An Instrument and Technique for Measuring the Anisotropy in the Cosmic Microwave Background Radiation

by

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B. S., Tufts University, 1991

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Thesis

Submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy in the Department of Physics at Brown University

May 1998
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by
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1997
This dissertation by Grant W. Wilson
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Vita

Grant Wilson was born on September 4, 1969 in Worcester, MA. He has spent most of his life living in Annapolis, MD where he was an active part of the sailboat racing community. He was introduced to physics by the late Professor Hall of the Naval Academy who taught the AP Physics course at Annapolis Senior High school. His professional interests and activities are summarized below.

Physics Research Interests

- Cryogenics: development of Adiabatic Demagnetization Refrigerators
- Astrophysics and Cosmology: structure formation, the search for dark matter, studies of the Cosmic Microwave Background Radiation.
- Optics: near-infrared detectors, single mode optical systems, millimeter quasi-optics

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Teaching Experience

- 1994–1995 Tutored undergraduates and custodial staff in beginning physics
1992–1993 Gave 1 hour lectures in public classrooms around Rhode Island as a part of community outreach by the Rhode Island Space Grant program.

1993 Began the Astrophysical Journal Club to enable Brown physics faculty and students to keep abreast of current progress in the field.


Awards

1993–1996 Graduate Student Researcher Fellow with the National Aeronautics and Space Administration (NASA) Goddard Space Flight Center.

1992–1993 Rhode Island Space Grant Fellow with the NASA Rhode Island Space Grant program

1991 Teaching Assistant in the department of physics at Brown University.

1990 Elected to Tau Beta Pi - Engineering Honor Society

1988–1991 Three year varsity letter earner on Tufts Sailing Team

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Acknowledgments

Having spent far more time sailing than studying physics as an undergrad, I came to graduate school a bit more green than your average graduate student. Thankfully, Peter Timbie kindly and patiently took me under his wing and taught me how to look at physical problems objectively, methodically, and with a good sense of detachment. Thank you Peter for being such an attentive and good-natured mentor.

Thanks also to Victor and Pete from the Brown shop. Victor, I will always fondly remember the hours and hours we spent talking over mechanical ideas, finding the most clever ways to do something, and generally learning about life. I never tired of your stories. Thank you Pete for taking machine drawings and making them real. I will always be amazed by your gifts. Finally, thank you Bob Dean for teaching me things I could never have learned from a textbook.

Of course, I must also thank the other people of the MSAM collaboration — without whom none of this is possible. While Palestine, Texas is not the oasis their theme song implies, it is good to be working among friends when the buffalo gnats are biting and dinner at Ryans is inevitable. Each member brings their own gifts to the project and shares them selflessly. Ed Cheng always showed up with my ramping coffee and doled out wise advice and support when life got hard. Dave Cottingham gave me something to do after the failed ’96 launch attempt and later provided a roof over my head. The rest continue to be both co-workers and friends. I am very lucky to be part of such a wonderful group of people.

Thank you parents and my sister Lee for your strength and help. I love you all dearly.
In Memory

This thesis is written in fond memory of Professor Ed Hall of the United States Naval Academy. Ed Hall taught the AP physics class at Annapolis Senior High school with passion and vigor. He passed away late last year.

I was not Ed’s best student. On the contrary, I struggled quite hard in his class and through a lot of work and a lot of help from my friends (help which he always encouraged) I managed to make good marks despite the fact that I was not interested in continuing to study physics after high school. In class, my behavior was not quite ideal either. Since the class was at the end of the day (and dealt with difficult concepts as well), we were often quite punchy and restless. While I could not take credit for being the class clown, I occasionally donated my fair share of wise-aleck comments that had the class laughing. Ed always took them with great poise, laughing at the truly funny ones and shaking his head vehemently at the failures.

The one day of class that will remain permanently a part of my memory was early in the second year of the course. Ed was lecturing at the board as we took notes and once again I found room to interject a wisecrack. Ed stopped lecturing and as the laughter died down he slowly turned to the class. Towering above me as I sat nervously at my desk he looked down and announced in his booming yet gentile voice, “Mr. Wilson, sometimes I lie awake late at night and wonder about you.” With that he turned back and began to lecture again as I sat still and quiet with the newfound realization that I would one day be a Physics professor too.

Professor Hall, I still wonder about you.
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Chapter 1

Introduction and Theory

The Cosmic Microwave Background Radiation (CMB) is arguably the most powerful tool of cosmologists since the invention of the telescope itself. Having last scattered only $\sim 3 \times 10^5$ years after the birth of the universe, the CMB carries the imprint of a time before the first clusters, before the first galaxies, and before the first stars. The radiation is rich with information. Its very existence is a principal pillar of the Hot Big Bang cosmology and, moreover, from its spatial power spectrum we are likely to learn the mass of the universe, the age of the universe, the rate at which it expands, and the amount of non-luminous matter (dark matter) - all to within 10% of their actual values. Furthermore, the CMB will help us to answer some of the more vexing questions of cosmology: Was there an inflationary period of growth in the universe? Are there topological defects present in the universe today? What is the fate of the universe?

Each of these questions can, in principle, be answered by observations of the CMB. We are, however, taking small steps. After its discovery in 1965 by Penzias and Wilson at Bell Labs [1], it was another 27 years before the COBE/DMR satellite discovered the first traces of anisotropy (for spatial frequencies higher than the dipole) in the power spectrum of the radiation [2] at large angular scales. Soon after this discovery in 1992, a host of balloon-borne and ground-based experiments began detecting the faint signal at intermediate angular scales ($\frac{1}{2}^\circ$ and greater). Currently, the field is scrambling to acquire data with the second generation of
CHAPTER 1. INTRODUCTION AND THEORY

CMB experiments which exploit advances in detector technology and have even finer angular resolution than their predecessors. Concurrent with this effort, a new generation of “long duration” experiments are being designed and built. These experiments will use long duration ballooning to increase the available observation time by an order of magnitude over conventional ballooning. Finally, two satellite missions - the MAP satellite in 2000 and the Planck Surveyor satellite in 2005 - are now being constructed to make the first full-sky maps of CMB anisotropy with angular resolutions of $\sim 1^\circ$. The sum of these experiments, combined with small-scale ground-based instruments, promise to shed new light on age-old questions about the universe.

This work describes a second generation CMB experiment which was built as the second stage of a program to measure the angular power spectrum of the CMB by the Medium Scale Anisotropy Measurement (MSAM)/Tophat collaboration. MSAM/Tophat is an international group composed of scientists and students from the Goddard Space Flight Center under Dr. Robert Silverberg, the University of Wisconsin under Dr. Peter Timbie, the University of Chicago under Dr. Stephan Meyer, and the Danish Space Research Institute under Dr. Niels Lund. The goal of MSAM/Tophat is to measure, unambiguously, the anisotropy in the CMB at sub-degree angular scales $[3]$. To date, the collaboration has accomplished three successful balloon flights of its first CMB telescope, the MSAM I instrument, and one flight of the second generation MSAM II instrument.

Herein I describe the design and principles of the MSAM II instrument for measuring anisotropy in the CMB. After both a cursory and technical discussion of CMB anisotropy in Chapter 1, Chapter 2 gives a historical account of the collaboration’s work and an overview of the MSAM II instrument. In Chapter 3, the cryogenics are discussed, including a detailed description of the 100 mK Adiabatic Demagnetization Refrigerator used to cool the detectors. The telescope
and cryostat optical systems are described in Chapter 4 and finally, in Chapter 5 we describe a new computational technique for extracting bounds on cosmological parameters from a CMB experiment. This technique is used to place limits on a number of cosmological parameters from the three flights of the MSAM I instrument.

1.1 A Pedestrian View of the CMB and its Measurement

The Hot Big Bang theory of the early universe is beautiful in its simplicity and remarkable in its predictive powers. To date it has been responsible for predicting the primordial abundance of the elements, rationalizing the expansion of the universe, and predicting the existence of the 2.7 K blackbody radiation which permeates the universe. The Cosmic Microwave Background (CMB) is the thermal relic of the Hot Big Bang. Approximately 300,000 years after the universe was formed, its temperature dropped below the ionization temperature of hydrogen and soon after underwent a quick transition from being optically thick to being optically thin. This epoch is known as the “last scattering surface” since the mean free time between scattering of the photons went from being very short to being effectively infinite. With no scattering media, these CMB photons continued through space and time, being uniformly redshifted along the way by the expansion of the universe, and carrying information about the spectrum of density inhomogeneities at the last scattering surface in their slight differences in temperature.

Both the FIRAS experiment aboard the COBE satellite [4] and the rocket-borne experiment of Gush, Halpern, and Wishnow [5] have shown the spectrum of the CMB to be that of a near-perfect blackbody. The current best-measurement
of the temperature of the radiation gives $T_{\text{CMB}} = 2.728 \pm 0.004$ K [6]. This radiation is spatially isotropic to one part in one hundred thousand — demonstrating the global isotropy of our universe.

In spite of this overwhelming isotropy of the radiation, it is its underlying spatial anisotropy which can shed new light on structure formation and the detailed nature of our universe. At large angular scales, the COBE-DMR experiment first measured anisotropy in the CMB at a level of $\sim 20 \, \mu\text{K}$ [2]. In the five years since this monumental discovery a host of experiments probing the spatial spectrum at smaller angular scales have also detected fluctuations in the mean temperature [7, 8, 9, 10, 11].

Because the CMB photons originate at the last-scattering surface, their spatial distribution carries information about physical processes occurring at that time. As is shown in the next section, for example, in the standard cold dark matter model of structure formation, acoustic oscillations in the baryon-photon fluid at the last-scattering surface produce peaks in the radiation power spectrum at the scale of the horizon. The sizes of these peaks depend on the value of the Hubble constant and the mass-fraction of baryons in the universe. The positions of these peaks depend on the total density of the universe. Thus, by mapping the power spectrum of the fluctuations, we are able to determine the values of several cosmological parameters [12].

Figure 1.1 is a plot of most of the pre-1997 experimental results. Overlaid are predictions from a variety of different viable cosmological models. While one can see that there is a qualitative similarity between the data and the theoretical spectra, it is evident that they are inadequate to stringently test the theories.

1. The errors are too large.

2. No single experiment covers a large portion of the spectrum.

The second problem is especially troubling for those wishing to extract bounds
Figure 1.1  This figure shows the CMB anisotropy (intensity fluctuation) power spectrum for a range of popular models together with a compilation of existing measurements. Note that the plot has linear scales in both directions. The data points shown are a summary of recent measurements (see [2, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 7]) with the exception of the recently flown BAM experiment [23] which adds another point at $l \sim 76$. The filled-in dots represent confirmed measurements over the same sky coverage with two or more experiments. The matter power spectrum for standard CDM, normalized to COBE-DMR, has the wrong shape and amplitude for agreement with galaxy surveys. There are several different simple adjustments to the standard theory that can be made to fit the data ([24]). We show the $C_l$’s for standard CDM and several of these variants ([25]). The tilted model has a Hubble constant of $h = 0.45$ and a scalar power spectrum index tilted away from the scale-invariant value of $n_S = 1$ to $n_S = 0.8$. The $\Lambda$CDM model is a flat model ($\Omega_{tot} = 1$) with $\Omega_A = 0.7$, $h = 0.8$, $n_S = 0.95$ and $r = 0.23$, where $r$ is the ratio of tensor (gravity wave) to scalar contributions to $C_2$. The mixed dark matter (MDM) model has massive neutrinos contributing $\Omega_\nu = 0.2$. The open model has $\Omega_{tot} = 0.4$. For $C_l$’s from defect models see [26, 27].
on cosmological parameters. To make matters worse, each experiment has a roughly 10% error in calibration. However, there is hope. As shown in section 4.1.1, the $l$-space coverage of an experiment can be increased by minimizing the beamsize and optimizing the observing strategy. The first problem requires a combination of sensitive detectors and long observing periods. Advancements in the fields of microwave optics and detector fabrication have allowed the next generation of experiments, such as MSAM II to improve drastically on this first set of results.

The following section covers the theory of the CMB anisotropy in more technical detail and develops the mathematical tools and nomenclature to be used in the remainder of this work.

1.2 CMB Fluctuations - A Review

This section presents an overview of the cosmic microwave background radiation and its power spectrum while focusing on the effects on the radiation of the different cosmological parameters. This section is meant to be a pedagogical primer of the contemporary ideas about the CMB for those who are not already familiar with the field as well as those experimentalists who have been too busy with their experiments for the past three years to sit down and work through the most recent technical theoretical papers [12, 28, 29, 30, 31, 32]. The principal reference for this section is the work of Hu, Sugiyama, and Silk [33] which appeared as a review article in Nature. In an attempt to make the physics of the acoustical peaks in the CMB power spectrum more accessible, many of the arguments of [33] are re-iterated here in a slightly less formal language.

We begin this section with a review of what a CMB experiment measures - developing the mathematical tools and notation which will be used throughout this work. Following this introduction, in sections 1.2.2 and 1.2.3 we step back
to the early stages of our universe in order to postulate how the temperature fluctuations we measure today came to be. Finally, an example of the effects of the baryon content of the universe on the CMB power spectrum is given in section 1.2.4 in the context of an inflationary cosmology.

1.2.1 What We Measure

For the moment, take as an article of faith that the goal of our experiment is to measure the temperature distribution of a field of radiation on the sky. For this discussion, the origin of the radiation is not important, yet by going through these steps, we will develop a language for discussing the CMB in the following sections.

Each point on the sky can be represented by a temperature difference between the true temperature and the mean temperature of the sky, $\Delta T(\vec{n}) = T(\vec{n}) - T_0$, where $\vec{n}$ is the polar and azimuthal vector of the observation

$$\vec{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$  \hspace{1cm} (1.1)

and $T_0 = 2.726$ in the case of the CMB. If we were working with a flat sky, we would take the Fourier transform of this temperature distribution in order to construct its power spectrum. Instead, since we are working on the surface of a sphere, we expand $\Delta T(\vec{n})$ in spherical harmonics:

$$\Delta T(\vec{n}) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm}Y_{lm}(\vec{n}).$$  \hspace{1cm} (1.2)

For a temperature distribution, like the CMB, which is formed by a set of random fluctuations, the expansion coefficients, $a_{lm}$, are random variables. Furthermore, since the sky is a sphere with rotational invariance, their ensemble average obeys

$$\langle a_{lm}^* a_{l'm'} \rangle_{ens} \equiv C_l \delta_{ll'} \delta_{mm'}.$$  \hspace{1cm} (1.3)
Equation 1.3 is the Legendre transform of the two-point autocorrelation function and, for random fluctuations which obey Gaussian statistics, completely describes the temperature distribution. As a result, any Gaussian model can be described by the two-point correlation function $C_t$.

So in what way does a CMB experiment probe the correlation function? Unfortunately, no CMB telescope has infinite angular resolution. Consequently, we must consider the angular response function, known as the antenna pattern or beammap, of the telescope in our analysis. A CMB experiment measures a point on the sky with beammap, $B(\vec{n})$, and derives a signal, $\Delta t(\vec{n})$ given by

$$\Delta t(\vec{n}) = \int_{\text{sky}} d\Omega B(\vec{n}) \Delta T(\vec{n}).$$  

(1.4)

For an ideal experiment which measures a large number of points which are well separated on the sky\(^1\), we can construct the ensemble average of these measurements [34]

$$\langle \Delta t(\vec{n}) \Delta t(\vec{n}') \rangle_{\text{ens}} = \int d\Omega_{\eta} d\Omega_{\eta'} B(\vec{n}) B(\vec{n}') \langle \Delta T(\vec{n}) \Delta T(\vec{n}') \rangle_{\text{ens}}.$$  

(1.5)

Expanding the two-point autocorrelation function of the temperature distribution in terms of the $C_t$’s gives

$$\langle \Delta t(\vec{n}) \Delta t(\vec{n}') \rangle_{\text{ens}} = \int d\Omega_{\eta} d\Omega_{\eta'} B(\vec{n}) B(\vec{n}') \frac{1}{4\pi} \sum_{l=1}^{\infty} (2l+1) C_l P_l(\vec{n} \cdot \vec{n}')$$  

(1.6)

and moving the sum outside of the integrand we get

$$\langle \Delta t(\vec{n}) \Delta t(\vec{n}') \rangle_{\text{ens}} = \sum_{l=1}^{\infty} (2l+1) C_l W_l(\vec{n} \cdot \vec{n}')$$  

(1.7)

\(^1\) Chapter 5 describes a technique for understanding more realistic experiments.
where $W(t; \vec{\mu}, \vec{\mu}')$ is defined as the “window function” of a particular demodulation of the experiment. The window functions are functions of the telescope beam shape and observing strategy and are the spatial frequency filters of the experiment. In other words, each window function gives the sensitivity of a given observing strategy (or demodulation scheme) to the spectrum of $C_i$'s. Window functions are further discussed in section 4.1.1.

Now, knowing what the experiments are capable of measuring, we are ready to explore the theoretical side of the story and determine both the form of the fluctuation spectrum and how different cosmological parameters affect the values of the spectrum of $C_i$'s. Chapter 5 will later tie the experimental and theoretical sides together and show how to use real experimental data to extract bounds on the cosmological parameters in inflationary models.

### 1.2.2 In the Beginning

For the first $\sim 300,000$ years of its existence ($z \geq 1000$), the universe was made up of a combination of dark matter (either of the non-relativistic “cold” variety (CDM) or hot dark matter (HDM) such as massive neutrinos) and electrons and baryons. The electrons were tightly coupled to the radiation through Compton scattering and coupled to the baryons through electromagnetic interactions - leaving the baryons and radiation in thermal equilibrium. As the universe expanded with time, the radiation was red-shifted and its effective temperature dropped. In other words, as the universe aged, it both expanded and cooled.

Eventually the effective temperature of the plasma dropped enough to allow hydrogen to form. This is known as the period of “recombination” and happened around $z \sim 1000^2$. Recombination is most simply defined as the time when

\[ \text{Note that the term “recombination” is a misnomer since the universe had never been cool enough for hydrogen to form before this time.} \]
the number density of photons with energy greater than the binding energy of hydrogen (13.6 eV) drops below the level required to keep the universe ionized. Thus, the time of recombination depends on (among other things) the radiation density, $\rho_r$, and the baryon density, $\rho_B$, and can be expected to have happened over a finite (and non-zero) amount of time. After recombination, in the absence of free electrons the radiation and the matter were effectively decoupled and the optical depth of the universe became nearly infinite. Since the radiation did not scatter again after this time, we call this epoch the “last scattering surface.” Thus began the long voyage of the CMB photons towards our waiting telescopes.

Despite the overwhelming isotropy and homogeneity of our universe (which is reflected in the temperature distribution of the CMB on the sky today), we know that at the surface of last scattering there must have existed the seeds of the density inhomogeneities which led to the later formation of structure. It is in the description of these “seeds” where much of the debate in cosmology arises today. Inflationary cosmologies predict a spectrum of energy-density perturbations which come from quantum mechanical fluctuations in the energy-density distribution in the very early universe. These perturbations were stretched to macroscopic scales by inflation [35, 36]. On the other hand, there are a variety of models based on cosmic “defects” which result from phase transitions in the early universe [37, 38, 39, 26]. For the purposes of this work, we only consider cosmological models which stem from inflation. While cosmological defect models are still viable, because of their complexity the theoretical progress in calculating accurate power spectra quickly is lagging that of the inflationary models. For the analytical purposes of this work, only inflationary models will be discussed.

Inflation predicts the existence of a spectrum of density perturbations of all wavelengths. Perturbations with wavelength larger than the Hubble radius are frozen in amplitude until entering the Hubble radius. While these perturbations
carry energy density and pressure, causality prevents microphysics from influencing them. Fluctuations with wavelengths inside the Hubble radius, on the other hand, are free to act (and react) on the matter-radiation fluid. We study these fluctuations in the spatial-frequency domain where a normal mode analysis allows us to examine the system separately at each spatial frequency $k$.

1.2.3 Acoustical Fluctuations at Recombination

Prior to recombination, the evolution of a temperature perturbation, $\Theta = \Delta T / T$, is governed by the Boltzmann equation with a source from Compton scattering off the free electrons. The baryons, on the other hand, evolve like a fluid under the continuity and Euler equations of motion. Together, this system of coupled equations fully describes the evolution of the CMB fluctuation spectrum [32]. We can gain valuable insight into how changing various cosmological parameters affects the fluctuation spectrum by exploring the form of these equations in the limit where the photon-baryon fluid is in tight thermal equilibrium. This is known as the “tight coupling” limit. In this case, the evolution of each spatial frequency of fluctuations, $\Theta$, becomes that of a simple harmonic oscillator

$$m_{\text{eff}} \ddot{\Theta} + k^2 c^2 \Theta / 3 \approx m_{\text{eff}} g$$

(1.8)

where derivatives are taken with respect to conformal time, $\eta$. Here, the effective mass of the oscillator is $m_{\text{eff}} = 1 + R$ where $R$ is the scale factor normalized to $3/4$ at baryon-photon equality ($R = \frac{3\eta \rho}{\eta \gamma}$). The effective acceleration of the system, $g$, comes from the standard Newtonian gravitational potential, $\Psi$, and the resulting perturbation of the spatial curvature, $\Phi$,

$$g \approx -k^2 c^2 \Psi / 3 - \ddot{\Phi}.$$ (1.9)
Physically, what equation 1.8 represents is this: in a region of the universe where there is an over density of matter (baryonic or dark), the baryons begin to tumble into the gravitational potential well. This gravitational infall compresses the baryon-photon fluid and causes the local temperature to rise. Eventually, the force from the increasing photon pressure is enough to reverse the motion, lowering the effective temperature until the baryons begin to recoil a second time and the process repeats. At last scattering, the radiation decouples from the matter fluid and is imprinted with the phase of the oscillation. Since different modes of the fluctuation spectrum enter the Hubble radius at different times, the corresponding modes of temperature fluctuations exit the last scattering surface at different points in the oscillation.

The Simplest Model

To understand what is happening to the temperature perturbation during this time (as well as the effects of different cosmological parameters), it is useful to start out with a series of approximations and then slowly relax them to see their effects. First of all, we begin by considering only the potential associated with the Newtonian gravity (i.e., we assume $\Phi = 0$). Also, we begin by assuming that the baryons do not contribute significantly to the effective oscillator mass, that is, $R \rightarrow 0$. Finally, we need to choose initial conditions for our oscillator at the point where the fluctuation enters the Hubble radius. Inflationary cosmologies stipulate that $\Theta(0) = -\frac{2}{3} \Phi$ and $\dot{\Theta}(0) = 0$ [40].

