in Fig. 4.9. The sky coverage for the west scans is shown superposed on a dust map of the observed region in Fig. 4.10.
Chapter 4: MSAM2 Observations

Figure 4.9: MSAM2 north scan patterns superposed on the Schlegel, Finkbeiner, and Davis 100 μm dust maps [82]. Image is scaled logarithmically in intensity (in MJy Sr⁻¹). Scaling is the same as that in Fig. 4.10. The relative sizes of the mm-wave beam and the chop throw are indicated by the icon in the lower right.

Figure 4.10: MSAM2 west scan patterns superposed on the Schlegel, Finkbeiner, and Davis 100 μm dust maps [82]. Image is scaled logarithmically in intensity (in MJy Sr⁻¹). Scaling is the same as that in Fig. 4.9.
Chapter 5

MSAM2 Data Analysis

Since the instantaneous signal to noise in a CMB data set is much less than one, the bulk of the effort devoted to the analysis of archived CMB flight data is typically spent in calibration, data cutting, and consistency cross checks to ensure that systematic effects do not swamp the weak underlying cosmological signal. In the specific case of MSAM2, both sensitivity and offset concerns prompted an extremely detailed look at the Jupiter raster data in order to adequately quantify the in-flight radiometer performance.

5.1 Jupiter Observations: Instrument Calibration and Optical Characterization

The raw raster data is comprised of the time-ordered voltage signals from the 5 radiometer channels, a "dark" control channel, time-ordered pointing data from the gyros, and the chopping secondary position signal. Since Jupiter is a bright, unresolved source whose position is precisely known, the mapping of the sky signal from the Jupiter observations to the time ordered data provide an ideal calibration of the instrument’s optical and pointing parameters\(^1\). We adopt the approach of modeling the time-ordered data directly, rather than spatially binning the radiometer signals and fitting to this secondary quantity.

5.1.1 Calibration Radiometric Preliminaries

At the time of observation, 02 June 1997 at 7.75 UT, Jupiter was a 42.4" diameter, 172 K source at \(\alpha = 21^h37^m50^s, \delta = -14^\circ54'42''.\) Integrating the brightness temperature \(T_J\) of the planet over

\(^1\)Free parameters in the model used for the calibration are indicated in the following discussions by braces \{\}.
the solid angle it subtends yields its flux density [88]

\[ F = \int d\Omega \, T_J(\theta, \phi). \] (5.1)

Taking the brightness to be uniform across the planet’s disk, this integral is

\[ F = T_J \Omega_J, \] (5.2)

\[ = 67 \text{ K arcmin}^2 \text{ or } 5.7 \mu \text{K Sr} \] (5.3)

for all channels\(^2\), where \( \Omega_J \) is the solid angle subtended by the disk of Jupiter at the time of observation\(^3\). The flux density measured by an instrument with beam pattern \( P \), normalized such that \( P(0) = 1 \), with a source centered at \((\theta, \phi)\) and beam offset from the source by an angle \((\theta_0, \phi_0)\) is

\[ \tilde{F}(\theta_0, \phi_0) = \int d\Omega \, T_J(\theta, \phi) \, P(\theta_0 - \theta, \phi_0 - \phi). \] (5.4)

This may be recognized as the convolution of the beam pattern with the source brightness distribution. Since Jupiter is unresolved by the beam, this may be rewritten as

\[ \tilde{F}(\theta_0, \phi_0) = T_J \int d\Omega \, \Omega_J \delta(\theta, \phi) \, P(\theta_0 - \theta, \phi_0 - \phi) \] (5.5)

\[ = T_J \Omega_J P(\theta_0, \phi_0). \] (5.6)

In this case, if the offset from the source is 0, the measured flux \( \tilde{F} \) is equal to the actual source flux \( F = T_J \Omega_J \).

The observed brightness is obtained by dividing the observed flux density (Equation 5.4) by the solid angle subtended by the beam,

\[ T_A = \frac{\int d\Omega \, T_J(\theta, \phi) \, P(\theta_0 - \theta, \phi_0 - \phi)}{\int d\Omega \, P(\theta, \phi)}, \] (5.7)

which, in the case of an unresolved source becomes

\[ T_A = \frac{T_J \Omega_J P(\theta_0, \phi_0)}{\Omega_B}. \] (5.8)

This quantity, the antenna temperature, is directly related to the power at the detector, and hence the voltage output \( S \) of the detector is directly related to this quantity,

\[ S = A T_A, \] (5.9)

where \( \{A\} \) is a calibration constant with dimension V/K.

\(^2\)A constant brightness temperature across the 65-180 GHz frequency range has been used [81]. The calibration uncertainty is approximately 8%.

\(^3\)The Kelvin \times Steradian unit (K Sr) is dimensionally equivalent to the Jansky (Jy); the source flux for Jupiter at 100 GHz corresponding to the value given in K Sr is \( \sim 1700 \text{ Jy} \).
5.1.2 Telescope Optical Model

The instrument design suggests that the beam pattern $P$ should be well modeled by a 2D Gaussian distribution [45],

$$P(\theta, \phi) = e^{-(\theta^2/2\sigma_\theta^2)-(\phi^2/2\sigma_\phi^2)}.$$  \hspace{1cm} (5.10)

We incorporate independent beamwidth parameters $\{\sigma_\theta, \sigma_\phi\}$ in the $\theta$ and $\phi$ directions to test for beam ellipticity. Axial beam symmetry would be expected based purely on the radiometer feed horn design, but the off-axis arrangement of the primary and secondary mirrors may be expected in principle to introduce slight beam ellipticity.

The raster is centered roughly on Jupiter based on positioning obtained from the on-board star camera and the gyros, but since the pointing of the radiometer beam relative to the camera is imprecisely known, the measured position of Jupiter within the raster relative to the gyro coordinates provides the final absolute pointing calibration of the mm-wave beam. Hence, we incorporate additional free parameters $\{\bar{\theta}, \bar{\phi}\}$ to fit this offset of the mm-wave beam relative to the gyro coordinate system. In addition, the instantaneous position of the beam is modulated in cross elevation relative to the gondola by the chopper,

$$\theta = \theta_{gyro} - \bar{\theta} + A_c f_{chop},$$  \hspace{1cm} (5.11)

where $f_{chop}$ is the unit peak-to-peak normalized chopper position signal$^4$, and $\{A_c\}$ is a free parameter, with dimension arcminutes, describing the amplitude of the full chopper throw.

There is only one telescope elevation for which the beam elevation on the sky remains constant throughout the chopper throw; this is inherent to the off-axis optical design of the instrument. For all other telescope elevations, the chopper varies the beam elevation slightly as a function of chopper deflection angle. We model this effect in the fit by

$$\phi = \phi_{gyro} - \bar{\phi} + \delta_{el,0} f_{chop} + \delta_{el,1} f_{chop}^2,$$  \hspace{1cm} (5.12)

where $\{\delta_{el,0}, \delta_{el,1}\}$ are coefficients parameterizing the variation of the beam elevation as a function of chopper deflection to first and second order, respectively.

The angular pointing measures in Equations 5.11 and 5.12 vary parametrically in time (Fig. 5.1), $\theta = \theta(t), \phi = \phi(t)$; substitution of these quantities into Equations 5.10 and 5.8 yields the desired model for the time-ordered radiometer optical input signal, $S_0 = S_0(t) = AT_A(t)$.

---

$^4$Archive signal chopper.
Figure 5.1: Time-ordered pointing signals for the Jupiter raster. For purposes of analysis, the gyro signals are interpolated to match the 160 Hz sample rate of the chopper and detector signals. The width of the instantaneous "beam position" trace is indicative of the amplitude of the chopper throw.

5.1.3 Signal Chain Model.

The time-ordered model developed to this point represents the optical input to the radiometer. The electrical output signal is related to this optical input signal by the radiometer transfer function $H(\omega)$.\(^5\)

The optical signal is converted to a voltage signal at a bolometer in each channel. It was shown in chapter 3 that the bolometer transfer function is that of a single-pole low pass filter,

$$H_{bolo}(\omega) = \frac{1}{1 + j\omega\tau}. \quad (5.13)$$

We have ignored any frequency independent multiplicative terms (since they will be absorbed into the overall calibration constant). The bolometer time constant $\{\tau\}$ is unknown and is taken as a free parameter in the model.

The bolometer voltage signal is read out by the preamp circuit introduced in Fig. 3.17. The first amplification stage is at a cold JFET internal to the dewar. This FET provides gain that is flat over a bandwidth much wider than that of the subsequent electronics, so its frequency dependence can be neglected. The FET signals are amplified outside the dewar by ambient temperature preamplifiers, comprised of two cascaded band pass gain stages that give a peak amplification of around 30

\(^5\)Further discussion of the MSAM2 transfer function may be found in §3.2.4.4.
Chapter 5: MSAM2 Data Analysis

mid-band\(^6\). Each gain stage is characterized by the transfer function

\[ H_{\text{gain}}(\omega) = 1 + \frac{R_2 C_1 j \omega}{(R_2 C_2 j \omega + 1)(R_1 C_1 j \omega + 1)}. \]  
(5.14)

These gain stages are followed by an AC coupler, a single-pole high pass filter with transfer function

\[ H_{\text{AC}}(\omega) = \frac{1}{1 + 1/RC j \omega}. \]  
(5.15)

The composite radiometer transfer function is the product of the individual elements

\[ H(\omega; \tau) = H_{\text{bolo}}(\omega; \tau) H_{\text{gain}}^2(\omega) H_{\text{AC}}(\omega), \]  
(5.16)

where we've noted explicitly that the only free parameter for purposes of the fit is the bolometer time constant. The transfer function for each frequency dependent stage, as well as the composite radiometer transfer function, is shown in Fig. 5.2.

\[ \text{Figure 5.2: Transfer function magnitudes for each element in the signal chain model. The bolometer transfer function shown is for time constant } \tau = 2 \text{ms.} \]

\[ \text{5.1.4 Modeling the Time-Ordered Data} \]

With the radiometer transfer function \( H(\omega) \) in hand, we construct a model of the time-ordered radiometer output by convolving it with the input signal \( S_0(t) \). Compiling the free parameters from

\[ ^6\text{For purposes of calibration, the composite transfer function is normalized to unity gain.} \]
the previous sections, we see that the quantity $S(t)$ depends on a 9 element parameter vector,

$$\bar{a} = [\sigma_\theta, \sigma_\phi, \tau, \bar{\theta}, \bar{\phi}, A_c, \delta_{el,0}, \delta_{el,1}, A] \quad (5.17)$$

and the explicit expression for the model time stream is

$$S(t; \bar{a}) = S_0(t) * F^{-1}\{H(\omega)\} \quad (5.18)$$

$$= F^{-1}\{F\{S_0(t; \sigma_\theta, \sigma_\phi, \bar{\theta}, \bar{\phi}, A_c, \delta_{el,0}, \delta_{el,1}, A)\}H(\omega; \tau)\}, \quad (5.19)$$

where $F$ ($F^{-1}$) is the Fourier (inverse-Fourier) transform operator. The parameter ordering of Equation 5.17 is maintained throughout the following discussion, so that e.g. the correlation between the cross-elevation beam width $\sigma_\theta$ and the bolometer time constant $\tau$ is the element $R_{02}$ of the correlation matrix presented in §5.1.9. The convolution $S$ of a model optical signal $S_0$ (Fig. 5.3) with the composite transfer function $H$ is shown in Fig. 5.4.

Although the detector output signals ultimately relate to the antenna temperature, this quantity is not optimal for fitting purposes, since the measured signal

$$S_0 \equiv AT_A$$

$$= \frac{T_J \Omega_J}{\Omega_B} P(\theta, \phi) \quad (5.20)$$

depends on a ratio of free parameters: The calibration constant ($[A] = V/K$) and the beam solid angle ($[\Omega_B] = \text{Sr}$, $\Omega_B = 2\pi \sigma_\theta \sigma_\phi$). This results in large correlations in the parameter set $\{A, \sigma_\theta, \sigma_\phi\}$. Instead, we fit to the flux directly (Equation 5.6),

$$S_0 = AT_J \Omega_J P(\theta, \phi), \quad (5.22)$$

and obtain an intermediate calibration constant $A$, $[A] = V/K \text{ Sr}$, which is then converted to a $V/K$ quantity by multiplying by the best fit beam solid angle $\Omega_B$.

5.1.5 Alternate Models

Although the preceding model is well motivated by the instrument design, other parameters that phenomenologically describe specific effects may yield better goodness of fit measures. We considered simpler models (excluding the beam elevation variation with chopper position), as well as more complex, 11-13 parameter models that allow the beamwidth, in addition to the beam elevation, to vary parametrically with chopper position. The model of §5.1.4 is the most efficient of those investigated (lowest $\chi^2$/DOF).
Figure 5.3: Simulated raw time-ordered data: A model of an optical input into the radiometer.

Figure 5.4: Simulated time-ordered data: A model of the signal out of the radiometer, given the optical signal of Fig. 5.3 as an input and the transfer function of Equation 5.16.
5.1.6 Raster Data Cleaning

In addition to the effects contained in the optical and electrical signal model, the real data contain noise, AC offsets, non-stationary effects, and glitches. The stationary noise component is of course handled by the data fit; the other components must be fit out phenomenologically, edited out, or at least noted when evaluating the goodness of the final fit.

Since the signal to noise ratio in the Jupiter raster data is large, a 4 or 5σ data cut leaves a large amount of residual glitching, while tighter cuts risk removing signal. For this reason, an iterative approach is taken, with an initial parameter estimate being derived from a fit to a $\pm 7\sigma$ cut of the raw data stream\(^7\). This preliminary fit is then subtracted from the data, and any 4σ outliers from the residuals are cut. This edited data set is then rerun through the fitting procedure for the final parameter estimates. The data points cut during each editing stage, for each channel, are shown in Table 5.1. In all cases, the number of degrees of freedom in each data vector was reduced by less than 0.5% due to deglitching. The iterative deglitching process is illustrated in Fig. 5.5.

Table 5.1: Jupiter raster deglitching: data cuts by channel. Raw data vectors contain 142336 samples.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rough deglitching</td>
<td>57</td>
<td>33</td>
<td>69</td>
<td>199</td>
<td>128</td>
</tr>
<tr>
<td>Residual deglitching</td>
<td>160</td>
<td>613</td>
<td>321</td>
<td>249</td>
<td>532</td>
</tr>
</tbody>
</table>

The MSAM2 radiometer signals contain AC offsets at multiple of 0.625 Hz, with prominent spectral features at 2.5 and 5 Hz\(^8\). The features are sharp, with no resolved bandwidth at $\sim 1$ mHz resolution (Fig. 5.6). The approach to removing these spurious signals is analogous to deglitching in the frequency domain. We employ a multi-notch filter\(^9\) in software with a band reject bandwidth of 3.4 mHz centered at multiples of 0.625 Hz to completely remove these modes from the data, resulting in a further reduction of 300 in the degrees of freedom in the data vectors.

5.1.7 Fitting the Time-Ordered Data

We apply the Levenberg-Marquardt nonlinear fitting algorithm \([83]\) to find the parameter vector $\tilde{a}$ that optimizes the fit between Equation 5.19 and the flight data. Represent the data by a vector

\(^7\)This is due to the asymmetry in the AC voltage signals; see Fig. 5.4.

\(^8\)The fundamental and first harmonic of the chopper modulation signal.

\(^9\)Since a model signal power spectrum is readily calculated, use of an optimal (Wiener) filter is also a viable approach. In practice, the best fit parameters were completely insensitive to the choice of filtering, so notch filtering was used for calculational convenience.
Figure 5.5: Successive deglitching operations on the Jupiter raster data. The full raster time stream for channel 1 is shown. The first cut identifies and removes $\pm 7\sigma$ outliers in the raw data. The second cut identifies and removes $4\sigma$ outliers from the residuals to a fit to the first data cut. The residuals after final deglitching and subtracting the best-fit model are shown in the bottom panel.
Figure 5.6: Bandwidth of 5 Hz offset in channel 1. Other channels, including the dark channel, are similar.

$D_i$. The merit function to be minimized is

$$\chi^2(\bar{a}) = \frac{1}{\sigma^2} \sum_i (D_i - S_i(\bar{a}))^2,$$

(5.23)

where the model has been expressed with an index $i$ (rather than as a function of a continuous time parameter $t$) to reflect the discrete sampling of the data.

We require a noise estimate to evaluate Equation 5.23 and obtain a rigorous goodness of fit measure. The segments of the raster when the telescope was oriented well-off the planet contain no signal and are useful for this purpose. Although the deglitching procedure removes the dominant nonstationary components of the time stream, the residual noise still exhibits substantial time scale dependent variance$^{10}$. To quantify this, we divide the time stream into 9600 sample (1 minute) segments and calculate the variance for each segment. The minimum variance segment is then used for noise estimation purposes (Fig. 5.7). This approach mitigates the effects of glitch residuals, which inflate the time stream variance estimates and hence bias $\chi^2$ downward. Under the reasonable assumption that the glitch events are uniformly distributed throughout the dataset and are uncorrelated with the signal, this process will yield a workable noise estimate.

The partial derivatives of the model with respect to each of the parameters are required for the fitting routine to navigate the $\chi^2$ hypersurface most efficiently, and to determine the confidence
Figure 5.7: Signal variance by channel, time, and segment length for the deglitched Jupiter raster data. Segment lengths of 15s, 30s, and 1m are indicated by the blue, red, and black traces, respectively. The large variance in the middle of the traces is signal; the signal-free, low variance sections at the beginning and end of the scans are used for noise estimation: For each channel, the minimum variance measured in a 1 minute segment is used.

Similar calculations are performed for each of the nine parameters for each fit iteration, in addition to the calculation of the model itself, necessitating 20 FFTs of $10^5$ element arrays per cycle. A typical fit, including rough deglitching, an initial convergence to a model estimate in 5 or 6 iterations, residual deglitching, and convergence to a final fit, takes about 4 minutes per channel on an 800
MHz Linux workstation.

The curvature matrix $\alpha_{kl}$ is constructed from the partial derivatives of the model,

$$
\alpha_{kl} = \frac{1}{\sigma^2} \sum_i \frac{\partial S_i(\vec{a})}{\partial a_k} \frac{\partial S_i(\vec{a})}{\partial a_l}.
$$

This object is used in calculating the increments $\delta \vec{a}$ selected by the fitting routine for each iteration, and also yields confidence limits on the best-fit parameters, since it is the inverse of the estimated covariance matrix of the errors in the fit parameters,

$$
\alpha = C^{-1}.
$$

The confidence limit on an individual fit parameter, assuming a single degree of freedom in the fit, is

$$
\delta a_i = \sqrt{\Delta \chi^2 C_{ii}}.
$$

All confidence limits are stated at the 95.4\% level ($\Delta \chi^2 = 4$) unless otherwise stated.

### 5.1.8 Fitting Procedure Validation: Simulations

We validate the fitting procedure by generating a model, adding noise, glitches, and offsets, then processing the model with the fitting code and comparing the best fit parameters to the model input parameters. Noise is chosen to give a signal to noise ratio comparable to the data. Glitches, Poisson distributed in time, are added to the timestream, with a mean interval between events comparable to the data, and normally distributed amplitudes. Starting "guess" parameters for the fit are also varied, to ensure the optimal fit parameter vector $\vec{a}$ is independent of the choice of the fit initialization parameter vector $\vec{a}_0$.

A sample fit simulation is presented in Table 5.2\(^{(11)}\). The iterative deglitching procedure typically detected 105-115 events per 100 glitches, indicating a small number of errors of the first kind (false positives) in the detection process. This quantity is consistent with the number of samples likely to exceed $4\sigma$ in a normally distributed data vector with $\sim 10^5$ elements; the effect of this on the fit is entirely benign.

The confidence limits on the individual fit parameters, derived using the noise estimate prescription described above, were statistically consistent with the simulation input parameters. Typical $\chi^2$ values for the simulation fits were $140000 < \chi^2 < 144000$ for $\sim 142000$ degrees of freedom. With large numbers of degrees of freedom, the likelihood $Q$ of obtaining the data $D$ given the model

\(^{(11)}\)For the gaussian beamwidth parameters we quote the full-width half-maxima values $FWHM_\theta, FWHM_\phi$ rather than the standard deviations $\sigma_\theta, \sigma_\phi$. The parameters are related by $FWHM = 2\sqrt{2\ln 2} \sigma$
Table 5.2: A sample fit to simulated time-ordered planet raster data. Simulation input, fit initialization, and fit output vectors ($\vec{a}_i$, $\vec{a}_0$, $\vec{a}$ respectively) are shown. The input parameters used to generate the simulated time stream are recovered by the fitter with high fidelity, in the presence of flight-like glitch rates and signal to noise.

<table>
<thead>
<tr>
<th>Fit to Simulation data: $\chi^2/DOF = 140126/141938$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FWHM_\theta$</td>
</tr>
<tr>
<td>(\degree)</td>
</tr>
<tr>
<td>$\vec{a}_i$</td>
</tr>
<tr>
<td>$\vec{a}_0$</td>
</tr>
<tr>
<td>$a$ (95.4%CL)</td>
</tr>
</tbody>
</table>

$S$ is extremely sensitive to the noise estimate; while in some cases the formal likelihoods of the data given the model are small, the $\chi^2$ values are entirely consistent with the precision of the noise estimate.

The fit residuals are shown in figures 5.8 and 5.9. For the simulation, we know $a$ priori that the model is good; once the fit parameters converge to the known values of the parent distribution, the fitter has been validated and the residuals should be consistent with noise (as they are). As an additional check on the quality of the fit, we bin the fit residuals by chopper position (Fig. 5.10). Model inaccuracies will in general show up as residuals that are coherent with the chopper modulation. For the real data, such binning provides a useful additional cross-check on the quality of the fit.
Figure 5.8: The time ordered data for the fit simulation of Table 5.2. Top left: The raw data, with simulated noise and glitch rates. Top right: The simulated data, post iterative deglitching. Bottom left: The best fit model. Bottom right: Fit residuals.