The first assumption leaves us with a constant gravitational force

$$g \simeq -k^2c^2\Psi/3$$

(1.10)

which moves the zero-point of the oscillator to $\Theta = -\Psi$. That is, the temperature fluctuations oscillate around a non-zero value given by the gravitational potential
Ψ. In the approximation where $\rho_\gamma \gg \rho_B$ ($R \to 0$), the photons leaving the surface of last scattering suffer an additional redshift equal to $\Psi$ when climbing out of the gravitational potential. This effect exactly cancels the effect of the displaced zero point of the oscillator. The equation of the motion of the oscillator (with the adiabatic initial conditions) is then

$$\Theta + \Psi = \frac{1}{3} \Psi \cos ks$$

(1.11)

where $s$ is the sound horizon$^3$. At recombination the fluctuation spectrum is frozen so that the post-recombination temperature distribution is

$$\Theta = \frac{1}{3} \Psi \cos ks_*$$

(1.12)

where $s_*$ is the sound horizon at last scattering.

The sound horizon at last scattering thus provides a scale over which the fluctuations evolve. For wavelengths greater than $k_A = \pi/s_*$, oscillations do not set up and the fluctuations exit the last scattering surface with $\Theta = -\frac{2}{3} \Psi + \Psi = \frac{1}{3} \Psi$. The first term is the fluctuation from the initial conditions and the second is from the redshift as the photon leaves the last scattering surface. Fluctuations on scales less than $k_A$ oscillate as described above and are “frozen” at different points in the oscillation at last scattering. The net result is a harmonic series of peaks with the $m$th peak located at $k_m = m\pi/s_*$. Odd peaks represent the compression phase (temperature crests) and even peaks represent the rarefaction phase (temperature troughs) of the oscillating fluid.

Here we see the most basic dependence of the CMB fluctuation spectrum on the cosmological parameters. Under this simple model, we expect to see peaks in the power spectrum which are separated in spatial frequency by $\pi/s_*$. The

---

$^3 s = \int c_s d\eta$ where $c_s$ is the sound velocity in the fluid.
peak size is determined by the value of \( \Psi \) which depends on the baryon content of the universe, \( \Omega_B h^2 \), and the first peak position depends on the size of the sound horizon at last scattering which depends on the total density of the universe, \( \Omega_0 h^2 \), and \( \Omega_B h^2 \) where the Hubble constant is \( H_0 = 100 h \) km/s/Mpc. Here we begin to see the great information content of the CMB.

Adding Baryons

Let us now relax the assumption that the effective mass in the oscillator is dominated by the radiation and allow significant portions of baryons to be present. Since more baryons implies a larger gravitational potential, this further shifts the zero-point of the oscillator. This effect is known as “baryon drag” and because the additional shift is not compensated by the redshift of the photons as they leave the last scattering surface, the peaks associated with compression have a greater amplitude than those associated with rarefaction. Thus, by measuring the difference in peak heights, we can probe the baryon content of the universe, \( \Omega_b h^2 \), through the effective mass of the oscillator.

The Doppler Effect

The line-of-sight velocity of the fluid on the surface of last scattering with respect to the observer results in a Doppler shift of the radiation coming from the fluid. For the oscillating fluid, the velocity is 90° out of phase with the density - leading to a Doppler effect induced temperature variation which is 90° out of phase with the acoustic oscillation component (i.e., the fluctuations go like \( \sin ks \)) but with a smaller amplitude. This has the effect of filling in the vacant power between the adjacent acoustical peaks.
Photon Diffusion

Since recombination is not instantaneous, we must consider the effect of the gradually decreasing number of free electrons on the fluctuation spectrum. Fluctuations with wavelength shorter than the diffusion length, \(\lambda_D \sim \sqrt{c\eta\lambda_C}\) where \(\lambda_C\) is the mean free path to Compton scattering, are damped exponentially. As a result, the acoustic oscillations on scales smaller than the diffusion length are wiped out early on. As the universe recombines and the ionization fraction decreases, the diffusion length increases and fluctuations at larger and larger scales are smoothed. If recombination is sufficiently delayed (or never happens at all), nearly all the acoustical peaks can be destroyed.

Projecting the Fluctuation Spectrum to the Present

The angular scale which a particular fluctuation on the last scattering surface subtends depends on the geometry of the universe and the distance to the last scattering surface. For a closed universe \((\Omega_0 + \Omega_\Lambda > 1)\), the photons stream to the observer along geodesics analogous to lines of longitude on a globe. As a result, fluctuations at the last scattering surface subtends a larger angular scale in a closed universe than in a critical (flat) universe. The converse is true for an open universe. On the other hand, increasing the distance from the present to the last scattering surface decreases the angular scale of fluctuations at recombination. The distance to the last scattering surface depends on \(\Omega_0, \Omega_\Lambda,\) and \(H_0\). As a result, determining the locations and spacing of the acoustic peaks provides insight into the geometry of the universe as well as these physical parameters.

Secondary Anisotropies

During the time between recombination and the present, the CMB power spectrum is further altered by both the changing gravitational potentials due to structure
formation as well as the reionization of the universe. Since these effects are related to structure formation rather than the physics of recombination, they are known as “secondary” anisotropies. A review of the causes of these secondary anisotropies can be found in [33].

1.2.4 The Power Spectrum of Fluctuations

In order to have a base to work from, theorists have defined a particular set of parameter values as “standard” cold dark matter (SCDM) parameters. These parameters, and their SCDM values, are given in table 1.1. While these values do not create a universe that is strictly consistent with observations, they do form a starting point for searches in parameter space. A nice review of each of their effect on the anisotropy spectrum can be found in Jungman et al. [12].

Table 1.1 Standard cold dark matter (SCDM) model parameters [12]. \(Q_{\text{COBE}} = 20 \, \text{\mu K} \) is the COBE normalization.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>SCDM Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Density</td>
<td>(\Omega_0)</td>
<td>1</td>
</tr>
<tr>
<td>Baryon Density</td>
<td>(\Omega_B)</td>
<td>.04</td>
</tr>
<tr>
<td>Hubble Constant</td>
<td>(H_0)</td>
<td>50</td>
</tr>
<tr>
<td>Cosmological Constant</td>
<td>(\Lambda)</td>
<td>0</td>
</tr>
<tr>
<td>Scalar Spectral Tilt</td>
<td>(n_S)</td>
<td>1</td>
</tr>
<tr>
<td>Ratio of Tensor to Scalar Contributions to the Quadrupole</td>
<td>(r)</td>
<td>0</td>
</tr>
<tr>
<td>Tensor Spectral Tilt</td>
<td>(n_T)</td>
<td>0</td>
</tr>
<tr>
<td>Running of Spectral Index</td>
<td>(\alpha)</td>
<td>0</td>
</tr>
<tr>
<td>Optical Depth of Reionization</td>
<td>(\tau_{\text{reion}})</td>
<td>0</td>
</tr>
<tr>
<td>Quadrupole</td>
<td>(Q)</td>
<td>(Q_{\text{COBE}})</td>
</tr>
<tr>
<td>Number of Neutrino Flavors</td>
<td>(N_\nu)</td>
<td>3</td>
</tr>
</tbody>
</table>

We conclude this introduction to CMB physics with an illustration of the effects on the power spectrum, as an example, of the baryon fraction, \(\Omega_B\). Figure 1.2 shows the power spectrum of inflationary fluctuations for three cosmologies.
Figure 1.2  Three correlation functions with different values of $\Omega_B$. All other parameters are held fixed at their SCDM values. These correlation functions were computed using the CMBFAST [25] code.

with different values of $\Omega_B$. All other cosmological parameters are held at their SCDM values. These correlation functions were computed using the CMBFAST program [25].

Three distinct areas of the spectra are marked in figure 1.2 and serve to illustrate the physics discussed above. In section I we see the dependence on the size of the first acoustical peak (a compression peak) on $\Omega_B$. In our simplest model, the peak size depends on the amplitude of the Newtonian gravitational potential, $\Psi$, which in turn depends on the baryon content. Therefore, greater $\Omega_B$ implies a larger first peak. Section II of the plot shows the relative change in heights between the first and second peaks as a function of the baryon content. Recall that the first peak results from a compressional phase of the photon-baryon fluid whereas the second peak comes from a rarefaction phase. As is shown in the power spectra, increasing the baryon-drag (by adding more baryons) boosts the fluctuation amplitude in the compressional phases with respect to the rarefaction
phases. As a result, for larger $\Omega_B$, the first and third peaks are larger with respect to the second. Finally, the effect of photon diffusion in shown in section III where in all cases the third acoustical peak is suppressed compared to the first. For smaller values of $\Omega_B$, the mean free path to Compton scattering increases - reducing the photon diffusion.

1.2.5 Conclusion

Inflationary cosmologies predict a spectrum of CMB fluctuations which, for the primary anisotropies, are understandable in terms of the harmonic oscillation of the photon-matter fluid at last scattering. Given a set of values for the cosmological parameters, we now have the ability to calculate the power spectrum of fluctuations to an accuracy of 10% [25]. While this is only true for the Gaussian fluctuations predicted by inflationary cosmologies, work in the theoretical community continues in the hopes of being able to rapidly compute correlation functions for cosmological defect models as well.

Concurrently with this theoretical effort, experimentalists are struggling to measure the fluctuation spectrum. The following chapters in this work describe the instrumentation of a recent effort, the MSAM II telescope, which will potentially probe the fluctuation spectrum out past the first acoustic peak. The ability to extract cosmological information for this, and other, experiments is not yet clear. While the last section shows a very obvious variation in the power spectrum for a single parameter, we must remember that we are dealing with a multi-dimensional parameter space where there is much degeneracy in the predictions. MSAM II will be an important step in the search for knowledge from the CMB but it will require an experiment which can reach out to the third acoustical peak for us have hope of breaking the degeneracies.
Chapter 2

MSAM II — Telescope and Flight Operations

2.1 History of the MSAM Collaboration

The MSAM Telescope first flew in 1992 using a multi-mode, bolometric, four channel radiometer constructed at the Massachusetts Institute of Technology [7]. This instrument had sensitivities of 810 and 1190 $\mu K \sqrt{s}$ (single difference) CMB in channels 1 and 2 with an effective beamsize of 30' (FWHM). MSAM I enjoys the distinction of being the first sub-degree CMB experiment to make a positive and unambiguous detection of anisotropy - reporting fluctuations at the level of $\Delta T/T \sim 10\mu K$ . Observations of the same patches of sky during the second flight of the instrument in 1994 subsequently confirmed the initial detection of anisotropy [41] shortly before an independent confirmation by the ground-based Saskatoon experiment [16]. A final flight of MSAM I in 1995 doubled the effective sky coverage of the instrument and resulted in yet another detection of anisotropy at sub-degree angular scales [42]. Chapter 5 describes a technique used to combine the results of these three data sets to place bounds on the values of several cosmological parameters.

The ground-breaking discoveries of anisotropy by a host of experiments at the beginning of this decade [7, 8, 9, 10, 43] laid the groundwork for the next generation of CMB experiments. No longer was the goal of the experimentalists...
to “discover” anisotropy, rather they were challenged with the task of “measuring” the fluctuations with a sensitivity and resolution that would enable discrimination of cosmological theories. This was the presiding thinking behind the design and construction of the MSAM II radiometer. At Brown University, novel monolithic silicon bolometers, built at the Goddard Space Flight Center, were designed and tested. A cryogenic system was constructed to ensure a 0.1 K bath temperature for the bolometers during the balloon flight, and the beam size of the MSAM telescope was reduced by 33% to 20’ (FWHM) to provide sensitivity to the sky’s correlation function out past the first acoustic peak (l ~ 300). This radiometer was completed in the winter of 1995 and integrated with the MSAM telescope the following spring. The following summer, the package was taken to Palestine, Texas for launch at the National Scientific Balloon Facility. Unfortunately, summer-long low-level winds prevented the package from launching in 1996 and the long awaited flight of MSAM II was put off until its maiden voyage on 1 June 1997.

2.2 Ballooning

We perform our observations of the CMB from a balloon-borne platform in order to avoid the relatively opaque atmosphere of the earth. While not all CMB experiments are done from balloons [44], nearly a factor of five improvement in total system sensitivity can be achieved through the reduction in atmospheric noise at balloon altitudes. The problem is not only that the atmosphere is relatively opaque at microwave frequencies, it is also changing on variable time and spatial scales. This makes for a laborious task of removing possible atmospheric contamination. We choose, instead, the laborious task of putting our telescope “above” most of the atmosphere - reducing the possible systematic contamination and increasing the sensitivity of our radiometer.

The atmospheric emission at microwave frequencies comes from a forest of lines
Figure 2.1  The atmospheric brightness at millimeter wavelengths. This figure shows the zenith atmospheric emission at altitudes of 0 km (ground based observations), 1.4 km (mountain top observations), 14 km (observations from planes) and 44 km (balloon-borne observations). The data for this figure is taken from [45]. See [45] for a description of the atmospheric parameters which go into each model.

due to O$_2$, H$_2$O, and O$_3$ [45]. Other atmospheric constituents such as aerosols, CO, N$_2$O, and OH are assumed to be negligible. O$_2$ makes up 23% of the atmospheric mass up to 60 km, H$_2$O column densities are highly variable below 14 km, and O$_3$ is concentrated in the lower stratosphere at about 30 km. The O$_3$ densities vary with season and latitude [45]. Figure 2.1 (copied from [45]) shows the atmospheric brightness as a function of altitude for heights of 0, 1.4, 14, and 44 km. Shown for comparison are the Planck curves for 300 and 2.7 K blackbodies. It is clear why we go to the trouble of observing at balloon altitudes.

High altitude ($\geq$27 km) balloon flights are launched in the United States from Palestine, Texas in the summer and Fort Sumner, New Mexico in the fall and spring. Launch times are regulated by the predominant direction and speed of
the upper atmospheric winds which tend to travel due west during the summer with a short turn-around period in the late fall. During the winter and spring the upper-atmosphere winds blow predominantly due east and there is a second turn-around in the late spring (generally during mid-May). This high altitude ballooning is supported by the National Scientific Ballooning Facility (NSBF) which is run by NASA and the New Mexico Physical Sciences Laboratory.

The MSAM experiments traditionally fly from Palestine, TX and require a 40 million cubic foot \((1.1 \times 10^6 \text{ m}^3)\) balloon to reach the desired float altitude of 36 km. The geometry of the package and balloon are shown in figure 2.2. The 2300 kg balloon, filled with 720 kg of He gas, inflates to a roughly ellipsoidal shape which is 121 m high and 141 m wide. The 200 kg flight train is composed of a ladder line in series with a parachute. The gondola weighs 1880 kg and hangs from the elongated parachute.

Flight times for MSAM have ranged from 9.4 hours in ‘95 to 10.5 hours for the most recent MSAM II flight. Table 2.1 shows a breakdown of the total flight time for the ‘97 MSAM II flight. CMB observations account for 45% of the total flight time while detector and sensor calibrations take up \(\frac{1}{3}\) of the flight. This time spent calibrating the instrument is crucial for understanding the detector response as well as other systematic effects.

**Table 2.1** A breakdown of the ‘97 MSAM II flight by task.

<table>
<thead>
<tr>
<th>Task</th>
<th>Time [minutes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ascent</td>
<td>95.5</td>
</tr>
<tr>
<td>Calibration of Attitude Sensors</td>
<td>35</td>
</tr>
<tr>
<td>Diagnostic Tests</td>
<td>17</td>
</tr>
<tr>
<td>Observations of Mars</td>
<td>29</td>
</tr>
<tr>
<td>Observations of Jupiter</td>
<td>100</td>
</tr>
<tr>
<td>CMB Observations</td>
<td>282</td>
</tr>
<tr>
<td>Observations of Saturn</td>
<td>12</td>
</tr>
<tr>
<td>Observations of Moon</td>
<td>16</td>
</tr>
</tbody>
</table>
Figure 2.2  A scale drawing of the MSAM II gondola and balloon. Rays from the Earth which scatter from the balloon onto the telescope come from zenith angles of 23° or less. Balloon emission comes from zenith angles of 27° or less. This figure was prepared by Jason Puchalla.
Balooning offers a convenient and stable platform above most of the earth’s atmosphere at a small fraction of the cost of a satellite experiment. The limited observing time on a balloon is its chief drawback; however, future CMB experiments are planning on taking advantage of new long-duration capabilities of the ballooning community. Recently it has become possible to have a two week flight from Antarctica and by the end of the century, 100-day balloon flights from a variety of launch sites will begin to be available.

2.3 Overview of Instrument Components

The Gondola

The MSAM II telescope is an off-axis Cassegrain with a 1.3 meter parabolic primary mirror. The gondola has been used a number of times before for MSAM I flights as well as with a different telescope and radiometer to map the Galactic plane [46]. Figure 2.3 shows the gondola and its principal components configured for the MSAM I flights. For the ’94 and ’95 flights of MSAM I and for the ’97 MSAM II flight, the rigid support structure at the top of the gondola (the shaded section of the figure) was replaced with a set of flexible steel cables and the “jitter bar” was replaced with a jitter motor. In addition, the battery boxes were moved to the back shelf with the NSBF CIPs (communication electronics) to move them further away from the microwave beam.

With the exception of the new radiometer and support electronics, the gondola and its motion are thoroughly described by Fixsen in [47]. In this work I will focus on the new radiometer for MSAM II which was designed, built and tested at Brown University.
Figure 2.3 The MSAM gondola.
The Radiometer

The MSAM II radiometer is a single mode five-channel microwave receiver operating between 65 and 170 GHz. The detectors are monolithic silicon bolometers designed at Brown and fabricated at the Goddard Space Flight Center. These detectors are cooled by an Adiabatic Demagnetization Refrigerator (ADR) supported by a pumped liquid helium bath and two liquid nitrogen tanks. The bolometers are fed by two independent optical systems. The low frequency optical chain passes microwaves in the horizontal polarization to three separate bolometers at 65-80 GHz, 80-95 GHz, and 95-110 GHz respectively (channels 1–3). The high frequency chain feeds two other bolometers in the frequency bands 130-150 GHz and 150-170 GHz (channels 4 and 5) in the vertical polarization. A sixth bolometer is blanked off to the incoming radiation and acts as a dark channel (channel 0).

The radiometer electronics are designed to be low-noise and non-intrusive to the sensitive detectors. The detector signals are given a gain of 20 by 100 K JFET amplifiers whose signals are further amplified by low-noise 300 K preamplifiers. All radiometer electronics are housed in RF-tight boxes which are mounted to the cryostat feedthroughs. These boxes provide amplification of the signals and do some conditioning before passing the signals on to the principal gondola electronics box (BSB). All signals moving between the radiometer and the BSB are passed differentially to avoid ground loops.

Since we were concerned about radio frequency interference (RFI) of the detectors, all signals entering the support electronics boxes pass through large pi filters with greater than 80 dB of attenuation at frequencies lower than 100 MHz. During the ‘97 campaign in Palestine we tested the broadband RF pickup of the system by asking a welder to draw a series of sparks near the radiometer. No evidence of the sparks was seen by the bolometers implying that the system is RF
tight.

Figure 2.4 is a cut-away view of the MSAM II radiometer and its sub-systems: the “warm” cryogenics, the “cold” cryogenics (the ADR), the bolometer mount and band-defining optics, and the cold/internal optics. Each of these components is described in detail in this work.

2.4 The Bolometers

Bolometers are thermal detectors which measure power from electromagnetic radiation using a temperature sensitive thermistor. When cooled below 1 K, bolometers are the most sensitive broad-band detectors available from millimeter to sub-millimeter wavelengths [48]. In conjunction with a group at the Goddard Space Flight Center, we have developed 100 mK monolithic Si bolometers for the MSAM II experiment. In this chapter I will briefly discuss their design, optimization, and performance. A much more detailed account of these bolometers can be found in [49]. A more comprehensive review of bolometer theory and practice can be found in Zhou et al. [50], on which much of this section is based.

2.4.1 Bolometers: The General Idea

A bolometer is a resistance thermometer (thermistor) which is coupled to a cold thermal bath by a weak link of thermal conductivity, $G$, and strongly coupled to an incoming power source via an absorber. The power, $Q$, which is absorbed by the bolometer causes its temperature to rise. This change in temperature is detected in a change in the resistance, $R$, of the thermistor. Figure 2.5 shows the principal elements of a bolometer with the corresponding noise sources (discussed below) given in parenthesis.

The noise in a bolometric measurement comes from the power landing on the
Figure 2.4  This figure shows the radiometer and its sub-systems. Each component is described in detail in the text. Signal conditioning boxes are omitted for clarity.
Figure 2.5  Schematic diagram of a bolometer. The electrical and thermal elements of the bolometer are shown with the corresponding noise sources given in parenthesis. Power, $Q$, incident on the bolometer absorber deposits energy and heats the detector with heat capacity $C$ and thermal conduction, $G$. The temperature change is detected through the change in resistance, $R(T)$, of the thermistor.
bolometer, the intrinsic noise of the detector materials, and the detector readout electronics. The “photon noise” is caused by statistical variations in the background loading of the detector. Intrinsic to the bolometer are the Johnson noise, resulting from electron shot noise in the thermistor, and phonon noise from thermal fluctuations in the thermal link. Finally, the readout electronics, if designed with care, contribute only a small amount of amplifier noise and can usually be neglected.

We define the Noise Equivalent Power (NEP) of a detector as the smallest amount of power a device can detect with a signal-to-noise of one and a one hertz post-detection bandwidth. The units are \(W/\sqrt{Hz}\). To put everything on an even footing we can express the noise terms above in terms of power and define a total NEP for the system as

\[
NEP_{\text{bolo}} = \sqrt{NEP_{\text{photon}}^2 + NEP_{\text{phonon}}^2 + NEP_{\text{Johnson}}^2}.
\]  

(2.1)

The NEPs all add in quadrature since the noise sources are uncorrelated [51]. Furthermore, note that even for an “ideal” detector with no phonon and Johnson noise, the NEP of the detector is limited by the external photon noise. This limit is the Background Limited Infrared Photon (BLIP) limit and is the design goal of bolometer builders.

Finally, the time constant of the bolometer is determined by the conductivity of the thermal link, \(G\), and the heat capacity, \(C\), as

\[
\tau = C/G.
\]

(2.2)

Since during observations of the CMB we would like to chop between points on the sky as fast as possible in order to bring the signal level above the \(1/f\) noise of the detector system, we desire a detector with a short time constant. In addition, the
shorter the time constant of the detector, the less data is lost during the removal of cosmic ray events during flight.

2.4.2 Monolithic Silicon Bolometers

The MSAM II monolithic Si bolometers, introduced by Downey et al. [52], are fabricated at the Goddard Space Flight Center [53]. The bolometer is made up of a micromachined thin Si film which is suspended from a Si frame by Si legs which also function as the thermal link to the temperature reservoir. What makes this bolometer “monolithic” is that the thermistor is ion-implanted in the Si film. The radiation absorbing surface is a layer of Bi which is evaporated onto the Si film. In this way, the thermistor, the absorber, and the thermal link can be optimized independently.