Figure 5.9: Detail of Fig. 5.8.
Figure 5.10: Residuals for the fit to the simulated data of Table 5.2, binned by chopper position.
5.1.9 Fits to the Radiometer Data

The optimal parameters from the fit to the Jupiter raster data for each radiometer channel are presented in Table 5.3 \(^{12}\). The $\chi^2$/DOF values for the fit are presented in Table 5.4. These values are very sensitive to the noise estimates, which are in turn susceptible to bias from known nonstationary contaminants in the data stream, including incompletely removed glitches as well as other effects of unknown origin. Given this constraint on the goodness of fit determination, the values obtained are reasonable; channel 1 is a formally likely result, channels 2, 3, and 5 are marginal, while channel 4 is unlikely, but also the most afflicted with nonstationary noise. In conjunction with examination of the variance of the residuals (appendix C) and the residuals binned in chopper position (Fig. 5.11), we argue that the fits provide a persuasive description of the data.

We find a considerable amount of ellipticity ($\varepsilon=0.44-0.60$) in the beam cross sections, with the cross elevation axis in all cases larger than the elevation axis. The beams show substantial frequency dependence, ranging from $\approx30'$ FWHM in channel 1 to $\approx20'$ FWHM in channel 5. The resulting variation in antenna temperature as a function of frequency is presented in Table 5.5

The bolometer time constants range between 1-4 ms; these numbers are comparable to the values measured pre-flight. The measured position of Jupiter in channels 1-3 is consistent, while channels 4 and 5 agree but show a small offset from the other channels. Since channels 1-3 share one feed horn, while channels 4 and 5 share another, the small discrepancy ($\approx4'$) between these two values indicates the precision with which the low and high frequency feed horns were aligned on the sky.

The chopper amplitude was statistically consistent in channels 1,2,3, and 5. The channel 4 data prefers a $\approx1'$ smaller throw, but the chopper-binned residuals show some structure consistent with an error in the chopper amplitude. This is likely due to contamination from nonstationary noise with a large $1/f$ component around records 40000-60000 and 120000-140000 in the channel 4 data only. This may also be the source of the large beam ellipticity measured in channel 4. The $\chi^2$ of the fit to the channel 4 data is the poorest (Table 5.4).

The beam deviation in elevation as a function of chopper position exhibits little linear dependence, but significant quadratic dependence, equivalent to a $1'-2'$ dip in elevation at the extrema of the chop. Some variation in elevation with chopper position is consistent with the optical design

\(^{12}\)The optical efficiency of channel 5 was poor, and would carry insignificant weight in the multi-channel CMB analysis, but we process the channel 5 Jupiter data in parallel with the other channels as a test for the robustness of the fit. In fact, although the signal to noise in channel 5 was low, the fit parameters were in most cases reasonable, and showed some consistency with the other channels. The exception was the channel 5 best fit bolometer time constant, which was a non-physical $-1.9$ ms.
Table 5.3: Jupiter raster data: Best fit parameters by channel (95% CL).

<table>
<thead>
<tr>
<th>Channel</th>
<th>FWHMθ (')</th>
<th>FWHMφ (')</th>
<th>τ (ms)</th>
<th>θ ('')</th>
<th>φ ('')</th>
<th>λc (')</th>
<th>ξc,0 ('')</th>
<th>ξc,1 ('')</th>
<th>A (V/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel 1</td>
<td>33.75 ± 0.33</td>
<td>29.96 ± 0.31</td>
<td>1.00 ± 0.18</td>
<td>-2.62 ± 0.13</td>
<td>1.87 ± 0.21</td>
<td>140.47 ± 0.43</td>
<td>0.334 ± 0.42</td>
<td>-8.1 ± 1.6</td>
<td>1.680 ± 0.012</td>
</tr>
<tr>
<td>Channel 2</td>
<td>28.86 ± 0.15</td>
<td>25.83 ± 0.14</td>
<td>1.680 ± 0.080</td>
<td>-2.715 ± 0.056</td>
<td>2.152 ± 0.092</td>
<td>140.70 ± 0.19</td>
<td>0.71 ± 0.19</td>
<td>-5.05 ± 0.70</td>
<td>1.5951 ± 0.0090</td>
</tr>
<tr>
<td>Channel 3</td>
<td>27.54 ± 0.17</td>
<td>24.59 ± 0.15</td>
<td>3.425 ± 0.098</td>
<td>-2.12 ± 0.066</td>
<td>2.48 ± 0.10</td>
<td>140.44 ± 0.22</td>
<td>0.33 ± 0.21</td>
<td>-4.86 ± 0.80</td>
<td>0.6369 ± 0.0043</td>
</tr>
<tr>
<td>Channel 4</td>
<td>27.83 ± 0.22</td>
<td>22.28 ± 0.18</td>
<td>4.047 ± 0.13</td>
<td>-2.37 ± 0.084</td>
<td>4.857 ± 0.12</td>
<td>139.32 ± 0.28</td>
<td>0.24 ± 0.24</td>
<td>-5.04 ± 0.90</td>
<td>0.1219 ± 0.0100</td>
</tr>
<tr>
<td>Channel 5</td>
<td>20.92 ± 0.55</td>
<td>20.05 ± 0.53</td>
<td>-1.96 ± 0.15</td>
<td>-1.01 ± 0.21</td>
<td>4.69 ± 0.34</td>
<td>141.20 ± 0.70</td>
<td>0.70 ± 0.70</td>
<td>-1.50 ± 1.3</td>
<td>1.272 ± 0.035</td>
</tr>
</tbody>
</table>

Figure 5.11: Fit residuals binned by chopper position for each channel. Coadded data for the entire calibration dataset (142336 samples) for each channel are shown.
Table 5.4: $\chi^2$/DOF results by channel for the fit of Table 5.3.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$/DOF</td>
<td>141868/141766</td>
<td>154639/141451</td>
<td>157611/141633</td>
<td>193390/141592</td>
<td>148237/141356</td>
</tr>
<tr>
<td>$\chi^2_{\nu}$</td>
<td>1.00072</td>
<td>1.09323</td>
<td>1.11281</td>
<td>1.35877</td>
<td>1.04868</td>
</tr>
</tbody>
</table>

Table 5.5: Jupiter antenna temperature by channel.

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_A$ (mK)</td>
<td>58.2</td>
<td>79.0</td>
<td>87.0</td>
<td>95.0</td>
<td>140.3</td>
</tr>
</tbody>
</table>

of the telescope; the value measured here will be used in §5.3 to reconstruct the absolute pointing of the telescope for all chopper positions at other elevation angles.

The Pearson linear correlation matrix $R$ for the fit, given by

$$R_{ij} = \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}, \quad (5.30)$$

is presented in Table 5.6. Significant correlations are found between the calibration constant $A$ and the beamwidths $\sigma_\theta$, $\sigma_\phi$ ($R_{08}$ and $R_{18}$, respectively), and between the position of Jupiter in elevation $\phi$ and the and the beam elevation variation as a function of chopper position $\delta_{el,1}$ ($R_{47}$). A small correlation exists between the bolometer time constant $\tau$ and the beam width in cross elevation $\sigma_\theta$, as might be intuitively expected. All other linear correlation coefficients are $< 0.05$.

5.2 Sensitivity Estimation.

With the calibration complete, the instrument’s sensitivity can be calculated. To do so, we estimate the instrument noise by examining the variance of segments of the CMB scan data. In principle, the

Table 5.6: Pearson correlation matrix of fit to Jupiter raster data.

<table>
<thead>
<tr>
<th>$\sigma_\theta$</th>
<th>$\sigma_\phi$</th>
<th>$\tau$</th>
<th>$\phi$</th>
<th>$\phi$</th>
<th>$A_c$</th>
<th>$\delta_{el,0}$</th>
<th>$\delta_{el,1}$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\theta$</td>
<td>1.00</td>
<td>0.00</td>
<td>-0.09</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.05</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>0.00</td>
<td>1.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>0.01</td>
<td>-0.00</td>
<td>0.01</td>
<td>-0.00</td>
</tr>
<tr>
<td>$\tau$</td>
<td>-0.09</td>
<td>-0.00</td>
<td>1.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>1.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.01</td>
<td>-0.00</td>
</tr>
<tr>
<td>$\dot{\phi}$</td>
<td>0.00</td>
<td>0.01</td>
<td>-0.00</td>
<td>-0.00</td>
<td>1.00</td>
<td>-0.00</td>
<td>0.02</td>
<td>0.77</td>
</tr>
<tr>
<td>$A_c$</td>
<td>-0.05</td>
<td>-0.00</td>
<td>0.02</td>
<td>-0.00</td>
<td>-0.00</td>
<td>1.00</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\delta_{el,0}$</td>
<td>-0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.02</td>
<td>-0.00</td>
<td>1.00</td>
<td>0.04</td>
</tr>
<tr>
<td>$\delta_{el,1}$</td>
<td>0.00</td>
<td>-0.00</td>
<td>-0.01</td>
<td>-0.00</td>
<td>0.77</td>
<td>0.00</td>
<td>0.04</td>
<td>1.00</td>
</tr>
<tr>
<td>$A$</td>
<td>0.58</td>
<td>0.48</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.00</td>
<td>-0.06</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
data contain signal and noise, but the instantaneous SNR is small enough that the bias introduced by the signal is insignificant. Data is deglitched, but otherwise unprocessed for this procedure.

The standard deviation for 1s (160 sample) segments of the West Scan 4 CMB data are plotted vs. time in Fig. 5.12 for each channel. In addition to the obvious data telemetry dropouts and small amounts of residual glitching, non-stationary periodic variations in the segment variance are apparent in channels 1, 4, and 5. These effects are intractable to subtract, but a sensitivity for the clean segments may be derived.

Given the standard deviation $\sigma_{V,i}$ of a segment of data in channel $i$, the sensitivity of channel $i$ is estimated by

$$
\sigma_{T,i}\sqrt{\tau_0} = \frac{G_{cal}}{G_{cmb}} \frac{\sigma_{V,i}}{A_i} \sqrt{\tau_0},
$$

(5.31)

where $G_{cal}, G_{cmb}$ are the postamp gain settings for the calibration and CMB scans, $A_i$ is the calibration constant for channel $i$ in V/K, $\tau_0$ is the sample time, and $\sigma_{T,i}\sqrt{\tau_0}$ is the channel sensitivity in K$\sqrt{s}$. The sensitivity for representative "clean" data segments, the calibration constant, and the gain settings for the calibration and the CMB scans are presented for each channel in Table 5.7. Since significant sections of the data in channels 1, 4, and 5 exhibit noise characteristics that deviate from this idealization, these numbers cannot be used to directly estimate per pixel sensitivity for these channels of the West Scan 4 data.

An illustration of how the noise integrates down by channel is shown in Fig. 5.13. We bin the data by chopper position, coadd for $\tau$ chopper throws, and calculate the variance of the coadded data vs. $\tau$. The slope of the log of the variance plotted against the log of $\tau$ is $-1/2$ if the noise is gaussian and stationary (c.f. the discussion of the radiometer equation in appendix A.4). For short time scales, the radiometer equation holds, but at longer time scales channels 1, 4, and 5 show significant excess variance, as would be expected from Fig. 5.12.

Table 5.7: MSAM2 Rayleigh-Jeans and thermodynamic sensitivities by channel, estimated from the West Scan 4 data. Nominal band centers used for thermodynamic corrections are those of Table 3.1.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma_V$ (V)</th>
<th>$G_{cal}$ (1)</th>
<th>$G_{cmb}$ (1)</th>
<th>$A$ (V/K)</th>
<th>$\sigma$ (mK)</th>
<th>$\sigma_{T,RJ}\sqrt{\tau_0}$ ($\mu$K$\sqrt{s}$)</th>
<th>$\sigma_{T,CMB}\sqrt{\tau_0}$ ($\mu$K$\sqrt{s}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.50</td>
<td>8</td>
<td>408</td>
<td>1.080</td>
<td>9.077</td>
<td>720</td>
<td>820</td>
</tr>
<tr>
<td>2</td>
<td>0.60</td>
<td>4</td>
<td>204</td>
<td>1.5951</td>
<td>7.376</td>
<td>580</td>
<td>710</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>4</td>
<td>204</td>
<td>0.6369</td>
<td>10.77</td>
<td>850</td>
<td>1120</td>
</tr>
<tr>
<td>4</td>
<td>0.27</td>
<td>4</td>
<td>102</td>
<td>1.219</td>
<td>8.69</td>
<td>690</td>
<td>1120</td>
</tr>
<tr>
<td>5</td>
<td>0.35</td>
<td>32</td>
<td>204</td>
<td>1.272</td>
<td>43</td>
<td>3410</td>
<td>6600</td>
</tr>
</tbody>
</table>
Figure 5.12: Standard deviation of 1s (160 sample) segments of the deglitched CMB data (West Scan 4) vs. time for each radiometer channel. A simulated pure gaussian noise channel with $\sigma = 0.5V$ is included for reference.
Figure 5.13: Variance of chopper coordinate binned data vs. integration time by channel. Variance of coadds from 1 to 300 chops (140 s), starting at record 19587 from the West Scan 4 data are shown. A simulated pure gaussian noise channel is included for reference.
The measured per channel sensitivities are two to four times the anticipated values (c.f. §3.2.4.4), even taking the known pre-flight degradation in optical efficiency into account; this would require approximately 5 times the integration time to achieve the per pixel sensitivity targeted by the initial observing strategy. Since the decreased sensitivity was noted in flight, the later CMB scans spent extended observing time on each field\textsuperscript{13}. The "West Scan 4", in particular, spent 40 minutes observing a \( \sim 2 \) square degree patch of sky, resulting in an average sensitivity on the order of 35 \( \mu K \) per 20\textdegree pixe. This is sufficient to detect excess variance due to a CMB signal with marginal significance, and the sky coverage is small. For this reason, we conclude the analysis by concentrating on the West Scan 4 data only, to determine if excess variance due to signal is indeed present.

5.3 Pointing Reconstruction

The gondola boresight orientation in right ascension and declination is determined by using the gyros to interpolate between reference photos of stars taken by the on-board camera, but the instantaneous position of the mm-wave beam must be determined for each detector sample by using the results of the calibration raster of Jupiter to add the motion of the chopper to the pointing.

5.3.1 Chopper amplitude in cross-elevation and azimuth

The chopper throw (peak-to-peak) is measured to be some angle \( \phi \) in the cross-elevation, elevation coordinate system. When the elevation of the telescope is zero, the cross-elevation great circle coincides with the horizon, and cross-elevation angular measure is equivalent to azimuth. The throw is related to the Cartesian coordinates in the 0\textdegree elevation plane by

\[
\tan(\phi/2) = \frac{y}{x},
\]

(5.32)

Now rotate the telescope up to an altitude \( a \) (Fig. 5.14). This transformation (a rotation about the y-axis) is

\[
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix} = \begin{pmatrix}
\cos a & 0 & -\sin a \\
0 & 1 & 0 \\
\sin a & 0 & \cos a
\end{pmatrix} \begin{pmatrix}
x \\
y \\
z
\end{pmatrix}.
\]

(5.33)

The azimuth angle \( \phi' \) subtended by a throw \( \phi \) in the cross-elevation plane at an altitude \( a \) is

\[
\tan(\phi'/2) = \frac{y'}{x'},
\]

(5.34)

\textsuperscript{13}The cost of this, of course, is reduced sky coverage.
and from (2) we have

\[ x' = x \cos a - z \sin a \quad (5.35) \]

\[ = x \cos a \quad (5.36) \]

\[ y' = y, \quad (5.37) \]

so

\[ \tan(\phi'/2) = \frac{y}{x \cos a} \quad (5.38) \]

\[ = \frac{\tan(\phi/2)}{\cos a}. \quad (5.39) \]

Note that as \( a \to 90^\circ \), \( \tan(\phi'/2) \to \infty \) or \( \phi'/2 \to 90^\circ \) independent of \( \phi \); any chopper throw in cross-elevation has a half-throw of \( 90^\circ \) in azimuth (or a full throw of \( 180^\circ \)) if the telescope was to be (hypothetically) pointed at the zenith, as is intuitively clear. This is illustrated in Fig. 5.15.

The chop throw is small (140'). For \( \phi \ll 1 \), \( \tan \phi \sim \phi \), and

\[ \tan(\phi'/2) \sim \frac{\phi/2}{\cos a} \quad (5.40) \]

and for \( a \) small (elevation not too near the zenith),

\[ \phi' \sim \frac{\phi}{\cos a}, \quad (5.41) \]

which is the desired relation between angular measure in cross elevation and azimuth.
5.3.2 Chopper modulation of beam elevation at varying elevation angles

As mentioned in §5.1.2, as a consequence of the off-axis Cassegrain optical configuration of the MSAM2 telescope, there is a single elevation at which the nutating secondary mirror (the "chopper") throws the beam at constant elevation. At all other elevations, as the chopper modulates the cross-elevation position of the beam, it also modulates the elevation of the beam slightly. The effect is quite small relative to the beam size, but since it was clearly detected in the calibration and is relatively straightforward to incorporate, we include it in the pointing reconstruction.

The gyros define an elevation (el), cross-elevation (xl) orthogonal spherical coordinate system. The origin of the coordinate system (xl=0, el=0) may be thought of as the intersection between two great circles: the local meridian, and the great circle orthogonal to the local meridian at some arbitrary elevation as determined by the observation. This circle orthogonal to the meridian through the xl, el origin is the cross-elevation great circle. For chopper throw measured in cross elevation, we expect some deviation in elevation from curvature effects alone, since cross-elevation
is a great circle but constant elevation (except at the horizon) is not. This effect, as viewed from the origin, is illustrated in Fig. 5.16.

Figure 5.16: Modulation of beam elevation as a function of chopper deflection. The actual path of the beam on the sky as it is deflected by the chopper is along an arc of a small circle that has a concave down projection onto the celestial sphere, with deviation -1.5' from constant elevation at the chop extrema, at an observation elevation of 20°.

From the Jupiter calibration data, the chopper throw is measured to be 140' in cross-elevation. The deviation of the cross-elevation great circle from constant elevation over an arc of this size, at the 20° elevation of the calibration raster, is -0.25'. The actual elevation deviation of the beam at the extrema of the chop was measured to be -1.5 ± 0.4', indicating that the beam deflection is along a small circle with a concave down projection onto the celestial sphere at the elevation at which the calibration was performed.

To calculate the beam deviation in elevation along the chop at arbitrary elevations, we first calculate the angle between the chop plane (the plane whose intersection with the celestial sphere is the small circle along which the chopper throws the beam) and the plane of the cross-elevation
great circle. A bit of trigonometry reveals that, for the measured elevation deviation at the chop extrema, this angle is $64^\circ \pm 5^\circ$, as illustrated in Fig. 5.17. Given this number, we have the following

prescription for adding the chopper motion to the instantaneous pointing in equatorial coordinates:

- Rotate the equatorial $(\alpha, \delta)$\textsuperscript{14} pointing vector to local alt-az $(a, A)$ coordinates.

- Interpolate the alt-az pointing vector up to match the dimension of the detector/chopper position data vectors.

- Add the chopper throw to the azimuth coordinate, using Equations 5.11 and 5.41 to convert from cross elevation to azimuth:

\[
A \rightarrow A + \frac{A_{c}f_{chop}}{\cos a}.
\]  

\[ (5.42) \]

- Find the elevation deviation amplitude $\delta_{el,1}$ of the chop at the elevation of observation by rotating the chop small circle of Fig. 5.17 to that elevation.

\[ \text{14We use the spherical coordinate system nomenclature and conventions of Green [84].} \]

Figure 5.17: Angle between the cross elevation plane and the chop plane.
Chapter 5: MSAM2 Data Analysis

Figure 5.18: Addition of the chopper throw to the pointing in equatorial coordinates for a small segment of the West Scan 4 data. The original gondola boresight pointing is shown in white. The instantaneous pointing of the beam, including the chopper deflection, is shown in black. The vertical axis has been greatly expanded to illustrate the slight curvature of the chopper throw.

- Calculate the elevation deviation for all points along the chop with Equation 5.12:

\[ a \rightarrow a + \delta_{el,1} f_{chop}^2. \]  

(5.43)

- Rotate the \((a, A)\) pointing vector back to equatorial \((\alpha, \delta)\) coordinates.

The effect of these operations is illustrated in Fig. 5.18. At this stage, for each detector sample we have a pointing datum, and sky maps can be constructed.

5.4 Constructing the CMB Maps

Our data vectors are comprised of sky signal, as well as noise and systematic instrumental offsets. The signal and noise can be described by a linear model

\[ d = Ps + \epsilon, \]  

(5.44)
where $d$ is a time-ordered data vector of dimension $n_d$, $s$ is the map comprised of $n_p$ pixels that we seek, $P$ is a "pointing matrix" of dimension $n_d \times n_p$ that encodes the observation strategy, and $\epsilon$ is the instrument noise (for a detailed discussion, see appendix B.3.) Since $d$ is of order $10^5 - 10^6$ (the number of points in the time ordered data), while $s$ is of order $10^2 - 10^3$ (the number of sky pixels observed), expression 5.44 represents an overdetermined system of equations; this simply reflects the fact that we integrate, i.e. observe each pixel many times.

The least square estimator (LSE) of $s$ is given by

$$\hat{s} = (P^T W P)^{-1} P^T W d,$$  \hspace{1cm} (5.45)

where $W$ is the weight matrix, $W^{-1} = (\epsilon \epsilon^T)$. It is instructive to note that for purely white noise, the weight matrix is the identity matrix multiplied by the inverse of the sample variance, and Equation 5.45 collapses down to a simple average of measured values for each pixel.