Waveguide Coupling

The waveguide-to-bolometer coupling scheme is similar to that introduced by Peterson and Goldman [54]. In this design, a 100 nm Bi film is deposited on a narrow Si substrate which is oriented along the E-plane of the waveguide. The absorber is backed by an adjustable backshort. The thermistor is located outside of the waveguide cavity and linked to the absorber via the Si membrane which passes through a hole in the waveguide walls (see figure 2.6). Since the thermal contraction of the Si bolometer frame is much smaller than that of the Al waveguide mount, the frame is glued to a small piece of Invar which has been press-fit into the mount. The proper Bi thickness for maximum absorption was found by first making reflection measurements on a scale model of the bolometer between 8 and 12 GHz. The finished bolometers were then tested at 4.2 K and the backshorts were tuned to minimize the reflection. We found coupling efficiencies \( \geq 90\% \) over the waveguide band for these devices.
Figure 2.6  Top open-faced view of the MSAM II monolithic silicon bolometer. (A) is the outer Si frame which is sunk to 100 mK. (B) is the thermistor and absorber. The thermistor is just above the end of the arrow while the absorber hangs over the open waveguide (C). The thermistor and absorber are suspended by legs (D), the upper two of which are ion-implanted to carry the signal from the thermistor. This signal is sent to the JFET amplifiers via the contact pads (E).
2.4.3 Design and Fabrication

The electrical and thermal properties of the bolometer are characterized by the operating temperature, the thermal sensitivity of the thermistor, the thermal conductance of the legs, the heat capacity of the detector, and the optical loading and modulation frequency. Bolometer optimization, including the non-equilibrium noise analysis of Mather [51], is described by Griffin and Holland [55]. For these bolometers, the behavior of the resistance with temperature of the thermistor is well described by

\[ R = R_0 e^{\sqrt{\frac{T_0}{T}}} \]  \hspace{1cm} (2.3)

\( R_0 \) is mainly given by the geometry of the thermistor and is typically around 0.5–2 kΩ. \( T_0 \) is extremely sensitive to the Si doping density and is chosen to have a very large \( \delta R/\delta T \) at the operating temperature, \( T \), while keeping \( R \) at a reasonable impedance (\( \leq 10^7 \Omega \)). The design goal for MSAM II was to have \( T_0 \sim 10 \) K. The heat capacity of the bolometer is dominated by the thermistor and goes as a geometry factor times the temperature. The thermal conductance from the heat reservoir to the thermistor can be described by

\[ G = G_0 T^3 \]  \hspace{1cm} (2.4)

where \( G_0 \) is mainly determined by the leg geometry.

Since there are run-to-run variations in the Si doping density, we choose our flight detectors from a large batch of bolometer candidates which come out of the processing. The cut is based on measurements of \( R_0 \) and \( T_0 \). The value of \( G \) is controlled by the manufacturing process and does not vary widely from device to device. Once the flight detectors have been chosen, the Bi is deposited on the unimplanted region of the Si substrate to act as the absorber. This Bi layer is
then covered by a thin layer of SiO to inhibit aging effects\textsuperscript{1}.

Jun-wei Zhou performed the optimization of our monolithic silicon bolometers based on an estimation of the background loading under flight conditions. For example, the expected in-flight loading of the channel 3 bolometer (95–110 GHz) comes from the 2.7 K CMB, the atmosphere, and emission from the optics. The total power on the bolometer is 0.3 pW and the photon noise is $6.6 \times 10^{-18}$ W/$\sqrt{\text{Hz}}$ (assuming a 30% coupling efficiency for the optical system). The optimal parameters for a 100 mK bolometer in these conditions are: $T_0 = 10$ K, $R_0 = 2000\Omega$, and $G = 2 \times 10^{-11}$ W/K. These parameters, along with the heat capacity of the detector, combine to give a receiver sensitivity\textsuperscript{2} of 160 $\mu$K$\sqrt{s}$ and a time constant of 3 ms.

### 2.4.4 Bolometer Readout and Performance

The bolometer signals are amplified by Si JFET amplifiers (2N6451) \[56\]. The FETs are placed 10 cm from the bolometer box in order to minimize the capacitive loading of the high-impedance bolometers as well as electromagnetic interference from the long wires to the 300 K amplifiers. The JFETs are mounted in a box near the bolometers and are temperature regulated to 110 K \[57\]. The flight JFET amplifiers are required to have voltage noise less than the intrinsic noise of the bolometer (that is, the sum of the Johnson noise, photon noise, and phonon noise). For our case, this implies that all JFETs must have voltage noises less than 5 nV/$\sqrt{\text{Hz}}$ and current noises less than $2 \times 10^{-16}$ A/$\sqrt{\text{Hz}}$. This condition was met by approximately 10% of the JFETs tested.

With a gain of 20 from the JFET amplifiers, the bolometer signal goes directly into the Signal Conditioning Box which is housed on the side of the cryostat. A

\textsuperscript{1}The effective impedance of the Bi absorber will change if exposed to air. A thin layer of SiO on top of the Bi has been found to retard this effect.

\textsuperscript{2}This is the sensitivity to the CMB spectrum for a single difference experiment.
preamplifier stage in this box gives a gain of 1000 to the signal before it is sent to be amplified again and then digitized by a final stage of amplifiers and electronics in the main gondola computer. The postamps filter the bolometer signal with an elliptical low-pass filter which cuts off at 80 Hz which is chosen to be the Nyquist frequency of the sampling rate.

We conclude this section with table 2.2 which gives the expected in-flight sensitivity for the five channels along with the expected noise contributions. The optical efficiency of the system is the fraction of the radiation entering the telescope that is absorbed by the bolometer and measurements of the optical efficiency of the system are described in Chapter 4. Variations in the predicted detector sensitivity arise from the differences in optical loading, the optical efficiency, and differences in $R_0$ and $T_0$ of the detectors. In fact, during the '97 flight the bolometers were not as sensitive as listed here. During the flight we measured excess loading on the detectors which did not show up in the dark channel (channel 0). As a result, the detectors were approximately a factor of four less sensitive than the calculated sensitivities given in table 2.2. We are currently searching for the cause of this problem and expect to have it solved for the '98 flight of MSAM II.

**Table 2.2** Expected Flight Performance of Bolometers

<table>
<thead>
<tr>
<th>Channel</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical Efficiency</td>
<td>0.26</td>
<td>0.24</td>
<td>0.23</td>
<td>0.15</td>
<td>0.06</td>
</tr>
<tr>
<td>Optical Loading (pW)</td>
<td>0.29</td>
<td>0.25</td>
<td>0.23</td>
<td>0.18</td>
<td>0.07</td>
</tr>
<tr>
<td>BLIP ($10^{-18}$ W/√Hz)</td>
<td>5.4</td>
<td>5.6</td>
<td>5.7</td>
<td>5.9</td>
<td>3.9</td>
</tr>
<tr>
<td>Detector Noise ($10^{-18}$ W/√Hz)</td>
<td>17.7</td>
<td>10.6</td>
<td>8.1</td>
<td>8.2</td>
<td>16.9</td>
</tr>
<tr>
<td>Total Noise ($10^{-18}$ W/√Hz)</td>
<td>18.5</td>
<td>12.0</td>
<td>10.1</td>
<td>9.4</td>
<td>17.3</td>
</tr>
<tr>
<td>Receiver Sensitivity (mK√s)</td>
<td>0.56</td>
<td>0.41</td>
<td>0.38</td>
<td>0.55</td>
<td>2.7</td>
</tr>
</tbody>
</table>
Chapter 3

MSAM II — Cryogenics

The six bolometers in MSAM II must be cooled to 100 mK in order to achieve near-background limited sensitivity. The goal is to lower the temperature enough to get the intrinsic Johnson noise and phonon noise below the photon noise from the optics and CMB. In addition, cooling reduces the bolometer heat capacity – speeding up the detectors. As a result, the cryogenic system becomes an integral part of the radiometer. We have designed and built a cryostat which cools the bolometers to 100 mK, some internal optical elements to 1.5 K, and some to 77 K. Each of these stages is critical to the successful operation of the radiometer and must survive through the extreme environmental conditions of a balloon flight.

The cryogenic goal is to provide a 100 mK thermal bath to the bolometers over the course of the balloon flight. We chose to do this by building an Adiabatic Demagnetization Refrigerator (ADR) which is thermally supported by a pumped liquid Helium (LHe) bath. An alternative approach would have been to use a dilution refrigerator. The principal advantages of the ADR over dilution refrigerators are: 1) there is no need for expensive 3He and 2) the final cooling process is electrical rather than mechanical. Unfortunately, users of ADR’s pay for these benefits with a system that has a low cooling power (the number of watts the system can absorb at a given temperature). As a result, the parasitic heat loads on an ADR must be absolutely minimized.

The following sections detail the thermal design and operation of the MSAM II
cryostat beginning with an overview of the “warm cryogenics” - the systems which provide the guard for the ADR - and then covering the design and performance of the ADR itself. A final section describes recent achievements and future prospects for new ADR designs.

3.1 Dewar Operations and Flight Requirements

The MSAM II radiometer (affectionately known as the “Blue Dewar”) was born a modified commercial I.R. Labs [58] HD-10 dewar. The principal differences between this version and the “standard” HD-10 include a set of 6 indium-sealed feed-thrus, back-up indium o-ring grooves, and a second fill tube to the liquid helium tank. The original dewar was cooled by a 4 liter liquid Nitrogen (LN$_2$) tank and a 9 liter L$^4$He tank. A second LN$_2$ tank and working volume was added in 1994 to cool the cold beam-forming optics.

Ballooning is especially challenging for a cryostat. The flight day begins with the dewar and telescope sitting snugly inside an air-conditioned staging building. About six hours before launch, the cryostat is brought into the hot and humid Texas air where it bakes in the mid-day sun while the gondola waits on the tarmac. Here it is not uncommon for the air temperature to exceed 100° F. This does not last long, however, for during the two hours of ascent the air temperature quickly falls below -60° C while the gondola passes through the troposphere. When the gondola reaches float altitude, the ambient air temperature has risen to -50° C and the pressure has dropped to 2.75 Torr. Figure 3.1 shows the ambient pressure along with the gondola altitude during the '97 flight. Figure 3.2 shows the ambient air temperature and dewar shell temperature during the flight.

To prevent the nitrogen from freezing, the two liquid nitrogen volumes are pressure regulated at 4 psig by commercial regulators [59] and reach an operating
Figure 3.1  Pressure and altitude profiles during the '97 flight.

Figure 3.2  Dewar shell and air temperatures during the '97 flight.
temperature of 66 K\(^1\). Thirty layers of aluminized mylar (MLI) \([60]\) cover both nitrogen tanks and their shields. This reduces the effective emissivity of the surface (as long as the MLI is replaced when it oxidizes) and provides a number of nearly isothermal shields between the 77 K surface and the 240 K cryostat wall.\(^2\)

In the lab, the hold times of the upper and lower \(\text{LN}_2\) tanks are measured to be 48 hours and 23 hours respectively. After launch, 10\% of the \(\text{LN}_2\) is lost as the pressure drops to 4 psi during the ascent; however, this loss is more than offset by the reduction in boil-off at float-altitude due to the lower dewar shell temperature. The lower nitrogen tank additionally benefits from the reduced optical loading through the cryostat window.

The \(^4\text{He}\) bath is unregulated and reaches 1.5 K during the flight. Lab tests show that the transition to superfluid \(^4\text{He}\) results in a nearly negligible increase in the \(^4\text{He}\) boil-off. In addition, the \(^4\text{He}\) boil-off is an extremely weak function of the dewar tilt angle. Unfortunately, the helium level sensor did not work properly during flight so we were not able to measure the actual flight hold time of the \(^4\text{He}\) tank. In the lab, however, we routinely measured >12 hour hold times with the \(^4\text{He}\) tank pumped to 3 Torr.

### 3.2 The ADR: Mechanical Design

The Adiabatic Demagnetization Refrigerator (ADR) is the heart of the cryostat. ADRs are electrical refrigerators which take advantage of the magnetic field dependence of the entropy in a paramagnetic salt. The following sections give a

\(^1\)Note that great care must be taken in choosing a \(\text{LN}_2\) regulator since failure results in the \(\text{LN}_2\) freezing and thermally detaching from the nitrogen tank walls. The Tavco regulators have now flown on four separate MSAM flights with no evidence of failure.

\(^2\)A general rule of thumb for MLI is that the effective IR loading reduces nearly linearly with the number of layers until \(N \sim 30\), when the conduction through adjacent layers begins to dominate. For the space between 77 K and 4 K surfaces it is possible to get a factor of 2 reduction in optical loading by using \(\sim 10\) layers of MLI.
design recipe for the ADR used in the MSAM II radiometer. Section 3.2.3 covers our procedure for growing the ferric ammonium alum salt crystal as well as the design of the salt pill housing. In section 3.2.4 we discuss our methods for thermally isolating the salt pill and the bolometer stage. The heat switch design is covered in section 3.2.5 and our interface electronics in section 3.3. Finally, I give a summary of the lab and flight performance of the ADR in section 3.4.

3.2.1 Principles of Operation

ADR’s cool by exploiting the magnetic field dependence of the entropy of a paramagnetic salt. Each ion in a paramagnet, such as Ferric Ammonium Alum (FAA), has a total angular momentum $J$. In the presence of a magnetic field, its magnetic moment will point in one of $2J + 1$ quantized directions with respect to the field direction. The probability distribution for the states of all the ions in the salt, at a temperature $T$, goes like the Boltzmann factor

$$e^{-m g \beta B / k T} \quad (3.1)$$

where $m$ is the level number, $g$ is the Landé splitting factor for an electron, $\beta$ is the Bohr magneton, $B$ is the magnetic field, and $k$ is Boltzmann’s constant. More specifically, we can write the expression for the reduced entropy, $S/R$, (where $R$ is the gas constant) of a paramagnetic salt with $n$ iron atoms [61] as

$$s = nk \left( \ln \frac{\sinh (2J + 1) \frac{x}{2}}{\sinh \frac{x}{2}} + \frac{x}{2} \coth \frac{x}{2} - \frac{x}{2} (2J + 1) \coth (2J + 1) \frac{x}{2} \right) \quad (3.2)$$

where

$$x = \frac{g \beta B}{k T} \quad (3.3)$$
and

\[ B = \sqrt{B_{\text{app}}^2 + B_{\text{int}}^2}. \quad (3.4) \]

There is a distinction between \( B \) and \( B_{\text{app}} \) because of the internal field \( B_{\text{int}} \) which is due to the mutual interactions of the dipoles as well as the interactions between the magnetic moments of the nuclei and the electrons. This internal field is material-dependent and is an important consideration when choosing a paramagnetic salt since it limits the lowest temperature achievable by the refrigerator.

Figure 3.3 shows a family of plots of the entropy of an FAA salt pill as a function of temperature. The highest entropy line corresponds to zero applied field and the lowest entropy line corresponds to an applied field of 3 Tesla. The dotted line in the figure shows the cooling cycle when starting from a base temperature of 2.4 K. The cooling cycle goes as follows:

1. Starting with the ADR in an isothermal state (heat switch closed), an external field \( B_{\text{ext}} \) is applied. This causes the state of the system to travel down the isothermal line labeled 1 in the figure.

2. With maximum applied field, the thermal switch is opened moving the ADR to an adiabatic (isentropic) state. The applied field is then removed and the system travels down the horizontal path, number 2, in the figure.

3. Finally, when the ADR reaches the desired temperature, the magnetic field is adjusted to balance the warming of the system by parasitic heat loads (the support structure, the thermometer wiring, etc.). The system then moves along line number 3 in the figure until reaching the point where \( B_{\text{app}} = 0 \).

It is interesting to note that the final step in this process not only keeps the ADR at a fixed temperature but also allows the ADR to absorb more heat before becoming warmer than the controlling temperature. Consider the case where the
Figure 3.3  Predicted entropy (normalized to the gas constant $R$) of an FAA ADR. The cooling cycle progresses through steps 1–3 as shown on the figure and described in the text.

applied field is reduced to 0 during the demagnetization stage. The heat that can be absorbed by the ADR before the salt exceeds a temperature $T_{hi}$ is given by

$$Q_u = \int_{T_{low}}^{T_{hi}} T \frac{\delta S}{\delta T} dT$$  \hspace{1cm} (3.5)

where $T_{low}$ is the temperature of the salt with 0 applied field. In comparison, when the ADR is being controlled at a fixed temperature $T_{hi}$, the heat that can be absorbed before the applied field is run to 0 is given by

$$Q_c = T_{hi} \Delta S$$  \hspace{1cm} (3.6)

In practice, for an FAA salt pill being controlled at 100 mK, the available enthalpy is roughly twice that of the uncontrolled pill (i.e., $\frac{Q_c}{Q_u} \sim 2$).
3.2.2 Tools of the Trade

The ADR was designed and assembled at Brown University. Figure 3.4 shows its principal components. The salt pill is suspended in the bore of a superconducting magnet purchased from American Magnetics [62]. This 15 henry magnet is capable of generating 3 T with 9 A of current. To shield the bolometers from stray magnetic field, the magnet and salt pill are housed in an annealed vanadium permendur shield. With the shield in place, the field near the bolometers is measured to be less than 1 mT with the magnet at full field. Thus, at the maximum operating field of 5.6 mT the stray field at the bolometers should be negligible. We are currently conducting tests to determine if the bolometers show any dependence on the local magnetic field so that we might remove the bulky magnet shield (and a substantial portion of the mass of the ADR).

We chose to make the ADR from ferric ammonium alum because of its low internal field and the simplicity of its growth. The dipole-dipole and exchange interactions in FAA are small due to the relatively large spacing between magnetic ions. Another alum with an even lower internal field is chromic alum (CCA). CCA has $J = 3/2$ and $B_{int} = 20$ mT. CCA is more suitable for space applications since a CCA crystal can survive a typical satellite vacuum bake-out without thermal damage. However, CCA crystals are more difficult to grow than FAA crystals due to the lower solubility of CCA [63]. Table 3.1 is a list of some of the physical characteristics of FAA.

<table>
<thead>
<tr>
<th>Chemical Symbol</th>
<th>$Fe(NH_4)_2(SO_4)_2H_2O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20° C solubility</td>
<td>1.2 g/cc</td>
</tr>
<tr>
<td>35° C solubility</td>
<td>4.0 g/cc</td>
</tr>
<tr>
<td>Crystal density</td>
<td>1.35 g/cc</td>
</tr>
<tr>
<td>Spin angular momentum $J$</td>
<td>5/2</td>
</tr>
<tr>
<td>Internal field $B_{int}$</td>
<td>50 mT</td>
</tr>
</tbody>
</table>
CHAPTER 3. MSAM II — CRYOGENICS

The salt is switched between the "isothermal" state and the "adiabatic" state by a mechanical heat switch (see section 3.2.5). Since the heat switch has a finite thermal conductance, as field is applied and the entropy of the pill decreases, the salt heats up. Typically, with the heat switch at 1.5 K, the salt reaches \( \sim 6 \) K when the magnet reaches full field. During this "soaking" phase, the salt is allowed to cool back to 2.4 K while the field is held fixed at its peak value\(^3\).

When the salt pill has reached 2.4 K again, the heat switch is opened and the magnetic field is slowly ramped down. Here it is important not to rush the process. High heat capacity objects in the thermal chain must be kept near the temperature of the salt so that the salt absorbs the heat at maximum enthalpy. We found differences in hold time as large as a factor of two depending on the rate at which we cooled the salt.

3.2.3 Salt Pill Design and FAA Growth

There are a number of constraints on a salt pill housing (SPH) which must be met to ensure a long lived and successful ADR. Below, I outline some of the major issues in SPH design along with the solutions which were implemented in making a number of 120 g FAA salt pills. Figure 3.5 shows the SPH which was used in the MSAM II radiometer and diagrams our solutions to the issues raised below.

SPH Materials

FAA, while being a particularly nice salt to work with in the lab (see below) is notorious for corroding copper-based alloys. This effect is not subtle. It only takes a few hours for a solution of FAA to dissolve a section of Cu wire. As a result, no copper alloys used in the SPH can be allowed to come in contact with the FAA.

\(^3\)We stop the cooling at 2.4 K simply for historical reasons. We could just as well continue to cool until the pill reaches 1.5 K.
Figure 3.4 The Full ADR System. Subsystems are described in the text.
We found that even gold plating the inner walls of a brass tube was not enough to prevent the salt from eating through the 500 $\mu$m tube wall within a few days. After numerous attempts using ceramics (which tended to crack when thermally shocked), we finally settled on type 304 stainless steel as the housing material. 304 stainless is non-reactive with FAA, welds easily, and is strong enough to allow us to reduce the wall thickness of the principal tube to 250 $\mu$m. The biggest drawback of using stainless steel is its large heat capacity.

Salt Dehydration

FAA crystals will dehydrate if exposed to the vacuum of a cryostat. A common method for sealing the ends of the SPH is to glue them closed using an epoxy which is well thermally matched to the SPH material. We avoid epoxy in the SPH as a mechanical link because of a general distrust of vacuum-tight epoxy joints. Instead, the end caps of our SPH are TIG-welded into place by a micro-welder. This has the advantage that once the joint is leak-tight, it will always be leak-tight. On the other hand, FAA undergoes a chemical transition which spoils the paramagnetism when heated above 40° C so the welding must be done with care to keep the heat away from the salt.

Thermal Link and Eddy Current Heating

Making good thermal contact to the FAA is one of the most difficult parts of SPH construction. Since the salt itself is a very poor conductor, a good conductor should occupy a substantial volume of the SPH to keep the time constant of the system at a reasonable level. On the other hand, the larger the effective cross-section of the thermal link, the larger the eddy current heating during times when the field is changing. We use a nest of $\sim$140 250 $\mu$m diameter 99.999% Au wire [64] which is suspended in the salt; and silver soldered with non-
superconducting cadmium-free silver solder at one end to a high purity 99.999% Cu rod [65]. The Cu rod and silver-solder joint are isolated from the FAA by a layer of Stycast 2850 epoxy [66]. Since the gold wires have a small diameter, the eddy-current heating is small. Also, gold does not interact with the FAA so there is no worry of corrosion. In fact, the number of gold wires we use is on the low side. Hagmann et al. [63] found that conduction along the gold is the limiting effect in their conduction path and we have since widened our Cu solder cup and begun to use ≥ 200 wires in our pills.