Instrumental offsets may be treated in various ways. Since they enter the time stream in a systematic way (as does the signal), they may be accommodated by adding metapixels to the pointing matrix. A complex, multiply modulated scan strategy will minimize the projection of simple time domain offsets onto the sky signal; indeed, this is the reason for choosing complex signal modulation schemes. Alternately, filtering may be used to remove contaminated modes from the data; the correlations introduced by the filtering must then be included in the subsequent analysis. For MSAM2, we adopt the filtering approach for channels 2 and 3. Channels 1 and 4 both exhibited significant nonstationary noise phenomena (Fig. 5.12) that are intractable to model, while channel 5’s sensitivity was too poor to merit further analysis.

5.4.1 The pointing matrix

The $n_p$ sky pixels we observe are mapped into $n_d$ time-ordered data points by the observing strategy we choose. Since MSAM2 is a single pixel instrument, each row of the pointing matrix contains a single nonzero element,

$$P_{ij} = 1 \iff \text{observation } i \text{ falls in pixel } j;$$  \hspace{1cm} (5.46)

all other elements are zero. Hence, instead of constructing the full $n_d \times n_p$ matrix, it is sufficient to construct a single vector of dimension $n_d$, in which the element $i$ contains the pixel index for data vector element $i$.

This process is illustrated in Fig. 5.19. Here, the West Scan 4 data is pixelized at 6’ resolution. The pixels are square, since curvature effects are negligible for the sky patch under consideration.
Figure 5.19: Pixelization of the West Scan 4 sky coverage. Each $6' \times 6'$ pixel in which the beam provides some coverage is assigned an index $k$. The beam size is illustrated in the top right corner as a scale reference.
To each pixel we assign an index $k$, where $0 < k < n_p - 1$. The pointing matrix is then represented by a vector with index $k$ in the $i$th element.

### 5.4.2 Matrix vector products

The LSE $\hat{s}$ may be viewed as the matrix product of two quantities, the map covariance matrix $(P^TWP)^{-1}$, and the noise weighted map $P^TWd$. The constituent matrices of these quantities are large, but since only matrix-vector products are required for the analysis, direct construction of the large matrices can be avoided [85]. For example, while the weight matrix $W$ is in principle of rank $n_d \times n_d$, the product $P^TWd$ is an $n_p$ element vector. The pixel/pixel covariance matrix $(P^WP)^{-1}$, of rank $n_p \times n_p$, can be directly constructed for observations with the sky coverage of MSAM2; for larger data sets more sophisticated techniques must be employed [64].

#### 5.4.2.1 Calculation of the noise weighted map

We begin with the calculation of the noise weighted map. We require the weight matrix $W$, which is the inverse of the covariance matrix $V$. For stationary noise, the correlation between samples depends only on the time interval between the samples,

$$\langle \epsilon(t)\epsilon(t') \rangle = a(t-t') = a(\Delta t),$$

(5.47)

where $a$ is the noise autocorrelation function. Since $a(t-t') = a(t'-t)$, the covariance matrix $V = \langle \epsilon\epsilon^T \rangle$ is symmetric, with constant diagonals\(^{15}\),

$$V_{ij} = V_{ji},$$

(5.48)

$$V_{ij} = V_{i+1,j+1}.$$  

(5.49)

A byte scaled generic image of such a matrix is shown in Fig. 5.20. In addition, if $a(t-t') \to 0$ for $t-t' >> 1$ (Fig. 5.21), $V$ is sparse. Since the correlation function is time translation invariant, its Fourier transform is diagonal,

$$\tilde{a}(f,f') = \delta_{ff'} F\{a(\Delta t)\}.$$  

(5.50)

Since $\tilde{a}(f,f')$ is diagonal\(^{16}\), it can be trivially inverted,

$$\tilde{a}^{-1}(f,f') = 1/\tilde{a}(f,f').$$  

(5.51)

\(^{15}\)Such matrices are termed circulant [86].

\(^{16}\)This object, the Fourier transform of the autocorrelation function, may be recognized as the power spectral density.
Fourier transforming back, the weight matrix is given by

\[ a^{-1}(\Delta t) = \mathcal{F}^{-1}\{1/\hat{a}(f, f')\}. \]  

(5.52)

Like the covariance matrix, the diagonals of the weight matrix are constant. Such matrices may be thought of as composed of \( n_d \) rows, each of which is a vector \( w \) of length \( n_d \), where row \( i \) is equal to row 0 shifted by \( i \) columns, that is,

\[ W_{ij} = W_{0,j-i}. \]  

(5.53)

The matrix vector product \( Wd \) may then be expressed as

\[
\begin{align*}
\tilde{d}_i &= \sum_j W_{ij}d_j \\
&= \sum_j W_{0,j-i}d_j \\
&= \sum_j W_{0,i-j}d_j \\
&= \sum_j w_{i-j}d_j \\
&= (w \ast d)_i, \end{align*}
\]

(5.54)

the convolution of the weight vector \( w \) with the data vector \( d \). This represents a vast simplification, since numerically an \( \mathcal{O}(n_d^2) \) operation has been replaced by 3 FFTs and a multiplication, an \( \sim \mathcal{O}(3n_d \log_2 n_d) \) operation.

Missing data due to glitch removal and telemetry dropouts introduce a bias in the weight vector estimation at low frequencies if Fourier techniques, which assume uniform sampling, are employed.
Figure 5.21: Autocorrelation function for the West Scan 4 channel 2 data. Correlations are small for lags $t - t' > 100$ samples.

For this reason, instead of evaluating Equation 5.50 directly, we obtain the power spectrum by calculating the Lomb Periodogram of the time stream,

$$\tilde{a}(f, f') = \mathcal{L}\{\epsilon, \tau\},$$  \hspace{1cm} (5.55)

where $\epsilon$ is the noise data vector, and $\tau$ is a vector of dimension $n_d$ that contains the time stamp for each sample in $\epsilon$. This approach accommodates the missing data problem optimally, in that the power spectrum estimate returned by the periodogram is equivalent to calculating a least squares fit of the irregularly sampled data to a harmonic model $d_i = A \cos \omega t + B \sin \omega t$ [83]. We then obtain the weight vector $w$ by inverting and inverse Fourier transforming the data’s Lomb periodogram,

$$w = \mathcal{F}^{-1}\{1/\mathcal{L}\{\epsilon, \tau\}\}. \hspace{1cm} (5.56)$$

The weight vector $w$ \footnote{It is again instructive to consider this object for the special case of uncorrelated white noise. With no correlations between samples, the weight matrix is diagonal, the diagonal elements are $1/\sigma^2$, and the convolution $w * d$ is simply the inverse variance weighting of the data $d$.} calculated for the channel 2 West Scan 4 cmb data is shown in Fig. 5.22. Although formally of length $n_d$, $w$ approaches 0 for large lags $\Delta t$; for this reason, some threshold value $\kappa \sigma^{-2}$ is chosen, below which samples are given 0 weight. Heuristically, operating
on \( W \) replaces datum \( d_i \) with a composite quantity that is a weighted average over the samples adjacent to \( d_i \), within a window defined by the weight vector threshold \(|w| > \kappa \sigma^{-2}\).

After calculating the convolution \( w * d \), multiplying by the transpose pointing matrix \( P^T \) is accomplished by a simple loop over the number of pixels \( n_p \), binning the vector \( Wd \) on the sky. This yields the desired noise weighted sky map \( P^TWd \).

### 5.4.2.2 Calculation of the map covariance matrix

The \( n_p \times n_p \) map covariance matrix \((P^TWP)^{-1}\) can be constructed directly for \( O(n_p) \sim 10^2 \) by direct calculation and inversion of the map weight matrix \( P^TWP \). This calculation involves an outer loop over the \( n_p \) pixels, each of which requires evaluation of the product \( WP \), a convolution of the \( n_d \) element weight vector with the pointing matrix. The pointing matrix is represented as an \( n_d \) element vector of pixel indices as described in §5.4.1, so this convolution may be handled by the methods of §5.4.2.1.

An inner loop, also over the \( n_p \) pixels, performs the matrix multiplication \( P^T(WP) \), which is a simple binning operation. Since the map weight matrix is symmetric, only \( n_p(n_p + 1)/2 \) unique elements are explicitly calculated; the remaining elements are determined by symmetry. The inversion of the map weight matrix is then readily performed via Cholesky decomposition [83],

\[
P^TWP = LL^T, \tag{5.57}
\]
\[
(P^TWP)^{-1} = (LL^T)^{-1}, \\
= (L^T)^{-1}L^{-1}. \tag{5.58}
\]
Cholesky decomposition is robust, but each inversion is explicitly checked by confirming that the identity

\[ P^T WP(L^T)^{-1} L^{-1} = 1 \]  

(5.59)
is satisfied.

With the map covariance matrix and the noise weighted map calculated, the LSE \( \hat{s} \) of the map is obtained directly from Equation 5.45; this is an easily handled product of a matrix of rank \( n_p \times n_p \) with an \( n_p \) element vector. The full algorithm for constructing the CMB temperature maps, including the method for generating the simulation cross-checks to be described in the following section, is depicted graphically in Fig. 5.23.

### 5.4.3 Fit simulations

To test the numerical implementation of the calculations of §5.4.2.1 and §5.4.2.2, we generate simulated map vectors, create simulated time-ordered data by observing them with the actual flight pointing matrix, add noise (white and 1/f components) and glitches, and then use them as input to the map-making procedure. Additionally, for small data vectors where such an approach is numerically feasible, we solve for the maps algebraically by actually creating the full pointing and weight matrices in Equation 5.45 and comparing the results with those obtained with the fitting code. This provides a cross check on the calculations of, and operations on, the compressed representations of the matrices.

An example of simulated time-ordered data, generated by applying 2048 samples of the West Scan 4 pointing matrix to a toy model sky pixelized at 12’ resolution, with pixel value \( k \) given by

\[ s_k = \sin(1.1k e^{-k/n_p}) \]

(5.60)
is shown in Fig. 5.24. The signal-to-noise for the simulation is set at 0.1. The modulation from the chopper encoded in the pointing matrix is obvious in the time-ordered signal component. The additional signal modulation at lower frequency due to the cross-elevation scanning of the gondola is apparent in the variation of the modulation envelope at longer time scales, as additional pixels come into view. Since the instantaneous signal-to-noise is low, the noise vector is indistinguishable from the signal+noise vector in the bottom panel.

We find the LSE of \( s \) by fitting the simulated time stream using the map fitting code. The results are shown in Fig. 5.25. The toy model sky is recovered with high fidelity.
Figure 5.23: Algorithm for constructing temperature maps.
Figure 5.24: Time-ordered data simulated from the West Scan 4 pointing vector and a toy model sky. The axis for the noise component is expanded relative to the signal component by a factor of 30.

### 5.4.4 Data fits

We apply the analysis algorithm developed in the preceding sections to the West Scan 4 channel 2 and 3 data. These channels have reasonable sensitivity and are relatively free of nonstationary signal perturbations, while this particular scan has the highest integration time per pixel of any of the CMB observations.

We deglitch the data by identifying $4\sigma$ outliers from the time-ordered data and excluding them from the analysis. Since the time domain transfer function was a poor fit to the glitch events, we simply exclude a fixed number of time constants of data following each tagged outlier. The consequence of this is higher pixel noise due to the additional rejected samples. The data cuts for each channel are presented in Table 5.8. Chopper synchronous offsets (amplitude 10-15 mK) were

<table>
<thead>
<tr>
<th>Channel</th>
<th>Glitch count</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>19685</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>27234</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Table 5.8: Data cuts for channels 2 and 3 due to deglitching.
Figure 5.25: Fits to the simulated time-ordered data of Fig. 5.24. Equation 5.45 is evaluated by performing multiplications of the explicitly constructed matrices, as well as by using the index matrix representations described in the text, for the small data set in the top panel. The results are identical. The full solution for the 65536 point data set is shown in the bottom panel. The error bars shown are the diagonal components of the map covariance matrix.
removed from the data using the periodic band-reject filter described in section 5.1.6.

Infrequent glitches in the pointing solution are a separate cause of missing data. Since the gondola attitude is a smooth, slowly varying function of time, these dropouts are simply interpolated over. The instantaneous pointing solution including the chopper deflection is then derived from this interpolated pointing vector.

The least square $\Delta T$ estimates for channels 2 and 3 are shown in Fig. 5.26. The plots suggest the presence of excess variance in the maps. We further find that the minimum variance sum of the thermodynamic temperature differences at different frequencies suggests excess variance, while the difference does not. A false color plot of the LSE for each channel is shown in Fig. 5.27.

The frequencies of observation for channels 2 and 3 yield a high ratio of CMB power to foreground power, so it is likely that a common sky signal in these channels would be largely cosmological (Fig. 3.5). In addition, the dust component for the west scans is known from composite DIRBE/IRAS maps to be low (Fig. 4.10). However, the presence of signal in only two channels rules out a useful multi-frequency fit, so attributing the excess variance to signal from the CMB must remain conjecture.

### 5.5 Conclusion

It is likely that the excess variance observed in the channel 2 and 3 data is due to temperature fluctuations in the CMB, but foreground contamination cannot be ruled out. Extending the fits to the other three west scans, as well as the north scans, would add significant sky coverage to the MSAM2 results, but since the detection in the fourth west scan was marginal, and this scan had the best per pixel sensitivity, the return would be minimal - too little in any case to yield a substantive cosmological result. We forego additional inquiry into the nonstationary effects in channels 1 and 4 for the same reason.
Figure 5.26: Least square estimate of $\Delta T$ for channels 2 and 3, West Scan 4, (pixelized at 6 minutes of arc resolution, 177 pixels total) vs. pixel index. The minimum variance sum and difference of the maps are presented in the following two panels. The bottom panel is a histogram of the samples per pixel for the scan; pixels with less than 320 hits (2σ) are omitted from the $\Delta T$ plots. Indicated errors are the 1σ diagonal components of the map covariance matrix.
Figure 5.27: Unfiltered map, West Scan 4, channels 2 and 3. The map pixel values of Fig. 5.26 are shown in two dimensions. Pixels are $6' \times 6'$; total sky coverage for the scan is $1.75^\circ$. 
Appendix A

Radiometric Fundamentals

We briefly review some standard radio astronomy results to provide a reference for the discussions of receiver design. For a more complete treatment, see Rohlfs [87] or Kraus [88].

A.1 Blackbody radiation and the Rayleigh-Jeans approximation

The CMB spectrum closely matches that of a blackbody at temperature $T=2.725$K. The brightness of a blackbody at a temperature $T$ is given by

$$B_{\nu}(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1},$$

(A.1)

where $B$ has dimension $[\text{Wm}^{-2}\text{Hz}^{-1}\text{Sr}^{-1}]$. In the long wavelength portion of the spectrum we speak of the Rayleigh-Jeans limit

$$B_{\nu}(T) = \frac{2\nu^2}{c^2}kT, \quad h\nu << kT.$$  

(A.2)

Note the proportionality between temperature and brightness in this limit. The linearity of the brightness as a function of temperature suggests the use of temperature as a brightness measure. This introduces the concept of the brightness temperature of a source; it is common to refer to the surface brightness of a source in terms of the temperature in K that would yield the measured brightness using the Rayleigh Jeans law, rather than with the units specified above.

CMB observers do not work strictly in the Rayleigh-Jeans limit. It is nevertheless still common to quote Rayleigh Jeans brightnesses, but a correction factor is required above $\nu \sim 30$GHz. To convert the Rayleigh Jeans brightness temperature to the thermodynamic temperature, we apply a correction

$$T = \frac{e^x - 1}{x}T_{\text{RJ}}, \quad x \equiv h\nu/kT,$$

(A.3)

which simply comes from relating A.1 to A.2 (see fig A.1). For anisotropy measurements, we are more concerned with the change in brightness with temperature. Taking the derivative of A.3, we
Figure A.1: Illustration of the use of the Rayleigh Jeans (RJ) temperature as a measure of brightness. When measuring the brightness of a thermal source, we can specify brightness in terms of the Rayleigh Jeans temperature at any frequency, but when the RJ approximation no longer holds ($h\nu \sim kT$), the RJ temperature will no longer correspond to the thermodynamic temperature of the source. In the example above, a brightness of 0.025 ergs/(cm² Sr s cm⁻¹) is measured at 18 cm⁻¹. A blackbody at 8K would yield this brightness at this frequency according to the Rayleigh Jeans law, but the actual thermodynamic temperature of the source is 18K.
Appendix A: Radiometric Fundamentals

find the absolute change in brightness temperature corresponding to a change in thermodynamic temperature,

\[ \delta T = \frac{(e^x - 1)^2}{x^2 e^x} \delta T_{RJ}. \]  

(A.4)

Dividing A.4 by A.3 we obtain the relative change,

\[ \frac{\delta T}{T} = \frac{e^x - 1}{xe^x} \frac{\delta T_{RJ}}{T_{RJ}}. \]  

(A.5)

It is important to keep these corrections in mind when, for example, quoting temperature fluctuations after an instrument has been calibrated from an astrophysical source of known Rayleigh Jeans brightness. Thermodynamic correction factors as a function of frequency for the 2.725 K CMB are shown in figure A.2.

Figure A.2: Thermodynamic corrections to Rayleigh-Jeans brightness temperature vs. frequency for a 2.725 K blackbody. Absolute \((T/T_{RJ})\), fluctuation \((\delta T/\delta T_{RJ})\), and fractional fluctuation \(((\delta T/T)/(\delta T_{RJ}/T_{RJ}))\) power corrections are shown.

A.2 Matched loads and the Nyquist theorem

We now consider issues involving the detection of radiated power from a thermal source. Consider the schematic representation of a radiometer in A.3. An antenna is coupled to a transmission line terminated with a resistor. If there is no impedance mismatch in the system, there is no power
Figure A.3: The relation between electrical power and temperature. An antenna observes a blackbody at temperature $T_0$. In thermal equilibrium, the net power flow is zero, so the input power from the blackbody equals the electrical power radiated out of the antenna due to thermal motion in the resistor.

reflection, the transmitted power is a maximum, and the loads are said to be matched. The power input at the antenna, in the RJ limit, is

$$P = \frac{1}{2} \frac{2kT}{\lambda^2} A_e \Omega_b \Delta \nu,$$

(A.6)

where $A_e$ is the effective aperture of the antenna, $\Omega_b$ is the solid angle subtended by the antenna beam, and $\Delta \nu$ is the bandwidth of the receiver. The prefactor of $1/2$ comes from the assumption that the antenna accepts a single polarization.

The analysis of the power per unit bandwidth in a resistor at temperature $T$ was first treated by Nyquist. We have (see Reif [89] for a derivation)

$$P = \frac{1}{4R} \langle v^2 \rangle \Delta \nu,$$

(A.7)

and the voltage noise $v$ is given by

$$\langle v^2 \rangle = 4kTR,$$

(A.8)

so the power is simply

$$P = kT \Delta \nu.$$  

(A.9)

This is the Nyquist theorem. In equilibrium A.9 must equal A.6, which requires

$$A_e \Omega_b = \lambda^2.$$  

(A.10)

---

1 We assume the thermal source completely fills the antenna beam.
Appendix A: Radiometric Fundamentals

The quantity \( A \) is called the optical throughput or the étendue\(^2\). The étendue is a fundamental quantity that is an invariant in an optical system as a result of the thermodynamic considerations above\(^3\). This scaling of \( A \) with wavelength illustrates how we come to the counterintuitive result that the power per unit bandwidth in a receiver observing a blackbody source is independent of frequency, even though the spectral power of a thermal source scales like \( \nu^2 \). The étendue scales inversely with \( \nu^2 \), exactly canceling the frequency dependence of the source. This can be understood in terms of diffractive effects at the antenna. This result was first derived by Dicke [90].

A.3 The antenna pattern and the antenna temperature

We now generalize the results above to the case where the source does not uniformly fill the beam of the antenna. Consider the output \( S \) of a receiver observing a point source. All antennae have finite beam sizes, so that

\[
S(\theta_0, \phi_0) \sim \int P(\theta, \phi) \delta(\theta_0 - \theta, \phi_0 - \phi) \, d\Omega \\
\sim P(\theta_0, \phi_0)
\]

(A.11) (A.12)

where \( P \) is the instrument beam pattern, characterizing the finite resolution of the antenna. We normalize this quantity, so that it has a peak value of 1, \( P(0) = 1 \).

The flux density of a source \( F_\nu \) is given by integrating the brightness of the source over the solid angle it subtends,

\[
F_\nu = \int_{\Omega} I_\nu \, d\Omega.
\]

(A.13)

\( F_\nu \) has dimension\(^4\) [Wm\(^{-2}\)Hz\(^{-1}\)]. Inserting this into the Rayleigh Jeans relation, it is seen that if we work in terms of the brightness temperature, the flux density has dimension [K Sr].

If we observe a source with flux density \( F_\nu \), we measure a weighted quantity that depends on the beamsizes

\[
T_A = \frac{\int I_\nu P \, d\Omega}{\int P \, d\Omega} = \frac{\int I_\nu P \, d\Omega}{\Omega_b}.
\]

(A.14) (A.15)

where \( \Omega_b \) is the solid angle of the beam pattern. This quantity is the antenna temperature. Note that it is an instrument dependent quantity, while the flux density is a characteristic of the source.

---

\(^2\)FR. “range, area, scope”.

\(^3\)We have derived the dependence of the throughput on wavelength in the case of an antenna that accepts a single mode and a single polarization. Multimode, dual polarization optical systems have correspondingly higher optical throughput, \( A \Delta_\nu \Omega_b = n\lambda^2 \), when \( n \) is the number of spatial modes that the optics couple to.