The amount of eddy current heating in our pill can be calculated following [61]. The power dissipated in a metallic rod of radius $r$, volume $V$, and resistivity $\rho$ is

$$P = \frac{1}{8} \rho^{-1} r^2 V \dot{B}^2 \text{ W}$$

(3.7)

where $\dot{B}$ is the change in magnetic field as a function of time in T/s. If we model the gold wires as independent rods aligned with the magnetic field, the eddy current heating for each wire is

$$P_{gold wires} = 4.3 \times 10^{-7} \dot{B}^2 \text{ W}$$

(3.8)

and for the stainless can (the outer shell)

$$P_{stainless can} = 0.12 \dot{B}^2 \text{ W}.$$  

(3.9)

Thus, during ramp down of the magnet from full field to the controlling field the 140 gold wires contribute 0.89 µJ while the stainless can adds 1.8 mJ - both insignificant amounts compared to the 134 mJ of enthalpy of the salt. During flight, the deviations in field while controlling at 100 mK are small compared to the ramp down (typical $\dot{B} = 1.2 \times 10^{-6}$ T/s). This results in many orders of
Figure 3.5  Salt pill housing for MSAM II ADR. The thermal path begins at the salt and ends at the threaded Cu rod which is thermally joined to the heat-switch and the bolometers.
magnitude reduction in the amount of eddy current heating. Consequently, this SPH has no eddy current heating worries.

Salt Pill Growth

The FAA crystal is grown from a heated saturated solution which is slowly cooled to room temperature. As mentioned above, FAA undergoes a chemical transition when heated above 40° C which destroys its paramagnetism permanently. Consequently, the growth of the pill requires a careful balance between keeping the FAA solution as close to 40° C as possible, to maximize the FAA in solution, without going over. We have developed a simple method for reliably filling our SPH with FAA crystals using a home-made oven and standard laboratory chemical equipment.

The “oven”, shown in figure 3.6 is composed of two concentric Plexiglas tubes. The inner tube has the same outer diameter as the SPH and is wound tightly with Cu magnet wire which acts as the heater. The SPH is attached to this tube by a greased semi-flexible rubber hose. Both the outer tube and the Styrofoam spacers act as baffles to reduce convective cooling to the heating element. The goal is for the oven to keep a column of FAA solution near 38° C for a week. The required heat input is calibrated by filling the inner tube and SPH with water and measuring the temperature as a function of height along the tube length. We found that with 7.07 W of input power over 91 cm of tube resulted in the temperature ranging from 36° C at the bottom of the tube to 38° C at the top.

The process for growth is straightforward. With the inner tube and SPH filled with FAA solution and the heater on, the growing assembly is lowered out of the outer tube in steps of 1 cm each 12 hours. Lowering the SPH into the cool lab air at this rate helps ensure that salt crystals will grow from the bottom to the top. If crystals do begin to form near the top, it is possible for them to seal off
the lower volume, starving it of fresh solute from the upstairs supply. Note also that because the gold wires are thermally anchored to the Cu post which is always near room temperature, crystals will begin to form on the gold - making better thermal contact.

When the SPH is fully retracted from the outer tube, the current through the copper heater wires is stepped down by 1.75 W each 12 hours until it is zero. Twelve hours after this the SPH is detached from the oven and excess salt is removed to make room for the second endcap to be welded in place.

We have used this method to grow 5 separate salt pills. In each case, density measurements showed that the SPH was completely filled with salt within the measurement error (a few percent). The history of these pills reads like the stories of fallen Greeks at Troy: the first pill, made with a 125 μm thick stainless wall, ruptured the side of the wall during a thermal cycle. The second pill (the first of the line to have a 250 μm thick wall) was operational for over a year before being heated to a fiery death inside of a cryostat which a graduate student was in a hurry to return to room temperature. The third pill had a brief existence and melted in the welder’s clutches. The fourth pill met its end in commercial transport while the fifth pill continues to cool the Blue Dewar to 100 mK today.

3.2.4 Thermal Isolation

The extent to which the 100 mK ADR and cold stage can be thermally isolated from their surroundings determines the hold time (and duty cycle) of the system. The goal is to have a stiff and stable suspension system while keeping the parasitic heat load to an acceptable level. We have constructed two thermal isolation systems using a network of Kevlar [67] string. One system supports the SPH between the jaws of the heat switch while the other system supports the detector module and is part of the optics chain. Below we describe the two systems in
Figure 3.6  Oven for salt pill growth. A tightly wrapped Cu wire heats the inner tube to keep the solution near 38°C. So that the crystals grow from the bottom up, the SPH is slowly lowered into the cool lab air over the course of many days.
detail.

Kevlar string is a nearly ideal substance for thermally isolating a cold stage. The spring constant of Kevlar is very large for a fiber and the creep ($\Delta L/L$) is logarithmic in time. One slightly odd feature of Kevlar is that it expands when cooled. Unfortunately the low temperature data on Kevlar remains sparse and is supplied mainly by ADR builders. Table 3.2 lists some physical characteristics of Kevlar we have either measured ourselves or found in the literature.

<table>
<thead>
<tr>
<th>Table 3.2 Some Physical Properties of Kevlar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
</tr>
<tr>
<td>Youngs Modulus</td>
</tr>
<tr>
<td>Creep ($\Delta L/L$)</td>
</tr>
<tr>
<td>Thermal Expansion ($\Delta L/L$)</td>
</tr>
</tbody>
</table>

SPH Suspension

The requirements for the suspension of the salt pill are not severe. The suspension must be stiff enough that eddy current heating due to vibrations of the pill in the controlling field are small. In addition, the pill must be geometrically located so that a point on the thermal path can be reached by the thermal switch. Finally, since Kevlar is a brittle fiber, any radii over which the Kevlar string turns must be kept large.

In suspending our salt pill we have adopted a method similar to that of Timbie [69]. Attached at either end of the SPH are two three-pronged aluminum fixtures with small pulleys emanating radially from the pill (see figure 3.4). These aluminum stars (and the salt pill) are supported by six Teflon washers that act as centering guides and spacers. In-between the 100 mK pulleys are another set of three pulleys which make up the “warm” end of the suspension. Kevlar string, terminated at one of the warm pulleys, is threaded between the 1.5 K and 100
mK pulleys as it makes its way around the pill. When the Kevlar returns to the initial pulley, it is wrapped on the pulley, secured with a number of half-hitches, and then tensioned by manually rotating and locking the pulley. The remaining pulleys are then locked into place with a nut which prevents rotation through friction. Once the strings are secured on both sides, the Teflon washers are removed from under the aluminum “stars” and the salt pill is isolated. In order to account for the creep in the Kevlar, the suspension system is checked and tightened 24 hours after it is first strung.

Because the Kevlar has such a large elastic modulus, the resonant frequencies of the suspension system are limited by the “springiness” of the aluminum stars. The lowest frequency mode of oscillation is the one in which the salt pill oscillates in the vertical direction. For a total suspended mass of 270 g, the resonant frequency is 170 Hz. The heat generated from this oscillation is produced in the aluminum stars and is dissipated into the stainless salt pill housing which acts as a low pass filter on its power spectrum. This aids our servoed controller which is stable at low frequencies at the expense of increasing the time constant of the pill.

**Bolometer Box Suspension**

In addition to thermally isolating the salt pill, it is necessary to thermally isolate all other 100 mK objects. We chose to build a separate suspension system for the bolometer box so that it could interface most easily with the other parts of the optical system. The requirements on this suspension scheme are much more severe than those for the salt pill. Here, two 1.5 K waveguide flanges are to sit opposite two matching 100 mK waveguide flanges with a 125 \( \mu \text{m} \) gap in between. The alignment of these flanges is critical to the optical efficiency (see section 4.2.2), requiring the suspension system to hold the bolometer box in all three dimensions to within 75 \( \mu \text{m} \).
Figure 3.7  Thermal isolation of the bolometer box. The box is suspended by Kevlar strings which are held under tension by weak springs which push on the upper support structure. The Kevlar is initially tensioned while the springs are held compressed by the jack screws.
Again we use Kevlar string as the support material. Figure 3.7 shows the bolometer box attached to the suspension system. Contact is made at four points along the bolometer box by eight separate strings. While this over-constrains the system, the simplicity of assembly more than makes up for the small increase in conductivity to the 1.5 K support plates. The Kevlar strings run over dowel pins which are sized to avoid mechanically weakening the brittle fibers and the strings are clamped by compression between two flat aluminum plates.

The suspension system is designed with the creep and thermal expansion of the Kevlar in mind. The bolometer box is initially set on an alignment jig to ensure proper waveguide alignment. With the box in place, the upper suspension plate is placed on the three guide-posts where it is screwed down onto the compression springs by the jack screws. At the point of nearly maximum compression the Kevlar strings are threaded through the box and around the dowel pins, pulled to an even tension, and secured to the upper and lower suspension plates. At this point the string clamps on the bolometer box are left open and the jack screws are released - freeing the compression springs and tensioning the Kevlar strings. With the strings under approximately equal tension, the alignment jig is removed and the bolometer box is lowered onto shim stock to set the appropriate gap thickness. The bolometer box is then at the proper height with respect to the facing waveguides, the final clamps on the bolometer box are tightened and the suspension is complete.

Because of the symmetry of the suspension system, there is no movement of the bolometer box in the horizontal plane as a result of creep in the Kevlar or thermal expansion. However, there is motion in the vertical direction which results in the gap between the bolometer box and waveguides getting larger\(^4\). The motion due to the creep is minimized by pre-stretching the Kevlar over a period of

---

\(^4\)Note that the compression springs keep the tension in the Kevlar string constant despite the change in length.
several days prior to the stringing. Unfortunately, the motion due to the thermal expansion is inevitable. Since we expect vertical motion of a few hundred microns, we compensate for this by setting the warm height of the bolometer box a few hundred microns below the optimal value. Bench-top tests at 77 K, where most of the expansion is expected to have occurred, show that this is a robust method for setting the gap size at low temperatures.

Fully loaded, the bolometer box weighs slightly over 700 g. The dominant mode of vibration is that where the bolometer box oscillates in the vertical direction against the compression springs. In this case, the resonant frequency is not set only by the spring constant of the compression springs. When the bolometer box is displaced in the downward direction, the Kevlar strings slacken and the compression springs push back with the standard force. After the bolometer box returns to the equilibrium position, however, the Kevlar strings pull with a spring constant many orders of magnitude greater than that of the compression springs - causing the bolometer box to reflect back in the downwards direction. If this were a lossless process, the resulting motion would be that of a rectified sine wave. In practice, there is loss from heat being produced in the aluminum supports and the copper bolometer box. The result is a dampened nearly-rectified sine wave. Ground measurements of microphonics in the bolometer system show a mechanical resonance of the system at 73 Hz. This is safely higher than the highest required bolometer sampling frequency of 40 Hz.\footnote{As shown in section 4.1.1, interesting CMB signals can be found out to 16× the chopping frequency (2.5 Hz) for a 20' experiment with a chopper throw of 80'.}

We are currently building a new suspension system for the salt pill which incorporates the ideas used in the bolometer box suspension. With a salt pill less than 200 grams, this suspension should have resonant frequencies well above 100 Hz.
3.2.5 Heat Switch

It is a common, yet difficult, task in experimental low temperature physics to make and break thermal contact between a heat reservoir and a sample. This holds true for the $^3$He tank in a closed-cycle $^3$He refrigerator, the salt pill in an adiabatic demagnetization refrigerator, heat capacity measurements, and any other process which requires low thermal conductivity at low temperatures. As a result, the heat switch is an important tool for the low temperature physicist.

There are three principal classes of heat switches in use today: the gas gap heat switch [70], material-based heat switches [71], and mechanical heat switches [72]. Gas gap heat switches rely on a small tube of gaseous $^4$He to make thermal contact between the sample and the bath. When thermal contact is no longer desired, the $^4$He is adsorbed onto a getter. Material-based heat switches exploit the properties of a material (magnetoresistance, superconductivity, etc.) in order to change the thermal conductivity. For example, at 1 K a non-superconducting tin wire has a thermal conductivity more than 100 times that of its superconducting state. Finally, a mechanical heat switch is simply a device which makes and breaks physical contact to a sample. While the thermal conductivity of a mechanical switch in the ON state is not as high as either a material-based switch or a gas gap switch, the OFF state of a mechanical switch is truly off - resulting in no heat flow from the switch to the sample$^6$. Table 3.3 summarizes the three types of heat switches and provides some measured data for existing switches in each class. Included is the figure of merit known as the on-off ratio which is the ratio of conductivities in the ON and OFF states.

The ADR puts two stringent constraints on the heat switch. Since the “soak” phase of the cycle requires high current to the magnet (typically 6 to 10 A), it

---

$^6$In fact, there will always be radiative heat transfer, however, most often this is negligibly small.
Table 3.3 Measured Data for Heat Switches

<table>
<thead>
<tr>
<th>Heat Switch Class</th>
<th>Gas-gap</th>
<th>Material Based</th>
<th>Mechanical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical On/Off Ratio</td>
<td>as high as 4000</td>
<td>as high as 2000</td>
<td>limited by radiation</td>
</tr>
<tr>
<td>Typical Mass</td>
<td>54 grams</td>
<td>mass dominated by magnet and shielding required to operate switch</td>
<td>600 grams</td>
</tr>
<tr>
<td>On Conductivity (4.2 K)</td>
<td>0.035 W/K</td>
<td>2000 W/m-K</td>
<td>0.01 W/K</td>
</tr>
<tr>
<td>Off Conductivity (4.2 K)</td>
<td>1.5 x 10^{-6} W/K</td>
<td>1 W/m-K</td>
<td>0 W/K</td>
</tr>
</tbody>
</table>

should be as short as possible to minimize the helium lost to Joule heating in the magnet leads. From Figure 3.3, a 120 g salt pill made from Ferric Ammonium Alum outputs 5.25 J of heat when brought to 3 T isothermally at 2.4 K. Therefore, to keep the soak phase below 10 minute duration, a heat switch conductivity of 10.9 mW/K is required. At the low temperature end, for a typical ADR made from FAA, only 200 mJ of heat may be absorbed before the salt can no longer operate at 100 mK. In the OFF state, today’s gas gap heat switches have a conductivity larger than .01 mW/K at 4 K. Since this would only allow the typical ADR to stay below 100 mK for a maximum of 5 ks rather than the 200 ks achievable with no heat switch conductivity, a mechanical heat switch which has infinite thermal resistance in the OFF state is preferable.

Mechanical heat switches have been designed for and used in ADR systems on a variety of platforms including the SIRTF satellite [69], a sub-orbital rocket experiment, and balloon-borne experiments [73].

Mechanical Heat Switch

There are two varieties of mechanical heat switches: those with drives external to the cryostat and those with drives internal to the cryostat. Externally driven mechanical heat switches rely on a mechanical drive linkage (drive shaft, string,
etc) which extends through the cryostat wall and necessitates the use of a bellows or soft o-ring joint. When the switch is activated (by pulling on the string or twisting the drive shaft), the sample is pinched between two contact points and thermal contact is made. Externally driven mechanical heat switches are simple and reliable and can provide force at the point of contact limited only by the size of the external activation motor. However there are two major problems associated with the use of an externally driven mechanical heat switch. First of all, the joint which seals the vacuum jacket of the cryostat at the exit point of the drive linkage must be leak tight. For a sliding or rotating shaft this is not a simple matter and for space applications where external temperatures fall well below -50° C, flexible vacuum seals become both expensive and difficult to realize. Furthermore, the use of an externally driven mechanical heat switch in an experiment introduces a major design consideration in the cryostat. Care must be taken in the placement of the drive linkage to avoid disassembly of the switch each time the cryostat is opened. Since heat switches are often added after a cryostat is constructed, this is not always a viable option.

Internally driven mechanical heat switches, on the other hand, are driven by a motor, spring, or solenoid which is internal to the dewar. This allows the switch to be compact and relatively unobtrusive to the remainder of the cryostat. Only a pair of electrical leads is required to activate the switch remotely. There are a number of drawbacks to internally driven mechanical heat switches, however. First of all, as is the case with the externally driven model, the force (and therefore the conductivity) produced by the heat switch at the contact point is limited by the size of the drive mechanism. Since cryostats generally are not built with space for the inclusion of a large motor or solenoid, high currents must be used to compensate for the reduction in mechanical advantage. Gearing can help to solve this problem but creating reliable moving parts at low temperatures where
standard oils and greases freeze can be quite difficult. Finally, an internally driven heat switch must be reliable, for if the heat switch should fail, the cryostat will need to be warmed and opened to diagnose and solve the problem.

Internally Driven Mechanical Heat Switch - A New Approach

We have designed, constructed, and used an internally driven heat switch which is small, reliable, and simple. The generic nature of the contact surfaces makes this switch quite versatile. The principle of operation is as follows: two stainless lever arms are pivoted near the point of contact to the sample such that when the long end of each lever is pushed outward from the switch, the short end of each lever moves to pinch the sample. A ratcheted 95 degree rotary solenoid [74] drives a peanut-shaped cam to move the lever arms. The solenoid is pulsed by passing 1 A through its windings for approximately 0.25 s. Pulsing the solenoid causes the cam to change positions consequently either opening or closing the switch (see figure 3.8). Once the solenoid has been pulsed, a return spring “reloads” the solenoid by rotating the solenoid drive shaft back to the initial position. A ratchet between the solenoid drive shaft and the cam, in conjunction with two trapping springs on the cam, prevents the heat switch from also returning to the initial position. Finally, a spring connecting the extreme ends of the lever arms returns the arms to the “low” position of the cam when the switch is opened.

There are a number of advantages of this design over typical internally driven mechanical heat switches. First and foremost, unlike most mechanical heat switches, no current is required to hold the heat switch in the ON position. As a result, the heat switch must only dissipate power from the sample and not from the solenoid itself. Secondly, since the activation of the heat switch is with a short pulse of current, large forces may be generated at the contact point with little power dissipation to the helium bath. Using 1 A of current for 0.25 s we are able to generate
Figure 3.8  Top views of the heat switch in the open and closed position. The peanut-shaped cam is turned by a ratcheted 95° rotary solenoid. The jaws of the heat switch make contact to flats milled into the copper post coming out of the ADR. The heat is then drawn down the copper arms, through the copper foil (not shown), and into the copper base of the heat switch.

greater than 200 N of force at the sample end while dissipating only 375 mJ of heat to the helium bath. The practical limit to the force achievable is set by the size of the solenoid’s leads. The Joule heating from the leads is minimized by running the leads through the helium tank (via the fill tube) and then into the vacuum space by way of a super-fluid tight feedthrough.

Finally, there are very few moving parts in this switch. The principal moving part is the drive shaft of the solenoid itself. Since the solenoid is designed to rotate 95 degrees, having the extra 5 degrees unloaded and at the beginning of the stroke allows the solenoid to easily break the static friction.

Once the solenoid has completed the stroke and the driving current is removed the most cryogenically challenging phase begins. Here, a balance must be achieved between the return spring and the ratchet. If drag on the ratchet is too great, the ratchet fails in the return and the return spring is unable to “reload” the solenoid. On the other hand, if there is too little drag on the ratchet, the ratchet can fail
Figure 3.9 Side view of the heat switch.

to activate on the forward stroke of the solenoid and no motion of the cam will occur. We have solved this problem by "floating" the cam assembly between the ratchet and a spring as shown in figure 3.9. By empirically adjusting the force generated by the spring with a set screw, it is simple to find the position which optimizes the mechanical performance of the switch.

**Thermal Characteristics**

Of course it is not sufficient for a heat switch only to open and close reliably. The heat switch must do this while maintaining high thermal conductivity between the thermal sink (in our case the $^4$He bath) and the sample. The thermal path begins at the sample where contact is made by the heat switch. The thermal conductivity across a pressed contact depends on both the bulk thermal conductivity of the materials in contact and on the force of the contact.

We have fabricated the contact points of the heat switch from high purity
oxygen-free high conductivity (OFHC) [65] copper (annealed at 800 C for one hour) with the mechanical supports for the contacts made from the same material and backed by stainless steel (grade 304). The stainless gives the necessary rigidity to the soft copper to provide the great force to the sample. The contact points themselves are threaded rods which are secured by set screws. Since the lever arms have a fixed throw, the threaded design allows the operator to choose the amount of force the heat switch will use to grab the sample. Once the heat has passed from the sample, through the junction, and into the heat switch, it must have a path off of the moving arms and into the thermal bath. The optimization here is to provide a thermal pathway that maximizes conduction while minimizing the restriction on the motion of the arms. This is done by having short pieces of 250 μm thick annealed copper foil soldered to both the arms and the copper body of the heat switch. The foil is soldered to the arms near to the pivot points to minimize the motion of the foil and is slit at the points of maximal stress to minimize the drag on the arms.

Performance

This switch was installed in the MSAM II cryostat for the first time in February of 1996. In May of 1996 the cryostat was sealed for a 105 day period during preparations for a balloon flight in Palestine, Texas. The switch was pulsed an average of four times per day, thermally cycled from 4.2 K to 1.5 K once per day, and thermally cycled from 300 K to 1.5 K six times in the 105 day period. Over this time there were fewer than ten misfires (where pulsing the heat switch failed to change the state of the switch) and three instances where the return spring failed to “reload” the solenoid. The failure modes only appeared when the cryostat was tilted more than 45 degrees past the vertical suggesting that the ratchet was not well constrained in off-axis directions. In all instances of misfire, a
second pulsing of the heat switch was sufficient for activation. In each case of the failure of the return spring, the cryostat was returned to the upright position and the heat switch was pulsed again. In over 400 pulsings of the heat switch there was no situation which warranted opening the dewar for heat switch servicing.

For the 1997 campaign the heat switch was fitted with a washer to provide more friction to the rotating peanut. In addition, the jaws were opened slightly and the force on the salt pill was reduced. The resulting unit was sealed in the dewar on 2 April, 1997 and retrieved after the flight on 3 June 1997. Again, the heat switch was fired an average of twice per day - resulting in over 120 cyclings without failure. None of the failure modes from the previous campaign recurred.

Cryogenically, the heat switch performed to expectation. Prior to cooling, the heat switch and salt pill were assembled together to “tune up” the clamping force. The heat switch was repeatedly pulsed while gradually increasing the force at the contact points until the heat switch either failed to close at 1 A pulsing current or failed to open at 1.2 A pulsing current (the opening current needs to be somewhat greater as the solenoid is weakest at the beginning of its travel). After extracting the heat switch from the dewar, we replaced the contact points (jaws) with steel spheres in order to measure the force of the pinching using the Brinell Hardness Test [75]. In this test, a hard spherical ball is pushed with a force \( F \) onto a soft metal. For a sphere with radius \( r \), Young’s Modulus \( E_1 \), and Poisson’s ratio \( \sigma_1 \) and a flat piece of metal with Young’s Modulus \( E_2 \) and Poisson’s ratio \( \sigma_2 \) the radius of the outer rim of the resulting “crater” \( a \) is given by

\[
a = \left\{ \frac{3}{4}Fr \left( \frac{1 - \sigma_2^2}{E_1} + \frac{1 - \sigma_1^2}{E_2} \right) \right\}^{\frac{1}{2}}
\]  

(3.10)

We found that for a number of cycles of the switch, each making separate indentations into the copper plate, a variety of sizes of craters were formed. Taking this range (as well as the uncertainty of the physical properties of the materials...
used) we find the force exerted by the heat switch on the sample to be 294±98 N.

In principle it is possible to produce greater force than mentioned here. As a practical constraint, we desired to preserve the ability to open and close the heat switch at room temperature with less than 90 Vdc of activation voltage. Since the solenoid’s 300 K lead resistance is 75 ohms, this limits us to a maximum of 1.2 A driving current. At low temperatures, however, the lead resistance of the solenoid is only 1.3 ohms so a much larger driving current (and therefore force) may be obtained at a more reasonable voltage.

3.3 Refrigerator Electronics

Because an ADR is an electrical refrigerator by its nature, the electronics which run it are of paramount importance to the success in keeping a fixed temperature during the controlling phase. We have built and used a set of refrigerator control electronics based heavily on an original design by Dan McCammon [76]. The goal was twofold: First, in the event of a departure from the nominal flight-plan, it should be possible to cycle the ADR remotely. Secondly, for optimal bolometer performance, the bolometer base temperature should be stable at 100 mK to 50 μK over 60 s time periods.

There are two methods for controlling the temperature of an ADR. Recall from equation 3.1 that in the idealized case, the entropy behaves as

\[ S \sim \frac{B_{app}}{T} \]  

(3.11)

so in an adiabatic state (such as during controlling at a fixed temperature) the field required to go from a temperature and field \((T_1, B_1)\) to a temperature \(T_2\) is

\[ B_2 = B_1 \frac{T_2}{T_1} \]  

(3.12)
Thus, at any controlling field,

$$B_{app} \sim T$$  \hspace{1cm} (3.13)

Now, the quiescent power loading on the ADR, $P_0$, leads to a change in temperature $dT/dt$. From eq 3.13

$$P_0 \sim dT/dt \sim dB_{app}/dt \sim di_{mag}/dt \sim V_{mag}$$  \hspace{1cm} (3.14)

where $i_{mag}$ and $V_{mag}$ are the current and voltage associated with the magnet.

Thus, we can go about controlling the ADR temperature in one of two ways. We can either control the current (and therefore the temperature) directly, or we can control the voltage across the magnet and attempt to compensate for the power deposited on the system. We chose the latter method. While controlling the current is a much more direct way to control the temperature, a desire for a fast change in the temperature results in a large voltage across the magnet due to the magnet’s large inductance. On the other hand, in an idealized system with constant parasitic heat loads, setting a constant (and slightly negative) voltage across the magnet results in a fixed salt pill temperature.

The feedback loop of the controller is shown in figure 3.10. The ADR temperature is monitored by a Germanium Resistance Thermometer (GRT) \cite{77} whose resistance (which is a strong function of temperature) is measured by a low-noise readout circuit which outputs a voltage proportional to the GRT resistance. An error signal is created by differencing this voltage with a reference voltage. This error signal is passed along to a PID controller which inverts the sign, applies gain, applies filters, and sets the resultant voltage across the leads of the magnet. This causes the magnet to either ramp up or down in such a way as to decrease the error signal.

The PID circuit has the largest flexibility of any element in the controller
Figure 3.10  Thermal Control Loop for the ADR. The thermal impedance “R” is representative of the conductivity between the salt and the GRT.
chain. The proportional element of the circuit simply adds gain to the incoming error signal. The integrator applies a low-pass filter to the signal and defines the zero (controlling) point and the differentiator adds a high-pass filter. In order to achieve stability, it is necessary to choose the proper time constants and gain for the different elements of the system. In our case we followed the analysis of Forgan [78] and chose a programmable range of time constants and gains once the time constant of the ADR system was understood.

The goal in choosing the time constants of the controller is to have the largest open loop dc gain as possible without inducing self-sustaining oscillations in the system. An arbitrarily large open-loop dc gain will make the system stable from thermal runaway; however, as the open-loop dc gain is increased, system stability around the set point is jeopardized and the controller tends to overshoot, compensate, overshoot on the other side, compensate, etc. With too little gain, the controller is overwhelmed by deviations in the temperature and stability is lost.

The differentiator time constant should be chosen to be equal to one of the time constants in the thermal system (not the longest). In this way, the effects of the two time constants will cancel and effectively reduce the number of time constants by one. Reducing the number of time constants tends to make a system more stable [78]. Further stability (as well as a fixed zero point) can be achieved by adding integral control where the time constant of the integrator is set equal to the longest time constant of the thermal system. It was not practical for us to implement integral control due to the long time constant of our thermal chain. Instead we controlled with a proportional and differential controller.

In figure 3.11 I show the closed-loop response of the control system to a delta-function input of 7 μW. This power level was chosen to be approximately 10 times the quiescent loading on the system. The power is applied electrically via a resistor bolted to the top of the bolometer box and next to the GRT. In the figure
**Figure 3.11** Thermal response of the controller in the presence of a 7 μW pulse of heat to the bolometer box. Shown is the GRT response as well as the current in the magnet. The controller overshoots the set point due to the thermal time constant in the ADR. Increasing the conductivity between the GRT and the salt pill would result in a shorter recovery time.

The effect of the finite thermal conductivity between the GRT and the salt is clear from the delay in the response of the GRT to the changing magnet current as well as from the overshoot of the system. Since our flight is many hours in length and relatively free of events that dump power on a very short time scale, this level of control is more than acceptable.

Figure 3.12 shows an example of the in-flight performance of our controller. Plotted is the temperature of the GRT and the magnet current over several minutes in the middle of the flight. During the flight, measurements of the stability of the refrigerator were limited by the noise on the GRT signal. Section 3.4 gives more information on the in-flight performance of the controller.

The drifts in temperature on time scales of a few minutes in figure 3.12 would be eliminated in the presence of integral control. As mentioned above, we eschewed integral control due to the long time constant required for the integrator.
Figure 3.12  In-Flight stability of GRT temperature. Shown are the GRT temperature and magnet current during a portion of the flight. The error in the measure of the stability is dominated by the noise in the GRT circuit. For the flight, the RMS deviations about the control temperature are 35 μK. The magnet current slowly decreases to match the quiescent thermal loading on the salt pill.
In this case, a digital computer would be an appropriate and convenient choice for a controller. While we are very pleased with the results of our analog controlling circuit, we recognize that using a computer would allow for many improvements over this hardware design. First of all, a clever computer program would empirically find the optimal gains and time constants for a particular system. In this way, the experimenter would be free of the chore of searching for optimal control parameters as the experiment evolved. Secondly, while it is difficult to create a long time constant for an integrator in hardware (the capacitors and resistors quickly become too large for time constants of more than a few seconds), it is only mathematics for a computer. Finally, when the transition from the ramp-down state to the controlling state occurs, the integrator in a hardware system is charged up to its rails. This causes an initial jump of the magnet voltage which can send the system into oscillation while the integrator discharges. A computer could initially implement proportional and derivative control and then switch in integral control when the system had achieved some stability.

### 3.4 ADR Characteristics and Performance

This section gives a summary of the characteristics of our flight ADR. To begin with, I demonstrate the typical cooling process. Section 3.4.2 gives some thermal characteristics of the refrigerator. I conclude with some brief comments on how this system could be improved and what ADR innovations are on the horizon.

#### 3.4.1 ADR Cycling

The ADR is cycled via a subsection of the controller electronics. Figure 3.13 shows a typical cycling of the refrigerator on the morning of a flight opportunity. At the beginning of the cycle, the L$^4$He bath is still being pumped down and
Figure 3.13  Cycle of ADR. This is a typical cycle of our flight-configured ADR.

the cold plate has only reached 2.0 K. By the end of the cycle the cold plate has reached 1.5 K. Following the steps laid out in section 3.2.1, the magnet is ramped at a constant rate to full field. When the magnet current has reached its “soak” value of 7.82 A, the heat switch is closed and the salt pill is left to cool. The heat switch is opened when the salt has cooled to 2.4 K and the magnet is ramped down at a sequence of rates designed so that the ADR remains nearly isothermal (see below). As the GRT approaches 100 mK, the voltage across the magnet is set to zero so that the field is fixed and the remaining thermal gradients in the system are allowed to vanish. When thermal equilibrium has again been reached, the ADR is ramped down until the GRT reaches 100 mK at which time the controller is activated.
3.4.2 Physical and Thermal Characteristics

Contribution of Parasitics

Table 3.4 gives the mass budget and heat capacity estimates of the refrigerator. Heat capacity values for Cu and Al are taken from Lounasmaa [79] while values for stainless steel come from [80]. Certainly, the stainless steel dominates the heat capacity (and therefore the time constant) of the system while the large bolometer box dominates the mass. In the table, “Thermal Post” refers to the Cu post which is soldered to the gold wires leading into the salt. The “Linking Rod” is a 13 cm threaded stainless steel rod which is used in the final stages of making the thermal connection between the bolometer box and the salt pill.

<table>
<thead>
<tr>
<th>Part</th>
<th>Material</th>
<th>Mass [g]</th>
<th>Heat Capacity [mJ/K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Post</td>
<td>Cu</td>
<td>54</td>
<td>1.62T</td>
</tr>
<tr>
<td>SPH tube</td>
<td>S.S. 304</td>
<td>24.2</td>
<td>11.25T + 0.0135T^{-2}</td>
</tr>
<tr>
<td>SPH endcap (top)</td>
<td>S.S. 304</td>
<td>11.5</td>
<td>5.35T + 0.0064T^{-2}</td>
</tr>
<tr>
<td>SPH endcap (bottom)</td>
<td>S.S. 304</td>
<td>3.5</td>
<td>1.63T + 0.002T^{-2}</td>
</tr>
<tr>
<td>Stars for Kevlar</td>
<td>Al</td>
<td>8.2</td>
<td>0.336T</td>
</tr>
<tr>
<td>Salt</td>
<td>FAA</td>
<td>113.1</td>
<td>(see text)</td>
</tr>
<tr>
<td>Thermal Link to Bolos</td>
<td>Cu</td>
<td>124.4</td>
<td>3.73T</td>
</tr>
<tr>
<td>Linking Rod</td>
<td>S.S. 304</td>
<td>24.12</td>
<td>11.25T + 0.0135T^{-2}</td>
</tr>
<tr>
<td>Bolometer Box</td>
<td>Cu</td>
<td>500.8</td>
<td>15.02T</td>
</tr>
<tr>
<td>6 Bolometers</td>
<td>mostly Al</td>
<td>156.1</td>
<td>6.40T</td>
</tr>
<tr>
<td>WR-06 diplexer</td>
<td>mostly Al</td>
<td>10.7</td>
<td>0.49T</td>
</tr>
<tr>
<td>WR-10 triplexer</td>
<td>mostly Al</td>
<td>20.3</td>
<td>0.83T</td>
</tr>
<tr>
<td>GRT + wiring</td>
<td>mostly Cu</td>
<td>41</td>
<td>1.23T</td>
</tr>
</tbody>
</table>

We can compare the parasitic loading on the salt pill in cooling the elements in table 3.4

\[ Q_{parasitic} = \int_{1}^{2.4} \left[ 59.14T + 0.0354T^{-2} \right] dT = 170.3 mJ \quad (3.15) \]
with the heat removed from the lattice of the salt during the demagnetization

\[ Q_{\text{salt}} = \int_{1}^{2.4} C_R \, dT = 3.38 \, J \]  \hspace{1cm} (3.16)

So the heat capacity of the objects cooled to 100 mK is only a small perturbation on the cooling capacity of the system. Note that this is only as long as the system remains in near thermal equilibrium during the cooling process. Since the enthalpy of the salt is a strong function of the salt temperature, it is important to extract the heat from the elements being cooled at as high a temperature as possible.

**Cooling Capacity**

The ADR can operate either with or without controlling the temperature of the salt pill. Without control, the magnetic field can be reduced adiabatically to zero and the salt pill temperature bottoms out at the temperature given by the internal field of the salt (for FAA this is \( \sim 35 \, \text{mK} \)). The amount of heat that the salt pill can absorb before warming to 100 mK is given by equation 3.5; for the 113 g FAA salt pill this turns out to be \( Q_u = 71.7 \, \text{mJ} \). For the controlled pill, the heat which can be absorbed before the applied magnetic field is run to zero is given by equation 3.6 which gives, for the same FAA salt pill, \( Q_c = 134 \, \text{mJ} \). Note that the controlled cooling capacity is nearly twice the uncontrolled cooling capacity.

**Quiescent Heat Leak - Suspension System**

For our suspension system, we use Kevlar 49 string with a 10 kg breaking strength. Twelve connections between 1.5 K and 100 mK are made to support the salt pill and sixteen more are made to support the bolometer box. Kevlar 49 has a thermal conductivity of [68]

\[ K = 48 T \frac{\mu\text{W}}{\text{cmK}} \]  \hspace{1cm} (3.17)
The 10 kg breaking strength braid that we use has a cross-sectional area $A_0 = 5.65 \times 10^{-4}$ cm$^2$ and each section has a length of 3.05 cm. This leads to an overall thermal leak from the suspension of 0.28 $\mu$W.

**Quiescent Heat Leak - Wiring**

The low temperature wiring for the refrigerator is composed of a set of twisted pairs of 75 $\mu$m manganin wires glued between two 15 $\mu$m thick Kapton sheets with an additional 15 $\mu$m thick layer of Teflon glue [81]. Using low temperature conductivity numbers for manganin and Teflon from [82] and for Kapton from [83] we calculate the heat leak from the 1.5 K bath due to the wiring to be 0.17 $\mu$W.

The total calculated heat leak for the ADR system is then 0.45 $\mu$W. During the ‘96 campaign, we measured the thermal loading on the system in flight configuration to be 0.54±0.005 $\mu$W which is in fairly good agreement with the calculated value. From the quiescent heat loading and the value of the cooling capacity at 100 mK, we can estimate the hold time of the ADR. For a 113 g FAA salt pill this is 68.9 hours. During the ‘97 campaign and flight, the quiescent heat leak was a factor of two larger (0.97±0.02 $\mu$W) - resulting in a hold time of 34 hours. This is most likely due to a small light leak from the 77 K walls surrounding the 1.5 K shields. Even in the high thermal leak configuration the duty cycle of the ADR remains above 97%.

**Time Constant and Thermal Conductivity to Salt**

We have measured the time constant of the ADR system at 100 mK as well as the thermal conductivity to the salt. Figure 3.14 shows the data used to calculate the two quantities. With the ADR at 96 mK and the voltage across the magnet fixed at zero, a small amount of power is added to the bolometer box by a fixed resistor. Removing the linear offset from the quiescent warming of the system
Figure 3.14  Time Constant and Thermal Conductivity Measurement of ADR.  
0.1 $\mu$W of input power is applied and allowed to heat up the coldstage. After thermal equilibrium is achieved, the power is removed. From this test, the relaxation time constant is measured to be 51 s and the conductivity between the GRT and the salt is 77.7 $\mu$W/K at 96 mK.

and measuring the change in temperature gives the conductivity to the salt pill. Removing the heat and observing the relaxation of the system gives the time constant. From the data in the figure, we calculate the relaxation time constant of the system to be 51 s and the conductivity between the GRT and the salt to be 77.7 $\mu$W/K at 96 mK.

Summary of Characteristics

Table 3.5 gives a summary of the characteristics of the ADR discussed above.
### Table 3.5  Physical and Thermal Characteristics of ADR

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Salt Mass</td>
<td>113 g</td>
</tr>
<tr>
<td>SPH Volume</td>
<td>51 cc</td>
</tr>
<tr>
<td>Heat Capacity of Parasitics</td>
<td>$59.14T + 0.0354T^{-2}$</td>
</tr>
<tr>
<td>Thermal Time Constant</td>
<td>51 s</td>
</tr>
<tr>
<td>Duration of Cycle</td>
<td>2.4 ks</td>
</tr>
<tr>
<td>Quiescent Thermal Load (flight)</td>
<td>0.97 $\mu$W</td>
</tr>
<tr>
<td>Cooling Capacity at 100 mK (Controlled)</td>
<td>134 mJ</td>
</tr>
<tr>
<td>Hold Time at 100 mK (Controlled)</td>
<td>138 ks</td>
</tr>
<tr>
<td>Duty Cycle</td>
<td>98.3%</td>
</tr>
</tbody>
</table>

### 3.4.3 Future Improvements and Innovations

As is the case with many working subsections of instruments, the ADR described here suffered from its own utility. Once the refrigerator was created, the demand on its availability prohibited the design improvements that might have substantially improved the performance. We list here some of the design changes that we are in the process of implementing in ADRs we are currently building.

**Time Constant**

The principal drawback of our ADR is its long time constant. This can be attributed to the following design flaws:

1. The heat capacity of the parasitic mass connected to the salt pill could be reduced by 30% by removing the stainless rod which is used to connect the thermal strap coming from the bolometer box to the thermal strap coming from the salt pill. We are currently replacing this rod with a brass version.

2. The poor thermal conductivity along the SPH body (a stainless steel tube with 10 mil walls) can be improved by adding longitudinal copper strips to the outside of the stainless can. This could be done by soldering the copper...
to the stainless or by electroplating the copper directly. The small increase in eddy current heating would be negligible.

3. Finally, more gold wires are needed to make thermal contact to the salt. Current versions of this SPH have twice the number of gold wires.

While an increase in refrigerator time constant would not affect the operational performance of the ADR in our application, in systems where the external heat loadings occur at higher frequencies, a shorter time constant will be required if the controller is to maintain the same level of temperature stability.

Heat Switch Conductivity

Despite the fact that the duty cycle of the refrigerator is greater than 98%, the 40 minutes required for the refrigerator cycling is slightly on the long side for day-to-day ADR operations. Additionally, because of the large heat capacity of the FAA, the salt pill requires 8 hours to cool from 300 K to 77 K.

It is at the contact at the salt pill itself where the greatest improvements in the conductivity of the heat switch can be made. Here, gold plated copper jaws clamp onto the bare copper post with a force greater than 200 N. A substantial increase in conductivity can be achieved by gold plating the rod coming from the salt [72]. This was not done earlier because of an oversight in scheduling. Additionally one could consider increasing the conductivity by increasing the force of the contact. However, the payoff here only goes as a weak power of the force [84] so substantial modifications to the heat switch design would be required.

Magnet Shielding

As mentioned above, it is not clear that the large magnet shield used in the ADR is required for our bolometer operation. Eliminating this shield would result in a
factor of 2 reduction in the mass of the ADR. We are currently investigating the use of “bucking coils” (shorted coils placed at the end of the magnet) to reduce the stray field. An ADR used to cool bolometers for a rocket-borne x-ray experiment has successfully used this technique with their bolometers [76], however, there is some loss of uniformity in the magnetic field in the absence of the shield.

Future ADR Innovations

Despite the many years that ADRs have been in use, we are currently undergoing a small revolution in ADR design. With the advent of cryocoolers which are capable of cooling a magnet and salt pill to 4 K with nearly 1 W of cooling power, it is only a matter of time before a 100 mK purely electrical refrigerator is built. We are now considering designing and building a second ADR system for operation in a long hold-time cryostat to be used during a 100-day balloon flight. Along with these innovations, it is certain that because ADRs now appeal to more and more physicists, we can expect the rate of new innovations to increase with the demand for higher cooling powers and ease of operation.
Chapter 4

MSAM II — Optics

The MSAM telescope is a 1.372 m, 51° off-axis Cassegrain with a nutating secondary which moves the beam on the sky by \( \pm 80' \). Photons traveling along the optical axis of the telescope are reflected by the fixed primary mirror onto the secondary mirror and then into the radiometer (see figure 4.1).

Inside the cryostat, the internal optics system is composed of two independent (yet parallel) optical chains. The low frequency system is responsible for channels 1–3 (65–110 GHz) while the high frequency system is responsible for channels 4 and 5 (130–150 GHz). In the case of the low frequency system (see figure 4.2), photons enter the cryostat through a 0.5 mm polypropylene window and are split by their polarizations by a wire grid polarizer. The horizontally polarized portion of the beam travels into the high frequency system where the out-of-band portion is rejected (i.e., reflected) into the walls of the cryostat. Meanwhile, the vertically polarized component reflects at 90° off the polarizer, reflects off an elliptical tertiary mirror, and enters a corrugated horn antenna. From here the radiation travels in waveguide and passes through a low-pass quartz-bead filter. The low-passed radiation then crosses a 125 \( \mu \)m gap which separates a 1.5 K waveguide flange from the 100 mK bolometer box. After crossing the gap, the photons are sorted into frequency bins (channels 1–3) by a microwave multiplexer and each group is absorbed by a separate bolometer. The high frequency system works in an exactly analogous manner with the exception that it is horizontally polarized.
Figure 4.1 Schematic of the MSAM II telescope. The gondola support structure, momentum wheel, ground shielding and electronics are not shown for clarity.
Figure 4.2 The low frequency optical chain. The polarizer and high frequency tertiary mirror are not shown for clarity.

radiation which is absorbed by the bolometers.

Before going into the details of the design of the MSAM II optical system, it is worthwhile to examine the constraints placed on CMB optical systems to better understand the various motivations involved in the design. In the following sections, I describe each element of the optical chain in detail — highlighting the various properties which help to satisfy the constraints outlined in section 4.1. Finally, I conclude this chapter by presenting results from measurements of the main beam and sidelobe response of the telescope and commenting on what may be done in the future to improve the instrument.
4.1 Optical System Goals and Constraints

Even before the first detections of CMB anisotropy, our knowledge of the frequency spectrum of the CMB and the confusing astrophysical foregrounds influenced the design of experiments. With the first detection and nearly a dozen others at a variety of angular scales, we now have some degree of knowledge about the spatial spectrum of anisotropy as well. The goal of the design of the MSAM II optics is to exploit this knowledge in order to maximize the scientific return of the instrument. Consideration must be made of the resultant beamsize, its frequency dependence, the sidelobe contamination to the measurement, and the optical loading on the bolometers. Any one of these factors, if mishandled, can hinder the scientific output of an otherwise viable experiment.

4.1.1 Beamsize and Chopping Strategy

As shown in Chapter 1, while most cosmologies predict roughly the same behavior of the correlation function out to the first acoustical peak \(l \sim 300\), beyond this angular scale the degeneracy in model predictions begins to be detectably broken. With this in mind, we have designed MSAM II to have a 20' FWHM beam with the goal of probing the power spectrum out to the second acoustic peak and placing much tighter constraints on various cosmological parameters than possible with earlier 30' experiments.