\(^4\)Or Janskys, (Jy), 1 Jy \( \equiv 10^{-26}\)Wm\(^{-2}\)Hz\(^{-1}\).

---
If we assume the source solid angle is small compared to the beam size, we may approximate the flux density distribution as

\[ F_\nu = T_s \Omega_s \delta(\theta, \phi). \]  

(A.16)

Observing this source with our beam \( P \) we measure

\[ T_A(\theta_0, \phi_0) = \frac{1}{\Omega_b} \int P(\theta_0 - \theta, \phi_0 - \phi) I_\nu \delta(\theta, \phi) \, d\Omega \] 

(A.17)

\[ = \frac{T_s \Omega_s}{\Omega_b} P(\theta_0, \phi_0). \]  

(A.18)

When the beam is centered on the source,

\[ T_A = \frac{T_s \Omega_s}{\Omega_b} \]  

(A.19)

and the antenna temperature \( T_A \) is scaled down by a factor \( \Omega_s/\Omega_b \) from the Rayleigh Jeans brightness temperature of the source. This effect is called beam dilution.

In the opposite extreme, where a uniform flux density fills the beam, A.14 simply yields\( T_A = T_s \). The source brightness temperature equals the thermodynamic temperature of the source in this case if:

- The RJ approximation applies,
- the source is a thermal source,
- and the source is optically thick.

If these conditions hold the antenna temperature and the thermodynamic temperature of the source observed are the same, recovering the result of the previous section.

### A.4 The radiometer equation, the system temperature, and the noise equivalent temperature

Consider a radiometer with predetection bandwidth \( \Delta \nu \). The bandwidth may be regarded in a sampling sense as the rate of information acquisition; a large bandwidth implies many independent data samples in Fourier space in a given amount of time. If Gaussian statistics apply, and the variance on a single sample is \( \sigma^2 \), the variance of \( N \) averaged samples is \( \sigma^2/N \). The total number of independent samples given bandwidth \( \Delta \nu \) and postdetection integration time \( \Delta \tau \) is \( \Delta \nu \Delta \tau \), so the variance on an averaged quantity in this case is \( \sigma^2/\Delta \nu \Delta \tau \), and the standard deviation is \( \sigma/\sqrt{\Delta \nu \Delta \tau} \).
Appendix A: Radiometric Fundamentals

Applying this to the error in a radiometric temperature measurement, we may write

\[ \Delta T = \frac{T_{\text{SYS}}}{\sqrt{\Delta \nu \Delta \tau}} \]  

(A.20)

where the system temperature \( T_{\text{SYS}} \) clearly parameterizes the instrument noise performance; a sensitive receiver will have a low system temperature. One imagines the output of a receiver observing an object at zero temperature: A noiseless receiver would present no power at its output terminals. In practice, no system is noiseless, and the output noise with no input power may be related to temperature as described above. This is the system temperature.

Since the bandwidth of a receiver is usually fixed and known, we may also characterize performance in a more useful way by

\[ \Delta T = \frac{T_{\text{SYS}}}{\sqrt{\Delta \nu \Delta \tau}} \]

(A.21)

\[ = \frac{\text{NET}}{\sqrt{\Delta \tau}} \]  

(A.22)

where the noise equivalent temperature (NET) has dimension (KHz\(^{-1/2}\)) or (Ks\(^{-1/2}\)). The minimum detectable temperature difference may then be quickly calculated from this quantity by dividing by the square root of the integration time. Note that the NET expressed in K Hz\(^{-1/2}\) is \( \sqrt{2} \) larger than the NET expressed in K s\(^{1/2}\), since a 1 s average is associated with 1/2 Hz of audio bandwidth (this is just power spectrum normalization given the Nyquist sampling theorem).

If the NEP of the detection system, the instrument’s optical throughput, and the instrument’s optical transmission spectrum have been determined, the NET can be related to these quantities directly. Let \( \eta(\nu), 0 < \eta < 1 \), represent the transmission spectrum of the optical chain, with the optical efficiency determining the overall scale. The optical power incident on the detector is given by

\[ P_i = \int A \Omega \eta(\nu) B_\nu(T) \, d\nu. \]  

(A.23)

Note that this is a generalization of Equation A.6, and acceptance of both polarizations is assumed.

The measured power difference due to a temperature difference \( dT \) is

\[ \frac{dP_i}{dT} = \int A \Omega \eta(\nu) \frac{dB_\nu(T)}{dT} \, d\nu. \]  

(A.24)

Recall that the NEP specifies the signal power necessary in a 1 Hz bandwidth to achieve an SNR equal to one. Using the equation above, we relate the NEP, a component (detector) figure of merit, to the NET, a system figure of merit,

\[ \text{NET} = \frac{\text{NEP}}{dP_i/dT}, \]  

(A.25)
Appendix A: Radiometric Fundamentals

\[ \frac{\text{NEP}}{\int A\Omega \eta(\nu) dB_\nu(T)/dT \, d\nu} \quad (\text{K Hz}^{-1/2}) \]  
(A.26)

\[ \frac{\text{NEP}}{\sqrt{2} \int A\Omega \eta(\nu) dB_\nu(T)/dT \, d\nu} \quad (\text{K s}^{-1/2}) \]  
(A.27)

Equation A.27 provides a concise, reliable top-to-bottom figure of merit for a CMB anisotropy experiment. Keep in mind, however, that the detector NEP itself depends on the throughput and optical efficiency because of photon noise; if the detector NEP is split into an intrinsic term \( \text{NEP}_d \) and a photon noise term (using Equation 3.7), we obtain the explicit but somewhat cumbersome

\[
\text{NET} = \sqrt{\frac{\text{NEP}_d^2 + \int 2h\nu A\Omega \varepsilon \eta(\nu) \frac{2h \nu^2 \varepsilon^2}{e^{h\nu/kT} - 1} \left[ 1 + \frac{\varepsilon \eta(\nu)}{e^{h\nu/kT} - 1} \right] \, d\nu}{\sqrt{2} \int A\Omega \eta(\nu) dB_\nu(T)/dT \, d\nu}}. \]  
(A.28)

Note that in the detector noise limited case (\( \text{NEP}_d^2 > \text{NEP}_{PHOT}^2 \)), NET scales like \( 1/A\Omega \), while in the BLIP limit NET scales like \( 1/\sqrt{A\Omega} \). This figure of merit, which alone would tend instrument design toward large \( A\Omega \), competes directly with keeping the beamsize small enough to probe angular scales of interest.
Appendix B

Derivations of Some Useful Expressions and Results

B.1 Entropy of interacting dipoles in a magnetic field

It is useful to have an analytic expression for the entropy of the paramagnetic salt commonly used in ADRs as a function of magnetic field and temperature, in order to evaluate refrigerator performance in advance, and make decisions about the $B$ field necessary to achieve a target hold time, etc. We do not provide a derivation (see [92] or [49]), but rather quote the result along with the relevant input quantities.

The electronic Landé factor is

$$g = 1 + \frac{J(J + 1) + S(S + 1) - L(L + 1)}{2J(J + 1)}, \quad (B.1)$$

where $J$ is the total angular momentum of the paramagnetic ion. The effective field on the dipoles depends on the applied field $B_a$ and the intrinsic field $B_{in}$ (the interaction term), and is given by

$$B = \sqrt{B_a^2 + B_{in}^2}. \quad (B.2)$$

Now define the intermediate quantity

$$x = \mu_B g B / kT, \quad (B.3)$$

where $\mu_B$ is the Bohr magneton, $\mu_B = 9.27 \times 10^{-24}$ J/T, and $k$ is the Boltzman constant, $k = 1.38 \times 10^{-23}$ J/K. The entropy of the system, normalized to the gas constant $R$, $R = 8.3143 \times 10^7$ ergs deg$^{-1}$ mol$^{-1}$, is then given by

$$\frac{S}{R} = \frac{x}{2} \coth \left( \frac{x}{2} \right) - (2J + 1) \frac{x}{2} \coth \left( (2J + 1) \frac{x}{2} \right) + \ln \left( \frac{\sinh((2J + 1)x/2)}{\sinh(x/2)} \right). \quad (B.4)$$
Note that the entropy depends only on the ratio of $B$ and $T$. The family of curves given by B.4 for $S(B,T)$, using the parameters for FAA, is what was used to generate Fig. 3.8. Note that departures from adiabaticity, such as those caused by Eddy current heating or internal temperature gradients, may need to be included to accurately model refrigerator performance.

### B.2 Bolometer responsivity

A bolometer acts as a transducer, converting optical power into a voltage signal. An expression describing the transfer function of this transducer, the *responsivity* in volts output per watts input, as a function of frequency, is useful for characterizing the device. In a typical application, a bolometer absorbs a modulated optical power in the presence of a fixed optical background\(^1\). The total optical power absorbed is then

$$P_{OPT} = P_0 + P_{me} e^{i\omega t}, \quad (B.5)$$

resulting in a time varying bolometer temperature with temperature amplitude $T_m$ above a baseline $T_0$

$$T_{BOLO} = T_0 + T_m e^{i\omega t}, \quad (B.6)$$

where $\omega$ is the angular modulation frequency of the optical power. The bolometer is biased with a voltage $V_{BIAS}$ through a load resistor $R_{LOAD}$, with $R_{LOAD} \gg R_{BOLO}$, so the current through the device is constant,

$$I = \frac{V_{BIAS}}{R_{LOAD} + R_{BOLO}} \sim \frac{V_{BIAS}}{R_{LOAD}}. \quad (B.7)$$

The thermistor is a resistive element subject to Joule heating - expanding $R(T)$ to first order in $T$, it dissipates a power in the bolometer

$$P_{elec} = I^2 R(T) \quad (B.8)$$

$$\quad = \left. I^2 \left( R(T_0) + \frac{dR}{dT} T_m e^{i\omega t} \right) \right|_{T_0}. \quad (B.9)$$

The bolometer cools through a weak link to a bath at temperature $T_B$. The power flow conducted through the link is

$$P_{out} = G(T_0 - T_B), \quad (B.10)$$

where $G$ is the thermal conductance as usually defined for a thermal path of cross-sectional area $A$, length $l$, and conductivity $k(T)$,

$$G = \frac{A}{l} \left( \frac{1}{T_0 - T_B} \int_{T_B}^{T_0} k(T) dT \right). \quad (B.11)$$

\(^1\)This analysis follows that of Mather [60].
The time varying component of the input optical power causes a time-varying power in the thermal link

\[ P_m = G(T_0)T_m e^{i\omega t}; \]  

(B.12)

Additionally, power is stored in the detector heat capacity \( C \)

\[ P_C = C \frac{dT}{dt} \]

(B.13)

\[ = i\omega CT_m e^{i\omega t}. \]  

(B.14)

Consider the box in Fig. 3.15 a thermodynamic control volume. The power in must then equate to the sum of the power out and the stored power; that is

\[ P_{OPT} + P_{ELEC} = P_{OUT} + P_m + P_C. \]  