Probing the sky with a small beam is not enough to make a successful measurement of the power spectrum, however. Since the signal in an experiment which measures total-power from the sky is dominated by fluctuations in the atmosphere and the emission of the optical surfaces, we must create spatial filters by sweeping the beam across the sky via a nutating secondary mirror and assigning a demodulation vector to the resulting data stream. For example, a simple demodulation
would be to take the signal from the time when the beam is to the left of its center position and subtract it from the signal from the time when the beam is to the right of the center position. This “chopping” of the experiment is what allows us to probe higher spatial frequencies of the fluctuations without “contamination” from the lower spatial frequencies (e.g., the monopole). Furthermore, the chopping of the sky signal aids us in stabilizing the instrument against drifts in the detector response.

The beamsize and the parameters of the chopping — the half-amplitude of the chopper throw \( \theta_c \), the chopper waveform, and the demodulation vector\(^1\) — determine the shape of the spatial filter on the sky. The beamsize sets the high frequency (small spatial scale) cutoff of the filter while the value of \( \theta_c \) sets the low frequency cutoff. The chopper waveform and demodulation vector combine to determine the “windowing” of the filter. As in any other case of windowing, a poor choice of chopper waveform and demodulation — combined with the sharp cutoff at the edges of the chopper throw — results in ringing of the filter which can sensitize a demodulation to undesired parts of the fluctuation spectrum as well as increase the correlation with other demodulations.

To calculate the spatial filters or “window functions” of our experiment, I follow the prescription described in appendix A of [85]. Assuming that the telescope is pointed at \( \theta_{ia} \) during the \( d^{th} \) sample of the \( i^{th} \) secondary cycle, the two-point theoretical correlation function of the observations can be related to the intrinsic two-point correlation function by

\[
C_{ij}^S = \left( \frac{1}{n^2} \right) \sum_{a} \sum_{b} \langle T(\theta_{ia}) T(\theta_{jb}) \rangle w_{\mu a} w_{\nu b}. \tag{4.1}
\]

where \( T(\theta_{ia}) \) is the CMB signal, \( w_{\mu a} \) is the \( \mu^{th} \) demodulation vector, and the sum

\(^1\)The demodulation vector is the vector of weights as a function of the beam position on the sky.
is performed over the number of samples in a chopper throw, $n$.

By isotropy in the mean, $\langle s(\theta_{ia})s(\theta_{jb}) \rangle$ only depends on the angular distance, $\theta_{ia,jb}$, between $\theta_{ia}$ and $\theta_{jb}$, so we can decompose equation 4.1 into Legendre polynomials and rewrite it as

$$C_{ij,\mu\nu}^S = \sum_l \frac{2l + 1}{4\pi} C_l W_{ij,\mu\nu}$$

(4.2)

where

$$W_{ij,\mu\nu} \equiv \left(\frac{1}{n}\right)^2 \sum_a \sum_b B(l) P_l(\cos \theta_{ia,jb}) w_{i\mu} w_{j\nu}$$

(4.3)

is the window function associated with the $(i, j)_{th}$ element of the covariance matrix and $B(l)$ is the Fourier transform of the beam pattern.

It is important to note here that this is not an accurate method for determining the expectation value of an experiment. The window functions defined in equation 4.3 are representative of an experiment where the measured points on the sky are statistically descriptive of the underlying correlation function — that is, that they are well separated on the sky and uncorrelated! Since this is not true in practice for CMB experiments, we can only use equation 4.3 as a guide in choosing the parameters of the optical system. For accurate simulations of the experiment’s potential for extracting information from the underlying correlation function we need to implement the techniques described in Chapter 5. However, studying the “on-diagonal” $(\mu = \nu)$ elements of equation 4.3 is an excellent way to guide the choice of optical system parameters.

**Chopper Parameters**

Choosing the chopper parameters which will create optimal window functions not only requires an understanding of the total-power beammap $B(l)$, but the chopper mechanism’s mechanical capabilities and the microphonic interaction (if
any) between the chopper and the bolometers as well. The MSAM II chopper is described in detail in section 4.3.5. It is capable of moving the beam on the sky in a programmable waveform limited by the peak accelerations and frequency of the chopping.

As mentioned above, the high spatial frequency cutoff of the window functions is determined by the beamsize while the low frequency cutoff, and hence the width of the window function, is set by the chopper half-amplitude, $\theta_c$. In rough terms, the number of frequency bands we can measure is given by $2\theta_c$/FWHM and the width of these bands is $\pi/\theta_c$. For the 20′ beamsize of MSAM II, we are able to chop the beam with $\theta_c = 80′$ before the edge taper of the beam — the power level of the beam at the edge of the optical surface — on the primary mirror becomes too large.

There are two methods for minimizing ringing in the window functions. In both cases, the goal is to reduce the weight of the data at the extreme ends of the chopper throw. One way we can do this is by applying an apodizing window (eg, a Hann window) to the demodulation vector. This method results in loss of precious data from the outer reaches of the chopper throw. This loss can be offset if the experiment spends less time observing at the far ends of the chopper and more time at the center. Knox has found that the optimal chopper motion, when applying a Hann window to the demodulation vector, is an inverse sine function of time [86]. Unfortunately, this is a very difficult chopper waveform to implement at 2.5 Hz (our desired chopping frequency) due to the infinite accelerations required at the endpoints. While the endpoints can be rounded (by low-pass filtering of the waveform), the force at the endpoints, which goes as $1/\tau_t^2$ where $\tau_t$ is the transition time, must be small enough to not significantly disturb the bolometers.

---

2 Our constraint on edge taper on all optical surfaces is -40 dB of the on-axis power. In fact, channel 1 suffers a 30 dB edge taper at full chopper throw - leaving us with the option of ignoring the outside ±20′ of the throw for that channel if contamination is suspected.
and must be realizable by the small motors driving the chopper. Neither of these conditions could be met with the inverse sine wave at 2.5 Hz.

A second method for minimizing ringing is to implement the “chirp” waveform first suggested to us by Fixsen [87]. Here, the chopper is driven by a sine wave whose amplitude is a sine wave of a higher frequency. The net motion of this waveform is maximized in the middle of the chop and minimized at the endpoints. The downside of this chopping scheme is the large number of transitions which may lead to excess microphonic bolometer noise. In addition, the chirp waveform requires a larger power draw on the motors than a “smoother” chopping scheme. Figure 4.3 shows a number of possible waveforms for 1/2 of the chop cycle.

For MSAM II we use a triangle wave chop which is low pass filtered to round the transition edges. The frequency of the chop is 2.5 Hz and the transition time is 8 ms. There is no evidence of microphonic response of the bolometers to this motion.

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{chirp_waveform.png}
\caption{A number of possible chopper waveforms.}
\end{figure}
At the time of the chopper construction, the microphonic coupling of the bolometers was unknown so we were not certain that a triangle wave could be used as the chopper waveform. A sine wave, with its smoother transitions, was the fall-back plan. To estimate the differences in the resulting science from the two chopper waveforms, we calculated the window functions in each case using simple sine wave demodulation vectors. Gaussian models were fitted to the resulting window functions in order to estimate a peak value and a width. The results are summarized in table 4.1.

Table 4.1 Window function parameters for sine and triangle wave chopper waveforms. $n$ is the mode-number where the demodulation is given by $\sin(2n\pi t)$ where $t$ goes from 0 to 1 over the chopper throw. The width of the window function is (arbitrarily) defined as the 1/e full-width of the best-fit Gaussian.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$l_{\text{Peak}}$</th>
<th>Width</th>
<th>$l_{\text{Peak}}$</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>118</td>
<td>155</td>
<td>103</td>
<td>137</td>
</tr>
<tr>
<td>2</td>
<td>176</td>
<td>173</td>
<td>155</td>
<td>152</td>
</tr>
<tr>
<td>3</td>
<td>238</td>
<td>177</td>
<td>203</td>
<td>162</td>
</tr>
<tr>
<td>4</td>
<td>301</td>
<td>177</td>
<td>249</td>
<td>169</td>
</tr>
<tr>
<td>5</td>
<td>364</td>
<td>176</td>
<td>294</td>
<td>175</td>
</tr>
<tr>
<td>6</td>
<td>428</td>
<td>174</td>
<td>338</td>
<td>180</td>
</tr>
<tr>
<td>7</td>
<td>491</td>
<td>173</td>
<td>382</td>
<td>185</td>
</tr>
<tr>
<td>8</td>
<td>555</td>
<td>171</td>
<td>425</td>
<td>189</td>
</tr>
</tbody>
</table>

As is clear from table 4.1, window functions constructed from the triangle wave chopper motion peak at higher $l$ values than the corresponding sine wave window functions. In addition, the calculated signal-to-noise, $s/n$, assuming a standard cold dark matter universe (SCDM) for the triangle wave window functions are higher than sine wave window functions with the same peak value. This tells us that efforts to ensure bolometer compatibility with a triangle wave chopper motion will be well rewarded in the scientific return of the instrument.
Demodulation Vector and Window Functions

The demodulation (weight) vectors, \( w_k \), are constructed to remove an arbitrary offset from the data

\[
\left( \frac{1}{n} \right) \sum_{k=1}^{n} w_k = 0 \quad (4.4)
\]

and so that the noise correlation between demodulations vanishes

\[
\langle N_1 N_2 \rangle = 0 \quad (4.5)
\]

where

\[
N_1 = \sum_{k} w_k n_k \quad (4.6)
\]

and \( n_k \) is the noise in sample \( k \). Note that this means that the “optimal” demodulation vector for an experiment depends on the characteristics of the noise of the experiment. Since this is never known \textit{a priori}, we design the experiment assuming that the noise is white and wait until after-the-fact to construct the proper vectors \( w_k \) which will hopefully not be too different.

As an aside, it is interesting to note that the demodulation vector and chopping frequency determine the required sampling rate of the bolometers. For example, for a chopper chopping at 2.5 Hz, the sky signal from a sine wave demodulation with mode-number, \( n \), appears in the detector signal at \( 2.5 \times 2 \times n \) Hz. For MSAM II the highest frequency demodulation of interest is the \( n = 8 \) sine wave which appears in the bolometer signals at 40 Hz.

Finally, note that as mentioned above, we make no implications that the window function formalism is a proper method for analyzing the data or results from an experiment. The window functions neither accurately reflect the observing strategy of the experiment nor the statistics of the results. Instead, the idea here is that by computing the window functions we can glean insight into how to
optimize some of the parameters of our optical system.

### 4.1.2 Frequency Dependence of Beams

MSAM II measures the CMB (as well as foreground sources) in five frequency channels spanning 65 to 170 GHz. If we are to combine the CMB measurements from the separate channels while adequately removing foreground contamination, the beamsize of the instrument must be roughly frequency independent. We can quantify this in the following way \[88\]. Consider two beams from an experiment \( B_1 \) and \( B_2 \) that may differ in size, be offset from each other in spatial coordinates, or both. The signal from the sky that is measured with each beam is given by

\[
t = \int d\Omega B(\Omega)T(\Omega)
\]

where \( t \) is the signal from the detectors and \( T \) is the “true” temperature distribution on the sky. The difference in what we measure between the two beams is

\[
\delta = \int d\Omega [B_1(\Omega) - B_2(\Omega)] T(\Omega).
\]

To get an idea of the size of this error we can assume that \( T \) is a Gaussian field and that we know the correlation function \( C \) of what we are observing\(^3\). The relationship between \( T \) and \( C \) is conveniently expressed in the Fourier domain\(^4\), as

\[
\langle T(\vec{k})T^*(\vec{k}') \rangle = \delta(\vec{k} - \vec{k}') C(\vec{k})
\]

so that

\[
\delta = \int d\vec{k} \left[ B_1(\vec{k}) - B_2(\vec{k}) \right] T(\vec{k})
\]

\(^3\)It is worth pointing out that this method for comparing beamsmaps explicitly depends on assuming the “answer”, that is, knowing \( C \).

\(^4\)For observations which are within \( \sim 10^\circ \) on the sky (such as the MSAM II observations), the flat-sky approximation can be used. This is further elaborated upon and justified in Chapter 5.
and

\[
\langle \delta^2 \rangle = \int d\vec{k} \left| B_1(\vec{k}) - B_2(\vec{k}) \right|^2 C(\vec{k}) \tag{4.10}
\]

where \( B_i(\vec{k}) \) and \( T(\vec{k}) \) are the Fourier transforms of \( B_i(\Omega) \) and \( T(\Omega) \) respectively.

To make this a relative error we divide by the size of \( B_1 \) and define

\[
\beta \equiv \sqrt{\frac{\langle \delta^2 \rangle}{\int d\vec{k} \left| B_1(\vec{k}) \right|^2 C(\vec{k})}}. \tag{4.11}
\]

The observational result of this is that if we measure the sky with two beams that differ (in this measure) by a factor of \( \beta \), then looking at the same field we would expect scatter in the measurements by the same factor \( \beta \) in addition to the instrument noise. Since our calibration is usually only good to 10\%, scatter in the measurements worse than 10\% would begin be noticeable depending on the signal-to-noise of the measurement.

### 4.1.3 Sidelobe Contamination

Sidelobe contamination of the CMB data is a major concern of experimentalists. The earth, moon, balloon, and telescope support structure — all at 300 K — each compete with the few tens of \( \mu \)K signal of the CMB. We cope with this by ensuring that our optics are “low noise”, that is, that the microwave beam is well defined, that the optical path is clear of obstructions, and that the edge illumination on the optical surfaces is low.

It is interesting to note here why it is so important to the far field sidelobes of the telescope for the edge illumination on the optical surfaces to be low. According to Kirchhoff [89], the field distribution at a point down-range of a surface \( S_1 \) is derived from the field distribution on the surface via

\[
\psi(\vec{x}) = \frac{k}{2\pi i} \int_{S_1} e^{iRk} \frac{\vec{n}' \cdot \vec{R}}{R} \frac{1 + i/kR}{R} \psi(\vec{x}') d\vec{a}' \tag{4.12}
\]
where \( \vec{n} \) is the local normal to the surface. In the far field (\( R \gg \) the characteristic size of the surface), equation 4.12 reduces to

\[
\psi(\vec{x}) = \frac{k}{2\pi i R^2} \int_{S_1} e^{ikR\psi(\vec{x}')} d\vec{a}'
\]

which is just the Fourier transform of the initial field \( \psi \) over the surface \( S_1 \) weighted by a factor of \( 1/R^2 \).

Now it is clear from equation 4.13 why low edge illumination is important. At the edge of an optical surface the field steps to zero in a discontinuous manner. As a result, ringing occurs in the Fourier transform of the field distribution which shows up as sidelobe contamination in the far field. Thus, a smooth taper of the field amplitude to zero at the edge of the optical surface is required to minimize the ringing (and therefore the sidelobes). For MSAM II, we have designed the optics with a minimum edge taper which is -40 dB of the on-axis power. While this seems like a conservative and costly approach, it is important to recognize that a -40 dB edge taper in power is only a -20 dB edge taper in the field amplitude.

We will refrain from making the standard arguments for what an appropriate sidelobe response is. The fact is that it is very difficult (and somewhat misleading) to come up with a reasonable definition for “appropriate” in this case. For example, the response from the earth is the integral over the demodulated sidelobe response and the contrast in surface emissivity and temperature. While the former quantity is in practice impossible to measure, the latter is a statistical quantity for the region below the experiment which has not been measured (and most likely never will be). Instead, we design the optics with a healthy combination of paranoia and experience with the goal of having an experiment with lower sidelobe levels than previously successful CMB experiments. Section 4.3.7 presents results from the ‘97 campaign sidelobe measurement tests and demonstrates our success with this aspect of the optical system.
4.1.4 Bolometer Optical Loading

Finally, the optical system must not substantially load the bolometers beyond the background loading of the CMB. This precludes “warm” optical elements which have moderate (≥1%) emissivities and places a limit on the total number of optical elements in general. Table 4.2 shows the expected in-flight optical loading on the bolometers for MSAM II. Note that the optical surfaces (in this case two 250 K surfaces and a 77 K surface) each contribute an equal amount of in-band power as the 2.7 K CMB.

**Table 4.2** The expected contributions to the bolometer optical loading in MSAM II’s channel 2 (80–95GHz). The optical efficiency is assumed to be 30%. Emissivity of the three aluminum mirrors is assumed to be 0.5%.

<table>
<thead>
<tr>
<th>Source</th>
<th>Power (pW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMB (2,726 K)</td>
<td>0.07 pW</td>
</tr>
<tr>
<td>Atmosphere (at 120 kft)</td>
<td>0.0003 pW</td>
</tr>
<tr>
<td>Optics (250 K)</td>
<td>0.078 pW</td>
</tr>
</tbody>
</table>

4.2 The Band Defining Optics

A constant concern for CMB experiments is the possible contamination of the data by foreground emission. For sources that are fixed with respect to the earth, we can demonstrate immunity by observing at different times of the year or in separate places on the earth. Galactic and extragalactic foreground sources are fixed with respect to the sky, however, and require multi-frequency measurements to disentangle them from the CMB.

There are three principal foreground contaminants that we must consider when designing our experiments. Galactic synchrotron emission, generated by relativistic electrons moving through the galactic magnetic field, has a frequency dependent intensity which goes as $\nu^\beta$ where $\beta$ ranges from -0.75 to -1.0 depending on the location in the sky [90]. While synchrotron radiation makes up the dominant
sky signal at 408 MHz [91] (where full-sky maps are available), it has fallen to a level below that of the CMB fluctuations by the MSAM II frequency bands.

Free-free emission (bremsstrahlung radiation) is generated by electrons scattering off protons in warm ionized galactic gas. Free-free emission also behaves as a power-law spectrum with a frequency dependent intensity characterized by \(\nu^\beta\) where \(\beta \sim -0.1\). Like synchrotron emission, free-free emission is expected to be nearly negligible for frequencies near the 2.7 K CMB peak. A useful tracer of free-free emission is available through optical observations of the H\(\alpha\) line from recombining hydrogen in the ionized gas [92].

Galactic dust dominates other galactic sources for frequencies above a few hundred GHz. The interstellar dust spatial and temperature distribution have been well mapped by the IRAS experiment [93] at 3000 GHz at arcminute angular scales and also at 7\(^o\) angular scales by the COBE/DIRBE experiment [94]. With the use of these maps it is possible to limit CMB observations to relatively dust-free regions of the sky.

Finally, as telescope beamsizes become smaller and smaller, extragalactic point sources become more and more of a worry. Our knowledge of these sources is severely limited by the lack of a flux-limited sky survey at millimeter wavelengths. While these sources are a fundamental problem for small scale experiments operating at frequencies less than \(\sim 30\) GHz, they are only expected to contribute \(\Delta T/T \sim 1\mu K\) at our wavelengths and with our beamsize [95].

For MSAM II, we adopt the strategy of observing at frequencies with the greatest contrast between the signal from the CMB and that of the known foreground sources. This is the frequency range of 65–170 GHz. At the back end of the optical chain, the copper bolometer box houses the six bolometers and two microwave multiplexers which define the five pass bands of the system. Channels 1–3 are mounted on a WR-10 triplexer which splits the WR-10 band (65–110 GHz)
into three distinct frequency bands (65–80, 80–95, and 95–110 GHz) while two more bolometers are fed by the WR-06 band splitting diplexer. In this case the bands are 130–150 GHz and 150–170 GHz. A final bolometer is blanked off at its waveguide input and serves as a dark channel for diagnostic purposes. While in principle, we should have no response to foreground sources in MSAM II, the availability of five separate frequency bands allows us the ability to spectrally remove any contamination without too great a sacrifice in system noise.

Below, I describe the band defining elements of the optical system with an emphasis on the frequency dependence of the transmission through the waveguide gap between the quartz filters and the bolometer box. At the end of this section, I give measured values of the band pass and optical efficiency of the system.

4.2.1 Stripline Multiplexers and Quartz Filters

The low and high frequency optical systems each span one waveguide band. For the low frequency system this is the WR-10 band which passes a single mode between 65 and 110 GHz. For the high frequency system this is the WR-6 band which passes a single mode between 130 and 150 GHz. In both cases, the incoming radiation is first high-pass filtered by the waveguide itself. An initial level of low-pass filtering is then done by a pair of quartz-bead filters, and finally, the five bands of the system are defined by two microwave multiplexers - a triplexer in the low frequency system and a diplexer in the high frequency system.

Stripline Multiplexers

One of the principal advantages of a single mode optical system over multi-mode systems is the large amount of engineering developmental work that has taken place in the field over the past 50 years. We take advantage of this work in the fundamental band-defining elements of MSAM II — the suspended stripline
circuit multiplexers.

The multiplexers are made by Al Hislop at Pacific Millimeter [96] and are composed of copper transmission lines which are suspended on a thin Teflon substrate. Coupling is made from waveguide to the suspended stripline circuit by a small probe transition which sticks into the waveguide opening. Once in the suspended stripline circuit, the radiation experiences a number of capacitive and inductive filters which either pass the radiation (if it is in the correct band) or reflects the radiation on to the next channel. Radiation which is out of the waveguide band is reflected out the front of the filter. For the WR-10 band we have a triplexer which splits the 65-110 GHz band into three channels (65–80, 80–95, and 95–110 GHz). Losses in the filtering are greater at higher frequencies so the WR-6 band is split into two equal bands (110–150 and 150–170 GHz).

We had a few problems with the filters after repeated thermal cycling between 300 K and 100 mK. After a number of cycles, we noticed that the coupling efficiency of the filter had dropped by nearly a factor of two. Upon inspection it was found that the copper transmission line had curled up under the differential contraction of the Teflon substrate. After straightening, the multiplexers had their original coupling efficiencies again. The second problem required involvement of the filter maker. Again, after a number of thermal cycles, it was found that one of the small traces making up the channel 2 filter had broken. Al Hislop repaired the trace immediately and the filter was returned within a few days. Unfortunately, one year later the same trace broke a second time. At this point the entire filter was replaced. While neither of these problems proved fatal to the experiment, both are quite annoying. We are currently searching for ways to increase the reliability and durability of these filters.

Hislop measured the insertion loss of the multiplexers to be between 2 and 3 dB in-band and below 40 dB out of band. The band edges are quite sharp
Figure 4.4  The coupling efficiency of the microwave triplexer as measured by Al Hislop.

(generally $\sim 2$ GHz to the 40 dB point) and the overlap between bands is small. Figures 4.4 and 4.5 show the coupling efficiency of the stripline filters as measured by Hislop.

Quartz Bead Filters

While MSAM II observes in frequency bands which are likely to be immune from Galactic foreground contamination, the existence of a high frequency leak (blue leak) in the optical chain could introduce an ambiguous signal from Galactic dust. Since the high frequency behavior of the stripline filters is not well measured, we employ a second level of filtering to ensure that any blue leaks in the system are small\(^5\). This filtering is done by quartz bead filters in waveguide at 1.5 K.

\(^5\) Actually, the quartz bead filters are also required to shield the ADR from the 300 K thermal load during ground tests of the instrument
Figure 4.5  The coupling efficiency of the microwave diplexer as measured by Al Hislop.

The MSAM II quartz bead filters were developed by Khurram Farooqui at Brown and are variations on a glass bead filter design by Sato et al. [97]. Granusil 4045 silica (fused quartz beads) [98], potted in polypropylene, fills a 1 cm length of waveguide. The polypropylene is used to improve the thermal conduction and cohesion of the filter. On either end, short Teflon stubs are used to provide an adiabatic transition into the quartz beads and thereby reduce in-band scattering from the interface. For details of the filter design see Farooqui’s thesis [49].

Two sets of tests were done to determine the filters’ insertion loss at high frequencies. First, a near infrared (TEA CO$_2$) pumped laser was used to measure insertion loss at 3313 GHz and at 866.5 GHz. In both cases, the measurement was noise limited at the -30 dB insertion loss level.

The second blue leak test of the filtering system is done by chopping the radiometer beam between a 300 K and 77 K load. This test is described by Ruhl
in [99]. A variety of high pass filters (thick grill filters constructed at GSFC) are used to determine the contribution to the signal from out of band radiation. If we define $\gamma$ as the ratio of signal with the filter in place to the signal with no filter,

$$\gamma = \frac{\Delta V_{\text{in}}}{\Delta V_{\text{out}}},$$

then following Ruhl’s thesis, in order to achieve an out of band dust contribution which is $10^{-5}$ that of the CMB, for each band we need $\gamma \geq 7$. Table 4.3 summarizes the results of these tests. Each channel, with the exception of channel 3, passes the blue-leak test. When these tests were made, channel 3’s optical efficiency was measured to be extremely high ($\sim 65\%$). Since no compensation to the extra optical loading was made to channel 3’s bias during the blue-leak test, the bolometer was significantly less responsive in the filterless measurement.

**Table 4.3** Results of the thick grill filter blue-leak test. A value of $\gamma \geq 7$ is needed to keep the signal from the CMB well above that of Galactic dust contamination. Channel 3’s incorrect bias setting leads to a lower-than-expected value for the out of band rejection.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Frequency [GHz]</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65–80</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>80–95</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>95–110</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>130–150</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>150–170</td>
<td>24</td>
</tr>
</tbody>
</table>

### 4.2.2 Waveguide Gap

Figure 4.2 shows the geometry of the free space gap in the optical chain which allows the bolometer box to be thermally isolated. Microwaves traveling along the low and high frequency optical chains must bridge this gap in order to enter the bolometer box. Consequently, the gap must be both a low loss conductor
and a stable conductor. The low insertion loss is needed since the sensitivity of the experiment is approximately proportional to the optical efficiency of the optical chain. Stability of the gap is required so that there is no false signal microphonically produced in the system.

Since the input to the bolometer box is machined to match a standard waveguide flange, the gap between the bolometer box and the waveguides below is accurately modeled by two pieces of waveguide which have been separated from each other at their flanges by some amount \( \epsilon \). As the photons propagate to the gap and enter free-space, they experience an impedance mismatch causing a small portion of the beam to be reflected while the remainder crosses the gap to enter the waveguide opening on the other side. Here the impedance changes again and there is another opportunity for loss due to reflections and leakage.

One way to minimize the loss from a small waveguide gap is by machining a groove into one of the facing waveguide flanges. The idea is that by properly placing and sizing the groove, it is possible to set up a reflection which is out of phase with the leakage. Of course, since this is a frequency-dependent effect, the “choke” groove will only be effective over a small part of the waveguide band.

We chose to machine choke grooves into the flanges of the 1.5 K quartz filters after experimentally seeing that there were distinct regions in each waveguide band where the insertion loss (the amount of radiation lost to reflection or absorption in an optical element\(^6\)) was worse than a few dB. In the low frequency optical chain this occurred at 90 GHz and in the high frequency optical chain at 140 GHz. Since we were interested in maximizing the gap distance (to avoid the possibility of a thermal short between the 100 mK bolometer box and the 1.5 K quartz filters) we scaled a choke groove design of Jeff Peterson [100] and then empirically determined the best choke groove depth using our insertion-loss setup.

\(^6\text{This is } 1-\tau \text{ where } \tau \text{ is the transmission.}\)
Figure 4.6  Design of the choke grooves (all dimensions are in inches).

and a number of waveguide flanges. The choke grooves which resulted in the lowest loss at a gap separation of 250 $\mu$m are shown in figure 4.6.

Figures 4.7 and 4.8 show the insertion loss across the gap as a function of frequency and gap size for the low and high frequency optical chains respectively. In the low frequency system there is a clear resonance that appears at the top of the band and moves to lower frequencies as the gap is widened. From the plot it is clear that the gap must be kept under 250 $\mu$m to keep the optical efficiency in channel 3 (95–110 GHz) acceptable. For the high frequency system, the optical efficiency with a 125 $\mu$m gap is quite good while gap sizes of 250 and 375 $\mu$m result in a degraded channel 5.

We performed the same measurements of the gap insertion loss at 77 K using the actual bolometer box and suspension plates. As expected, the gap widened at 77 K due to the thermal expansion of the Kevlar suspension strings. This effect can be reproduced and allows us to calibrate the change in gap size by monitoring the position of the resonance in the low frequency optical chain. The position of the resonance was determined as a function of gap size by making measurements
Figure 4.7  Insertion loss in choked gap between 75 and 110 GHz. The insertion loss was measured for gap sizes of 125 $\mu$m (5 mils), 250 $\mu$m (10 mils), and 375 $\mu$m (15 mils).

Figure 4.8  Insertion loss in choked gap between 130 and 170 GHz. The insertion loss was measured for gap sizes of 125 $\mu$m (5 mils), 250 $\mu$m (10 mils), and 375 $\mu$m (15 mils).
at 300 K with a gap set by a known shim thickness. The resulting gap size at 77 K agrees very well with the value predicted by assuming 0.1% expansion of the Kevlar strings. Because the gap widens by \( \sim 125 \mu m \) during cooling, the bolometer box is initially set 25–50 \( \mu m \) above the 1.5 K quartz filters before being closed in the cryostat and cooled.

### 4.2.3 Bandpass and Optical Efficiency Measurements

We performed bandpass measurements of the radiometer during the ‘97 campaign in Palestine, Texas. The measurement was made by actively quadrupling the output of a microwave sweeper, operating from 1–20 GHz, and then either passively doubling (low frequency system) or tripling (high frequency system) the result. For the low frequency system, we are able to probe the frequency response of the instrument from 75–110 GHz. With the tripler, we are able to measure frequency response between 125 and 170 GHz.

The setup of the measurement was as follows: The microwave sweeper, quadrupler, and doubler (or tripler) fed a small pyramidal horn antenna. This set of hardware was mounted to the top of a large aluminum ladder, carried 80 m from the gondola, and hoisted 5 m off the ground by a forklift. The source and antenna on top of the ladder were surrounded by Eccosorb foam\(^7\) to help reduce back-scattering effects. In addition, the source was amplitude-modulated at 52 Hz in order to improve the rejection of low frequency drifts in the system.

The gondola remained in the staging building for this test. The front wheels were removed to allow the elevation drive to reach low enough for the main beam to see the sources. After an initial detection of the sources, the signal was peaked by moving the gondola in azimuth and elevation until the maximum signal was

---

\(^7\)Eccosorb, made by Emmerson & Cuming [66] is an iron-loaded dielectric which has an emissivity near 1 in the microwave regime.
Figure 4.9  Results from the low frequency bandpass measurement of MSAM II. Each channel’s response has been (arbitrarily) normalized to the peak response of the band. Data taken below $\sim 75$ GHz is contaminated by detector noise.

received. Data was taken with the flight ground station. The sweeper frequency signal was brought from the sweeper via a very long section of coaxial cable and inserted into the channel 0’s postamp so that it could be a part of the telemetry stream.

Measurements were taken first for the low frequency system and then for the high frequency system. For each sweep of the frequency band, detector linearity was checked and more attenuation was added at the source if needed. Once a stable state was found, the source was allowed to sweep a number of times over each frequency band. In analyzing the data, a separate bandpass measurement was made for each sweep of the source and the set of measurements were averaged to make figures 4.9 and 4.10.

The bandpass of the instrument is shown in figures 4.9 and 4.10. Because the source has low power below 75 GHz, the measurements at those frequencies are contaminated by the bolometer noise. While there is quite a bit of structure
Figure 4.10 Results from the high frequency bandpass measurement of MSAM II. Each channel’s response has been (arbitrarily) normalized to the peak response of the band.

in the bands, in all they are well separated from each other with the exception of channel 3 which was found to have an electrically-induced coupling with the other channels at the 2-10% level. This effect has been removed in the making of this plot. We are currently planning another measurement of the bandpass of the system using a Fourier Transform Spectrometer at the University of Chicago during the fall of 1997.

Optical Efficiency

The optical efficiency of a telescope is the fraction of photons which, on entering the optical system, are absorbed by the bolometers. Raising the optical efficiency of a radiometer directly increases the “signal” while only slightly increasing the noise. While multi-mode systems can afford to have optical efficiencies as low as 10%; in order to remain background-noise limited, single-mode systems must have optical efficiencies $\geq 30\%$ since all of the power from the other modes has been
Figure 4.11  The setup for the optical efficiency tests. A total-power measurement is first made with the two variable attenuators in series and then compared to the measurement with the test piece in-line to determine its insertion loss.

reected by design.

Since the optical efficiency of a system is the product of the effect of its parts, we measure the total optical efficiency of the system both in parts, at 300 K for diagnostic purposes, and as a whole in flight configuration. Table 4.4 shows the measured optical efficiencies for a number of elements of the two optical systems. These measurements were performed using a 1–20 GHz microwave sweeper whose output was multiplied to achieve either 75–110 GHz or 130–170 GHz. Each test was a substitution measurement where the piece in question was inserted into the test setup. Since standing waves are a large source of systematic error for measurements like this, two waveguide attenuators and a directional coupler were used in the setup to attenuate reflections. The test setup is shown in figure 4.11.

We have also measured the optical efficiency of the radiometer as a whole in flight configuration. This test has been done in two ways. First, a blackbody calibrator, based on the design of the reference source in the FIRS experiment [101],
Table 4.4 Measured optical efficiencies for a number of pieces of the optical system. These measurements were done on the lab bench at 300 K and are averaged over the bandpass of the system. See figures 4.7 and 4.8 for plots of the insertion loss of the waveguide gap.

<table>
<thead>
<tr>
<th>Element</th>
<th>Optical Efficiency [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolometers</td>
<td>≥ 90%</td>
</tr>
<tr>
<td>Stripline Filters</td>
<td>≥ 50%</td>
</tr>
<tr>
<td>Waveguide Gap</td>
<td>≥ 93%</td>
</tr>
<tr>
<td>Quartz Filters</td>
<td>≥ 80%</td>
</tr>
<tr>
<td>Waveguide Bends</td>
<td>≥ 95%</td>
</tr>
<tr>
<td>Waveguide Flanges</td>
<td>≥ 95%</td>
</tr>
<tr>
<td>Total Piecewise Optical Efficiency</td>
<td>≥ 30%</td>
</tr>
</tbody>
</table>

was installed at the aperture of each of the horn antennas. These calibrators fill the microwave beam and act as a termination for each optical system. With the bolometers at 100 mK, a known amount of power was added to a heater on the calibrators — causing a temperature change in the bolometers. If we assume that the calibrators are black and that their temperature distribution is even and is accurately measured by the calibrator’s silicon diode thermometer, then knowing the difference in blackbody power delivered by the calibrators and the change in power measured by the bolometers gives the optical efficiency of the system. Table 4.5 gives the results of this test for each channel.

It is clear that tables 4.4 and 4.5 are not consistent. In fact, we have not been able to reproduce the warm measurements of the optical efficiency when the cryostat is cold. Furthermore, it has been shown that thermally cycling the dewar to 300 K and back to 1.5 K results in a change in the optical efficiencies. This problem is, in part, due to the systematic errors involved with measuring the system optical efficiency as well as the thermally-induced bending of the Teflon in the stripline filters described in section 4.2.1. We also have suspicions of water or nitrogen ice being trapped inside of the quartz filters on cooling.

As a diagnostic of this problem, we use a chopped cold load to measure the
optical efficiency of the radiometer without warming the dewar to 300 K. An Eccosorb coated “fan” is used to chop the bolometric signal between the 300 K blades and a 77 K piece of Eccosorb submerged in liquid nitrogen. Unfortunately, the rapid growth of frost on the cold surfaces makes this measurement less than ideal and as a result gives us only lower limits to the optical efficiencies. Results from this test are also reported in table 4.5.

Note that the two tests were taken several weeks apart. In between these tests, one of the transmission lines on the WR-10 tripler was found to be broken. This caused the optical efficiency in channel 2 to drop below 5% — necessitating the extraction and repair of the tripler. The 77 K test was done with the new tripler in place and after the last thermal cycling of the cryostat before the ‘97 flight.

Table 4.5 Measured optical efficiencies in each channel for the cold radiometer for the internal calibrator test, $\eta_{int}$, and the liquid nitrogen cold load test, $\eta_{77}$. These tests were taken several weeks apart. The 77 K chopped load test was performed after the WR-10 triplexer had been repaired. As mentioned in the text, these numbers are lower limits.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\eta_{int}$</th>
<th>$\eta_{77}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20%</td>
<td>26%</td>
</tr>
<tr>
<td>2</td>
<td>39%</td>
<td>24%</td>
</tr>
<tr>
<td>3</td>
<td>69%</td>
<td>23%</td>
</tr>
<tr>
<td>4</td>
<td>22%</td>
<td>15%</td>
</tr>
<tr>
<td>5</td>
<td>19%</td>
<td>6%</td>
</tr>
</tbody>
</table>

4.3 Beam Defining Optics

The desired beamsize of the MSAM II telescope evolved during the design and construction of the instrument. Initially, the goal of MSAM II was to independently confirm the results of the MSAM I measurements at the MSAM II frequency bands. This would require a 30$\prime$ (FWHM) beam and an observing strategy that mimicked that of the MSAM I observations. Fortunately, in 1995 the Saskatoon
experiment did this for us [16], freeing the MSAM II experiment to push the beam size down from 30' to 20' and potentially allow for tighter constraints on cosmological models. Of course, the price paid for a smaller beam size is either larger optical elements or a larger edge taper (the power striking the edge of the mirror) on existing optical elements. We were able to change the MSAM II beam size by redesigning the secondary mirror and loosening the constraints on the edge illumination on the primary mirror for channel 1\textsuperscript{8}.

A design goal of the experiment which remains constant is the desire to have a frequency independent beam. Since the foreground sources described in section 4.2 are not scale invariant, the beam size of the instrument must be roughly independent of frequency in order to have a successful spectral removal of contamination.

Below I describe the process of designing the beam forming optics. Beginning with a description of the theory used to describe the system, I go on to define and describe each of the optical elements of the telescope. In the concluding sections, I then give measured data on the beam size and side lobe level of the instrument in flight configuration.

4.3.1 The Theory of Gaussian Optics

Gaussian Optics provides a simple theory for predicting the output of single mode optical systems. Following, I give a review of the theory based on a discussion by Goldsmith [102].

The wave equation for a single component of a field $\psi$ in free space (ignoring the temporal component) is

$$\nabla^2 \psi + k^2 \psi = 0$$

\textsuperscript{8}In order to maintain the -40 dB edge taper constraint, the data from channel 1 for beam throws $\geq 60'$ would have to be thrown away. This accounts for 25\% of the total channel 1 data.
where \( k = 2\pi/\lambda \). We can hypothesize a solution to this equation as a plane wave traveling in the (arbitrary) \( z \)-direction with an amplitude \( u(x, y, z) \) that is a slowly varying function of \( z \) (that is, \( \frac{\delta^2 u}{\delta z^2} \ll \frac{\delta u}{\delta z} \)) where

\[
\psi = u(x, y, z)e^{-ikz}.
\] (4.16)

Substituting this into eq 4.15 we have

\[
\frac{\delta^2 u}{\delta^2 x} + \frac{\delta^2 u}{\delta^2 y} - 2ik \frac{\delta u}{\delta z} = 0.
\] (4.17)

This differential equation can be solved by a product of a Hermite polynomial, a Gaussian transverse amplitude variation, and a phase factor\(^9\). This solution set represents the various modes of the beam of radiation. The fundamental mode of propagation, which as shown below is the dominant mode in the beam produced by a corrugated horn antenna, is

\[
\psi = A \frac{w_0}{w(z)} \exp \left( -\frac{r^2}{w^2(z)} \right) \exp \left( -ikz \right) \exp \left( \frac{-i\pi r^2}{\lambda R(z)} \right) \exp \left( i \arctan \frac{\lambda z}{\pi w_0^2} \right)
\] (4.18)

where

\[
w(z) = w_0 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2}
\] (4.19)

is the beam amplitude “radius”, \( A \) is an arbitrary constant, and

\[
R(z) = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right]
\] (4.20)

is the radius of curvature of the phase front.

\( w_0 \) in equations 4.19 and 4.20 is called the “beam waist radius” and is the minimum size of the beam radius \( w(z) \). The point in space where this occurs is

\(^9\)In cylindrical coordinates the Hermite polynomials become Laguerre polynomials.
Figure 4.12  Geometry of some of the beam-defining parameters for Gaussian Optics.

the “beam waist” and holds an analogous relationship to a focus in a geometrical optical system. It is at the waist that we take \( z = 0 \) in the equations above. Here, the phase front of the wave is flat \((R \to \infty)\) as well as at large distances. A final interesting parameter is the asymptotic angle of growth of the beam radius when \( z \) is large. This is given by

\[
\theta_{w_0} = \frac{\lambda}{\pi w_0}.
\]  

(4.21)

It is very important to note at this point that the terms defined here correspond to the amplitude distribution of the field and not the power distribution\(^{10}\). Figure 4.12, reproduced from [102] shows the geometry of the terms defined above.

Assuming, for the moment, that our beam forming optics produce a Gaussian beam in accordance with equation 4.18 above, we now understand how to propagate the beam forward in the absence of optical elements. To propagate the beam through optical elements, we must have a formalism for the effect of a thin lens on a Gaussian beam.

Assume a Gaussian beam with waist, \( w_0 \), impinging on a thin lens from a distance, \( d_0 \), from the lens. The output of this system will have a waist, \( w_1 \), at a distance, \( d_1 \), from the lens. In the plane of the lens we can equate the input and

\(^{10}\)The amplitude waist radius is \( \sqrt{2} \) times the power waist radius.
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output beam radii $w_0$ and $w_1$ as well as the radii of phase front curvature, $R(d_0)$ and $R(d_1)$, to get the output beam parameters, $w_1$ and $d_1$, as a function of the input beam and focal length $f$ of the telescope:

\[
\frac{d_1}{f} = 1 + \frac{(d_0/f) - 1}{[(d_0/f) - 1]^2 + (\pi w_0^2/\lambda f)^2},
\]

\[
\left(\frac{w_1}{w_0}\right)^2 = \frac{1}{[(d_0/f) - 1]^2 + (\pi w_0^2/\lambda f)^2}.
\]

This formalism has the obvious limitations of being a paraxial theory which relies on thin-lens approximations. Since MSAM II is an off-axis telescope with four separate reflectors, it was not clear to us how inaccurate the paraxial and thin-lens approximations would be. However, because corrugated horn antennas produce a nearly perfect Gaussian beam, we did have some confidence that the output beam would be Gaussian. Since the theory of Gaussian beams simply allows us to quickly and easily analyze different optical systems where diffraction plays a non-trivial role, we felt that it was a good place to begin the design. In fact, as shown below we found that a Gaussian Optics treatment of the telescope consistently resulted in an overestimation of the size of the output beam by $\sim 15\%$. We later compensated for this when we chose to make MSAM II a $20'$ system rather than the originally intended $30'$ system.

4.3.2 Corrugated Feed Horns

Principles of Operation

The basis of the decision to model the MSAM II telescope with Gaussian optics relies on the output beams of the two corrugated horn antennas having a Gaussian amplitude profile. With a properly designed corrugated feed horn the production of a Gaussian beam is a simple matter.
Consider first the fields in the aperture of an open-ended cylindrical corrugated waveguide. The dominant mode in a hybrid-mode\textsuperscript{11} waveguide is \[ E_x = A_1 J_0(Kr) - \frac{(X - Y)}{kr_1} A_2 J_2(Kr) \cos 2\phi \] \[ E_y = \frac{(X - Y)}{kr_1} A_2 J_2(Kr) \sin 2\phi \] (4.24)

where \( J_0(kr) \) and \( J_2(kr) \) are Bessel functions of the first kind, \( K \) and \( k \) are the transverse and free-space wavenumbers, \( A_1 \) and \( A_2 \) are amplitude coefficients, \( X \) and \( Y \) are the impedance and admittance at the boundary \( r = r_1 \)

\[ X = -j \frac{E_\phi}{H_z} y_0 \] (4.25)

\[ Y = j \frac{H_\phi}{E_z} y_0 \] (4.26)

and \( y_0 \) is the admittance of free space.

To construct a far-field beam pattern which is symmetric and has a low level of cross-polarization, we wish to match the impedance and admittance at the aperture — that is, we want \( X - Y = 0 \). In this case, the \( \phi \) dependence is removed from the \( E_x \) portion of the field and the \( E_y \) component vanishes. This is known as the “balanced hybrid” condition and is satisfied in our case by setting \( X \) and \( Y \) to zero by design. \( X = 0 \) is achieved by ensuring that there are enough corrugations per wavelength that the azimuthal electric field vanishes at the edge of the corrugations (i.e., \( E_\phi = 0 \) at \( r = r_1 \)). Additionally, by having narrow corrugations which are a quarter-wavelength deep, the axial currents in the horn

\textsuperscript{11}In this case, “hybrid” refers to the fact that we are not exciting strictly TE or TM modes but a linear combination of the two. The hybrid modes in a corrugated waveguide are labeled HE and EH.
cancel and $H_\phi$ and therefore $Y$ vanish\textsuperscript{12}.

For the corrugated feed horn operating in the balanced hybrid condition, a phase factor is added to account for the flare angle and equation 4.24 becomes [104]

$$E_x(r) = AJ_0(K_c r) \exp \left( \frac{-jk\tau}{2R_0} \right)$$

$$E_y(r) = 0$$

(4.27)

where $K_c r_1 = 2.405$ is the root of the zero-order Bessel function $J_0(K_c r)$.