(B.15)

The time independent terms in the power balance determine the biased operating point of the detector

\[ P_0 + I^2 R(T) = G(T_0 - T_B); \]  

(B.16)

an intuitively clear result. Grouping the time dependent terms in the power balance we find

\[ P_m + I^2 \frac{dR}{dT} T_m = G(T_0)T_m + i\omega CT_m \]

(B.17)

\[ \downarrow \]

\[ \frac{P_m}{T_m} = G(T_0) - I^2 \frac{dR}{dT} + i\omega C \]  

(B.19)

Note that the term in \( dR/dT \) effectively modifies the conductivity \( G \); this is termed electrothermal feedback. It is clear such an effect must exist for a thermal detector: For a current biased bolometer that, e.g., decreases resistance with increasing temperature, increasing optical power will raise the temperature, hence lowering the resistance and lowering the electrically dissipated power. Here it is convenient to introduce a quantity \( \alpha \),

\[ \alpha = \frac{1}{R} \frac{dR}{dT} \]  

(B.20)

parameterizing the temperature sensitivity of the thermistor. The modified conductivity \( \tilde{G} \) may then be written as

\[ \tilde{G} = G(T_0) - I^2 R \alpha, \]  

(B.21)

so \( \alpha \) acts as a parameter that modifies the effective conductivity of a biased thermal detector by an amount proportional to the \( I^2 R \) Joule heating in the thermistor due to the bias current. Substituting this result in B.19 we obtain

\[ \frac{P_m}{T_m} = \tilde{G} + i\omega C. \]  

(B.22)
For the monolithic silicon detectors used for MSAM2/Tophat, we have the resistance vs. temperature parameterization
\[ R = R_0 \exp \sqrt{\frac{T_0}{T}}, \]  
(B.23)
which yields \( \alpha < 0 \), indicating that the electrothermal feedback modified conductivity of Equation B.21 is larger than the steady state conductivity. This restates the example described above exactly: A modulated power input yields a smaller temperature excursion above the quiescent point than would be expected from the steady state conductivity.

We can also use the time dependent power term to determine the change in voltage across the detector due to the modulated optical power input. Note that the change in electrical power in the time varying component is
\[ P_{m,ELEC} = I^2 \frac{dR}{dT} T_m \]  
(B.24)
\[ = IV_m \]  
(B.25)
\[ \downarrow \]  
(B.26)
\[ V_m = I \frac{dR}{dT} T_m. \]  
(B.27)
The bolometer responsivity to the modulated power is
\[ S(\omega) = \frac{V_m}{P_m} \quad (V/W) \]  
(B.28)
\[ = \frac{I(dR/dT)T_m}{P_m} \]  
(B.29)
Substituting Equations B.22 and B.20 into B.29 above, we obtain,
\[ S(\omega) = \frac{IR_0}{G + i\omega C}. \]  
(B.30)
The detector time constant is modified by electrothermal feedback as well, \( \tau = C/\bar{G} \). Substituting this into the equation above,
\[ S(\omega) = \frac{IR_0}{G(1 + i\omega \tau)}. \]  
(B.31)
which, aside from the effective conductivity introduced here, is as stated in Equation 3.4.

### B.3 Maximum likelihood estimators for linear models with known Gaussian errors

Here we derive the analytic expression for the maximum likelihood estimator (MLE) for a linear model given measured data \( d \) in the matrix-vector formulation of the signal detection problem, a
result commonly quoted in CMB work but rarely derived. Any linear model may be written as

\[ d = Ps + \epsilon \]  

(B.32)

where \( d \) is a vector of measured data of dimension \( n_d \), \( s \) is a vector of parameters to be fit of dimension \( n_p \), \( P \) is a design matrix of dimension \( n_d \times n_p \) describing the measurement process, and \( \epsilon \) is a vector describing the noise, the covariance matrix of which is \( V = \langle \epsilon \epsilon^T \rangle \). In the particular case of a cosmic microwave background anisotropy measurement, this problem arises when finding the best estimate for the temperatures of some set of sky pixels \( s \), given a timestep \( d \), an observing strategy \( P \), and a noise estimate \( \epsilon \). If the instrument views a single pixel at any given time, the design matrix \( P \) takes a particularly simple form: \( P_{ij} = 1 \) if observation \( i \) falls in pixel \( j \), 0 otherwise\(^2\).

If the errors are Gaussian distributed, determining the MLE reduces to the problem of minimizing a \( \chi^2 \) variable,

\[ \chi^2 = (d - Ps)^T W (d - Ps), \]  

(B.33)

where \( W \) is the weight matrix, \( W = V^{-1} \). The value of \( s \) that minimizes the \( \chi^2 \) is called the least square estimator (LSE) of \( s \); we write this quantity as \( \hat{s} \). It can be shown [93] that the least square estimator \( \hat{s} \) has the lowest variance of any estimator linear in the data \( d \).

To derive an expression for \( \hat{s} \), we rewrite (B.33) with indices explicitly noted,

\[ \chi^2 = (d_i - P_{ik}s_k)^T W_{ij} (d_j - P_{jk}s_k), \]  

(B.34)

and summation over repeated indices implied. To minimize we require the derivative of \( \chi^2 \) w.r.t. the set of parameters \( \{s_\beta\} \),

\[ \frac{\partial \chi^2}{\partial s_\beta} = -P_{ik}\delta_{k\beta}W_{ij}(d_j - P_{jk}s_k) + (d_i - P_{ik}s_k)W_{ij}(-P_{jk}\delta_{k\beta}s_k), \]  

(B.35)

where \( \delta_{ij} \) is the Kronecker delta, \( \delta_{ij} = 1 \) if \( i = j \), and we have implicitly used the linearity of the model expressed in B.32. Note that the Kronecker delta just serves to pick out the appropriate components of the design matrix for the derivative of \( \chi^2 \) w.r.t. \( s_\beta \) being evaluated. We set the derivative to zero as usual and isolate the data and least square estimator, yielding

\[ (P_{i\beta}W_{ij} + P_{i\beta}W_{ji})d_j = (P_{i\beta}P_{jk} + P_{j\beta}P_{ik})W_{ij}s_k \]  

(B.36)

\[ = (P_{i\beta}W_{ij}P_{jk} + P_{j\beta}W_{ij}P_{ik})s_k \]  

(B.37)

\[ = (P_{i\beta}W_{ij}P_{jk} + P_{i\beta}W_{ji}P_{jk})s_k. \]  

(B.38)

\(^2\)Note that the pointing matrix is a sparse bit matrix in this case, and can be represented in a radically compressed indexed form.
where in the last line we permuted the dummy indices $ij$. The weight matrix, like the covariance matrix, is symmetric, that is $W_{ij} = W_{ji}$. Applying this to B.38, we find

$$P_{ij}W_{ij}d_j = P_{ij}W_{ij}P_{jk}s_k,$$

which is equivalent to

$$P^T_{ji}W_{ij}d_j = P^T_{ji}W_{ij}P_{jk}s_k.$$  \(\text{(B.39)}\)

The index ordering is now that of standard matrix multiplication, so we drop the indices in B.40, and we see that the LSE $\hat{s}$, given the data $d$, design matrix $P$, and weight matrix $W$, is given by

$$\hat{s} = (P^TWP)^{-1}P^T W d.$$ \(\text{(B.41)}\)

The covariance matrix for the LSE $\hat{s}$ is

$$V_{\hat{s}} = \langle (\hat{s} - s)(\hat{s} - s)^T \rangle$$  \(\text{(B.42)}\)

$$= \langle (P^TWP)^{-1}P^T WW^{-1}P^T WP (P^TWP)^{-1} \rangle$$  \(\text{(B.43)}\)

$$= (P^TWP)^{-1}P^T W \langle \epsilon \epsilon^T \rangle W P (P^TWP)^{-1},$$  \(\text{(B.44)}\)

but $\langle \epsilon \epsilon^T \rangle = V$ and $W = V^{-1}$, so B.44 reduces to

$$V_{\hat{s}} = (P^TWP)^{-1}P^T W W^{-1}WP (P^TWP)^{-1}$$  \(\text{(B.45)}\)

$$= (P^TWP)^{-1}.$$  \(\text{(B.46)}\)

Typically, the time ordered data $d$ has been processed by some some known instrumental transfer function. This convolution of the data with a transfer function is equivalent to multiplication by a matrix $H$ in the matrix-vector formulation, where $H$ is a circulant matrix that encodes the time-domain representation of the instrument’s impulse response. This modifies the time-stream model, and introduces (additional) correlations in the noise. In this case, Equation B.32 becomes

$$d = H(Ps + \epsilon)$$  \(\text{(B.47)}\)

$$= HPs + H\epsilon.$$  \(\text{(B.48)}\)

Let $\tilde{P} = HP$ and $\tilde{\epsilon} = H\epsilon$. Then

$$\tilde{W}^{-1} = \tilde{V} = \langle \tilde{\epsilon} \tilde{\epsilon}^T \rangle$$  \(\text{(B.49)}\)

$$= \langle H\epsilon \epsilon^T H^T \rangle$$  \(\text{(B.50)}\)

$$= H \langle \epsilon \epsilon^T \rangle H^T$$  \(\text{(B.51)}\)

$$= HVH^T$$  \(\text{(B.52)}\)
Appendix B: Derivations of Some Useful Expressions and Results

and Equation B.41 for the LSE becomes

\[
\hat{s} = (\tilde{P}^T \tilde{W} \tilde{P})^{-1} \tilde{P}^T \tilde{W} d
\]  \hspace{1cm} (B.53)

\[
= (P^T H^T (HVH^T)^{-1} HP)^{-1} P^T H^T (HVH^T)^{-1} d
\]  \hspace{1cm} (B.54)

Although formidable in appearance, since the matrices \(H, V, H^T\) are circulant, the evaluation of the product is equivalent to a convolution, and is easily inverted using Fourier methods.
Appendix C

Detailed Residual Plots for Planet Transits
Figure C.1: Cross-elevation transit of Jupiter: Channel 1 data, model, residuals, and off-source noise.
Figure C.2: Cross-elevation transit of Jupiter: Channel 2 data, model, residuals, and off-source noise.
Figure C.3: Cross-elevation transit of Jupiter: Channel 3 data, model, residuals, and off-source noise.
Figure C.4: Cross-elevation transit of Jupiter: Channel 4 data, model, residuals, and off-source noise.
Appendix C: Detailed Residual Plots for Planet Transits

Figure C.5: Cross-elevation transit of Jupiter: Channel 5 data, model, residuals, and off-source noise.
Appendix D

MSAM2 Telemetry Signal Dictionary

Signals contained in the MSAM2 1997 flight data archive, with the signal dimension and a brief description, are provided below.

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<th>Signal</th>
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<th>Description</th>
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<td>value</td>
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<td>V</td>
<td>BSB elec bat volts</td>
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<tr>
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<td>V</td>
<td>Chopper drive bat volts</td>
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### Appendix D: MSAM2 Telemetry Signal Dictionary

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