To understand the modal-distribution of the power from the $E_x$ field, we expand the Bessel function in equation 4.27 in Gauss-Laguerre modes

$$J_0(K_c r) = \sum_{p=0}^{\infty} A_p L_p^0 \left[ \frac{2r^2}{w^2} \right] \exp \left( -\frac{r^2}{w^2} \right)$$

(4.28)

where $L_p^0(x)$ is a zero-order Laguerre polynomial and $w$ is the familiar beam radius defined in section 4.3.1. The value of $w$ is arbitrary and depends on the geometry of the horn. Wylde [105] shows how to choose $w$ to maximize the power in the fundamental (Gaussian) mode and finds the condition

$$\frac{w}{r_1} = 0.6435$$

(4.29)

where $r_1$ is the inner radius of the horn aperture. Table 4.6, reprinted from [104] gives the calculated power in each mode for the first 11 modes of the expansion in equation 4.28. The fact that 98\% of the power falls in the fundamental mode validates our use of Gaussian Optics to model the optical system.

One last element of the feed horn is the “mode converter” which lies in the

\textsuperscript{12}While this is a frequency dependent feature, it has been shown that the general properties of the horn remain invariant for wavelengths where it is slightly unbalanced.
Table 4.6 Normalized Power Coefficients for Gauss-Laguerre Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Power coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.9792</td>
</tr>
<tr>
<td>1</td>
<td>4.90 \times 10^{-9}</td>
</tr>
<tr>
<td>2</td>
<td>1.45 \times 10^{-2}</td>
</tr>
<tr>
<td>3</td>
<td>1.86 \times 10^{-3}</td>
</tr>
<tr>
<td>4</td>
<td>3.81 \times 10^{-1}</td>
</tr>
<tr>
<td>5</td>
<td>1.16 \times 10^{-3}</td>
</tr>
<tr>
<td>6</td>
<td>3.97 \times 10^{-4}</td>
</tr>
<tr>
<td>7</td>
<td>1.50 \times 10^{-8}</td>
</tr>
<tr>
<td>8</td>
<td>1.59 \times 10^{-1}</td>
</tr>
<tr>
<td>9</td>
<td>2.33 \times 10^{-1}</td>
</tr>
<tr>
<td>10</td>
<td>1.12 \times 10^{-1}</td>
</tr>
</tbody>
</table>

throat of the horn. The goal of the mode converter is to combine the TE_{11} and TM_{11} circular waveguide modes to create the HE_{11} corrugated waveguide mode. It is in the mode converter where the true bandwidth of the horn is defined (since the mode conversion is done with corrugations sized by the wavelength). The danger here is in exciting the EH_{11} mode which turns up when the surface reactance becomes positive. The EH_{11} mode radiates a large level of cross-polarization. Fortunately, there are a number of recipes and design techniques of mode converters which allow corrugated feed horns to be used over a factor of two in frequency without degradation in performance [104, 106, 107, 108].

Finally, there is a natural choice of horn designs that can be made on the basis of how the resulting beam scales with frequency. We define a characteristic quantity, \Delta, for a horn as the difference between the spherical wavefront emanating from the horn and the aperture plane as shown in figure 4.13.

\[
\Delta = \frac{R}{\lambda} \sin \theta_0 \tan \frac{\theta_0}{2}
\]  

(4.30)

where \theta_0 is the semi-flare angle of the horn (that is, \theta_0 = \arcsin \frac{r}{R})}. For horns with
**Figure 4.13** Principal components of a corrugated feed horn. $\Delta$ is the phase “error” at the aperture of the horn and $R$ is both the radius of the phase front curvature and, for a wide-band horn, the length of the cone which forms the horn.

$\Delta < 0.4$, the beamwidth is determined mainly by the aperture size (measured in wavelengths) and is frequency dependent while the beam waist produced by the horn is independent of frequency. For horns with $\Delta \geq 0.75$ the beamwidth is determined by the semi-flare angle of the horn and is nearly frequency independent. The former type of horn is called a “narrow-band” horn while the latter type is known as a “wide-band” horn [108]. For wide-band horns the beam waist is located in the throat of the horn and is given by

$$w_0 = \frac{\lambda}{\pi \theta_0}.$$  \hspace{1cm} (4.31)

**Design of the MSAM II Horns**

In the MSAM II optical system, the two corrugated feed horns illuminate two elliptical tertiary mirrors which feed the secondary mirror of the Cassegrain tele-
scope. To choose the aperture dimensions and horn lengths, a Gaussian Optics model was built of the telescope. Both horn and tertiary mirror parameters were varied to find the combination that would result in a beam which is frequency independent and a system that would fit inside the cryostat. This optimization was done by calculating a coarsely grided parameter space and then minimizing the viable parameter space on the basis of the resulting beamsizes and horn sizes.

Table 4.7 gives the principal dimensions of the two horns designed. The horns themselves where made by A.J. Tuck Company [109] by first creating an aluminum mandrel (negative) of the corrugations, electroplating a thin layer of gold onto the aluminum, and then electroforming a thick layer of copper onto the gold. The aluminum was then etched away with sulphuric acid. For the low frequency horn this process worked perfectly the first time, however, two attempts were required for the high frequency horn due to the many small crevices and sharp corners which make up the corrugations. In-lab measurements of both horns show that the reflection coefficient of the horns is less than 20 dB over most of the band with a maximum of 2.3% at the low end of the band.

<table>
<thead>
<tr>
<th></th>
<th>Low Frequency Horn</th>
<th>High Frequency Horn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>6.0 cm</td>
<td>3.5 cm</td>
</tr>
<tr>
<td>R</td>
<td>12.22 cm</td>
<td>8.96 cm</td>
</tr>
<tr>
<td>$2\theta_b$</td>
<td>13.12°</td>
<td>11.26°</td>
</tr>
<tr>
<td>$w_a$ (65 GHz)</td>
<td>0.904 cm</td>
<td>0.52 cm</td>
</tr>
<tr>
<td>distance to tertiary</td>
<td>27.1 cm</td>
<td>31.6 cm</td>
</tr>
</tbody>
</table>

As mentioned above, the mode converter is the most difficult (yet important) part of a wide-band corrugated feed horn to design. We followed a design recipe given by [106] and found it to work as advertised. Drawings of the low frequency horn mode converter and horn corrugations are shown in figure 4.14 and a corresponding table of dimensions is given in table 4.8.
Figure 4.14  A detail of the mandrel used to make the low frequency horn mode converter. The recipe for this mode converter comes from [103]. See table 4.8 for a list of dimensions of the grooves.

Table 4.8  Groove dimensions (in inches) for the low frequency horn. The mode converter runs from groove 1 to groove 11 and then the standard corrugations begin.

<table>
<thead>
<tr>
<th>Groove/Fin #</th>
<th>( b_i )</th>
<th>( a_i )</th>
<th>( g_i )</th>
<th>( t_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0658</td>
<td>0.1177</td>
<td>0.005</td>
<td>0.0380</td>
</tr>
<tr>
<td>2</td>
<td>0.0676</td>
<td>&quot;</td>
<td>0.0074</td>
<td>0.0356</td>
</tr>
<tr>
<td>3</td>
<td>0.0694</td>
<td>&quot;</td>
<td>0.0098</td>
<td>0.0332</td>
</tr>
<tr>
<td>4</td>
<td>0.0712</td>
<td>&quot;</td>
<td>0.0122</td>
<td>0.0308</td>
</tr>
<tr>
<td>5</td>
<td>0.0730</td>
<td>&quot;</td>
<td>0.0146</td>
<td>0.0284</td>
</tr>
<tr>
<td>6</td>
<td>0.0747</td>
<td>&quot;</td>
<td>0.0170</td>
<td>0.0260</td>
</tr>
<tr>
<td>7</td>
<td>0.0765</td>
<td>&quot;</td>
<td>0.0194</td>
<td>0.0236</td>
</tr>
<tr>
<td>8</td>
<td>0.0783</td>
<td>&quot;</td>
<td>0.0218</td>
<td>0.0212</td>
</tr>
<tr>
<td>9</td>
<td>0.0801</td>
<td>&quot;</td>
<td>0.0242</td>
<td>0.0188</td>
</tr>
<tr>
<td>10</td>
<td>0.0819</td>
<td>&quot;</td>
<td>0.0266</td>
<td>0.0164</td>
</tr>
<tr>
<td>11</td>
<td>0.0837</td>
<td>&quot;</td>
<td>0.0290</td>
<td>0.0140</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td>0.0290</td>
<td>0.0140</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td></td>
<td>0.0290</td>
<td></td>
<td>0.0140</td>
</tr>
</tbody>
</table>
CHAPTER 4. MSAM II — OPTICS

Indoor beam maps of the two horns were made in the lab at Brown during the winter of 1995. To make the maps, the horns were mounted to a rotation stage which was placed at the opening of an Eccosorb-lined shelter with another opening at the far end for the source. The Eccosorb-lined shelter resembles a large dog house in size and shape. The source used was a Hewlett-Packard microwave sweeper which was actively quadrupled and then either doubled or tripled depending on the frequency of the measurement. The sources ran at a fixed frequency and fed a short pyramidal horn antenna facing a Teflon lens designed to flatten the phase front. The source was amplitude modulated at 500 Hz and was referenced to a lock-in amplifier which read out the detector. For both the low and high frequency horns, the detectors were wide-band diode detectors from Millitech [110]. We found that it was necessary to use the lock-in to gain the long term stability needed to repeat the measurements.

Figure 4.15 shows the measurement of the beam pattern of the low frequency horn for the band centers of channels 1 and 3. Plotted are the beam pattern measurements taken at discrete angles along with the best-fit Gaussian. For channel 1 the best fit Gaussian has a 1/e beam width of 13.28° ±0.02°. The channel 3 measurement has a 1/e beam width of 12.76° ±0.04°. Both measurements show fine agreement with the design goal of 13.14°. Figure 4.16 shows the results of the same measurement for the high frequency horn in channels 4 and 5. Here, the 1/e beam widths are 11.86° ±0.03° and 11.84° ±0.03° for channels 4 and 5 respectively. The fact that these beams are 5% wider than the designed values poses no concern to the telescope.

4.3.3 Tertiary Mirrors

The tertiary mirrors couple the output of the scalar feed horns to the secondary mirrors. The two mirrors sit at right angles to each other — each facing a polar-
Figure 4.15 Indoor map of the response of the low frequency corrugated horn antenna. Shown is the horn response for channel 1 at 87.5 GHz and for channel 3 at 102.5 GHz. Overlayed are the best-fit Gaussians which provide the parameters used in the text for comparing the beams.
Figure 4.16 Indoor map of the response of the high frequency corrugated horn antenna. Shown is the horn response for channel 4 at 140 GHz and for channel 5 at 160 GHz. Overlaid are the best-fit Gaussians which provide the parameters used in the text for comparing the beams.
izer which is transmissive to the high frequency system and reflective to the low frequency system. Both mirrors are sections of ellipsoids.

The tertiary design is not quite trivial since the input and output beam waists do not fall at the foci of the ellipsoids. The design recipe for the two mirrors is as follows:

1. Use an optimization program (as described above) to find the desired focal length, \( f_t \), of the mirror given the input feed horn beam waist \( w_a \) and the desired output waist \( w_l \).

2. For the tertiary mirror to be in focus, the phase front curvature of the incoming wave should match the curvature of the mirror\(^\text{13}\). The mirror parameters are chosen by calculating \( l_1 \) — the distance from the first geometrical focus of the ellipsoid to the center of the mirror via equation 4.20,

\[
l_1 = d_1 \left[ 1 + \left( \frac{\pi w_a^2}{\lambda d_1} \right) \right]. \tag{4.32}
\]

3. The distance from the center of the mirror to the second geometrical focus is given by the lens equation

\[
l_2 = \left( \frac{1}{f_t} - \frac{1}{l_1} \right)^{-1}. \tag{4.33}
\]

4. Choose an angle of reflection of the beam from the horn off the tertiary mirror to determine the equation of the ellipsoid as well as the point on the ellipsoid which is the center of the mirror.

5. Finally, determine the outer rim dimensions of the ellipsoid by defining a

\(^{\text{13}}\)Since this is a frequency dependent effect, we design the mirrors using the center of each frequency band.
contour of constant power. In the case of MSAM II, this was chosen to be
the -40 dB off-axis power contour as predicted by Gaussian Optics.

The tertiary mirrors for MSAM II were built on a numerically controlled mill
by JMD Inc. [111]. The mirrors were each cut from a solid piece of aluminum
6061. Light-weighting of the back surface was done first and then the mirrors
were flipped and the front surfaces were cut. Because of the coarseness of the
cuts taken, the tool marks are clearly visible. Measurements with a dial indicator
and a translation table show that the structure is periodic with ~75 μm peak
heights and 2.5 mm spacing. We have some concern that this periodic structure
acts as a diffraction grating, producing sidelobes to the tertiary system which,
when integrated over the 77 K walls, may contribute a non-negligible amount of
bolometer loading. We are currently testing the mirrors to see if this is the case.
If so, we will polish the mirrors before the next flight.

Figure 4.17 shows the layout of the 77 K optical system. The view is looking
down into the lower liquid nitrogen tank with the horns and other 1.5 K opt-
tics removed. Shown are the two tertiary mirrors facing the wire grid polarizer
(described below).

The beams from the two tertiary mirrors were measured a number of times. Ini-
tially we attempted the measurements inside the lab with Eccosorb foam covering
the floor and other objects that were close to the beam axis. These measurements
were inconsistent and clearly contaminated by reflections. Outdoors we set up a
source consisting of two Gunn diodes and a Teflon lens on the roof of the physics
building at Brown. The lower nitrogen tank and optics assembly was mounted on
the ground on a camera tripod which has a geared elevation control. Azimuthal
motion was achieved by mounting the tripod on a 90 cm bearing obtained from
McMaster Carr [112]. Beammaps were made at the two Gunn diode frequencies
of 90 and 150 GHz. The predicted beamsize of the low frequency horn/tertiary
Figure 4.17  A view looking down onto the 77 K optical surface. This surface is actually the top plate of the lower liquid nitrogen tank. The two tertiary mirrors are mounted to the 77 K tank and face the polarizer. The high frequency tertiary mirror (to the right) transmits through the polarizer and out the snout. The low frequency tertiary mirror’s beam reflects off the polarizer and becomes co-aligned with the high frequency beam before exiting through the window on the left. For scale, the aperture diameter of the window is 15.2 cm.
system is 1.31° (this is the half-width 1/e size). Measurements showed the beam to be 1.13° which is 14% narrower. At 150 GHz, the beamsize is again smaller than the predicted value. While measurements show the beam to have a 1/e half-width of 0.68°, the design value is 0.89°. That the discrepancy in beamsizes can not be accounted for in the error in positioning of the optical elements suggests that the disparity between prediction and measurement must be in the failure of Gaussian optics to accurately model the off-axis nature of the telescope.

We exploited this discrepancy between the predicted and calculated beam sizes as an excuse to push the beamsize of the entire telescope from the original 30′ design down to 20′. Soon after the problem was discovered, a new elliptical secondary mirror was commissioned from JMD Inc. to create the smaller beam on the sky. The new focal length of the secondary mirror was calculated using Gaussian Optics with an emperically derived compensation for the off-axis nature of the telescope.

4.3.4 The Polarizer

The two beams from the tertiary mirrors are combined and co-aligned via the wire grid polarizer shown in figure 4.17. This polarizer was manufactured at JMD Inc. by mounting identical copper frames on a lathe and slowly wrapping even horizontal lines of the copper magnet wire. Once the wire was wrapped, the grid was epoxied to the frame using Stycast 2850 epoxy and the frames were separated.

It took three attempts to find a frame design which was stiff enough to keep the middle wires under tension. Early designs, which did not have the stiffening supports of the final version, all had wire resonant frequencies of a few Hertz. The final design has the calculated resonance frequency of 106 Hz. An important feature of the polarizer wires is that they are very close to their elastic limit. As a result, small forces can cause the wires to stretch and lose their tension. This
makes the polarizer very fragile!

To choose the wire diameter, $d$, and pitch, $S$, of the polarizer we followed the advice of Lesurf [113] who gives simple relations for the reflected and transmitted power as a function of the polarizer parameters. The normalized reflected power is

$$|r_p|^2 = \left[ 1 + \left( \frac{2S}{\lambda} \right)^2 \ln \left( \frac{S}{\pi d} \right) \right]^{-1}$$  \hfill (4.34)

and the normalized transmitted power is

$$|r_n|^2 = \frac{(\pi^2 d^2)^2}{(2\lambda S)^2 [1 + (\pi d)^4 / (2\lambda S)^2]}.$$  \hfill (4.35)

Figure 4.18 shows the predicted response of a wire grid polarizer with 75 windings per inch. The reflection improves as the wire diameter is reduced. The transmission, while more lossy than the reflection, improves with smaller wire diameter. Since the emissivity of the wire is low, the power which is not transmitted is lost in reflection as a cross-polarized component. Driven by practical considerations and the results of these calculations, we designed our polarizer to have 75 wires per inch with a 100 µm (4 mils) wire diameter.

Measurements of the cross-polarization sensitivity of both optical systems were made by transmitting polarized 90 GHz and 150 GHz sources in the direction of acceptance of the system and in the opposite polarization. Both optical systems rejected the cross-polarized radiation to better than 20 dB.

In summary, the polarization sensitivity of the optical system works as follows. For radiation that comes through the window of the dewar, the polarizer passes radiation with the correct polarization to the correct optical system. While radiation that comes off the 77 K wall behind the polarizer is passed in the “wrong” polarization, this is rejected by the (polarization sensitive) rectangular waveguide at the back end of the corrugated feed horn. This waveguide also rejects the
Figure 4.18  The predicted reflection and transmission of a wire grid polarizer. The polarizer shown here has 75 windings per inch. Shown are the reflection for channels 1 and 3 and the transmission for channels 4 and 5.

cross-polarized signal at the 20 dB level. Note that this scheme relies on the alignment of the two waveguides with respect to the polarizer wires. For alignment mis-matches which are greater than 9°, the passed component of the 77 K wall will dominate the optical loading on the bolometers. The error estimate in the alignment between the waveguide and the polarizer wires is 0.5°.

4.3.5 The External Optics

The Primary Mirror

The primary mirror is the sole optical element from the MSAM I series of balloon flights which was a part of the MSAM II system. The mirror is a section of a paraboloid with a focal length of 181 cm and a circular projection of 1.0 m diameter. The surface finish is of near-optical quality with the exception of the edges of
the mirror which are marked with some unknown substance. The calculated offset from the edge marks (assuming 100% emmissivity) is less than a few $\mu$K. During the landing of the telescope in 1995, the primary mirror was damaged when the gondola rolled onto its side. On close examination, one can see a small fold in the upper left corner of the mirror which is a chord extending approximately 3 cm from the outer circumference. We were not able to notice any asymmetry in the beam maps of the telescope which we could correlate with this feature.

Gaussian Optics gives us an approximation of what the field distribution of the beam looks like on the primary mirror without knowing anything about the telescope itself. Assuming we have a 20$'$ Gaussian beam in the far field, we can propagate the waist backwards to approximate the size of the beam on the primary mirror. The beam waist radius which creates a 20$'$ FWHM beam is given by

$$w_0 = \frac{\lambda}{\sqrt{2\pi}\theta_0}$$

where

$$\theta_0 = \frac{\alpha_{\text{FWHM}}^2}{\sqrt{2\ln 2}}$$

and $\alpha_{\text{FWHM}}$ is the FWHM of the beam. For a 20$'$ beam at 65 GHz (the low end of channel 1), the beam waist radius is 29.2 cm. If we propagate this back to the primary mirror via equation 4.19 we find a beam radius on the primary mirror of 33.3 cm. Bear in mind that this is the amplitude beam radius. The power beam radius is $\sqrt{2}$ smaller.

In order to chop the beam on the sky by $\pm 80'$ using the secondary mirror, the beam must move from side to side on the primary mirror. For MSAM II, this motion is made by nutating the secondary by $\pm 3.44^\circ$ which moves the beam on the primary by $\pm 16$ cm. It is this motion which, while allowing us to chop the beam on the sky, possibly introduces the systematic offset in our data. As the beam
moves across the primary mirror, the edge taper on the mirror (and therefore the far field sidelobe pattern) changes. Furthermore, since we are sampling various sections of the mirror at our chop frequency, we become sensitive to temperature and emissivity variations in the mirror surface. We should note that the offset in past MSAM experiments has never been directly attributed to the motion of the beam on the primary mirror, however, it is a source of concern.

**The Secondary and Chopper**

The secondary mirror is a section of a hyperboloid with a focal length \( f_s = -40 \text{ cm} \). The focal length and size of the mirror is chosen with Gaussian Optics and a guess for the correction factor to account for its off-axis nature. To get the parameters for the mirror, an optimization code was written to try different mirror geometries and focal lengths. This mirror, like the tertiary mirrors, was machined from a single piece of aluminum at JMD Inc. The back side was light-weighted with supporting ribs so that the resonant frequency of the mirror is above 100 Hz. The front surface was cut and then polished by Josh Gunderson to a near-optical finish. The smooth finish of the secondary is necessary for reflecting a pen laser during the cryostat alignment procedure. As shown below, the mirror is successful in producing an output beam of the telescope of 20 arcminutes.

The chopping mechanism was designed and built by Jason Puchalla and is described in detail in [114]. As mentioned above, the goal of the chopper is to move the beam on the sky by \( \pm 80' \) in a triangle wave at 2.5 Hz over the course of the flight. Because the chopper is suspended on the metering structure near the focus of the primary mirror, power dissipation and mass are critical constraints in a successful design.

Figure 4.19 shows the mechanical view of the chopping mechanism. Two drive coils (speaker coils) are driven asynchronously by a waveform from the gondola
Figure 4.19 The MSAM II Chopper. Two asynchronous coils drive the steel armature which turns against the four flex pivots. Feedback from the RVDT angle sensor and the velocity sensor is used to achieve maximum stability of the chopping rate and amplitude.

computer. The driving voltage is conditioned by feedback from the RVDT angle sensor and the velocity sensor to increase chopper stability. Moving the 500 g secondary mirror in a triangle wave at 2.5 Hz requires 3 W of power dissipation in the chopper. The complete chopper weighs 2 kg and is run by 0.75 kg of electronics which are suspended underneath the metering structure.

The chopper was run in the thermovac (Bemco) chamber at NSBF in Palestine to check for temperature dependence in the waveform and potential increase in friction due to thermal contraction. A small mirror mounted on the back of the secondary mirror, a laser, and a CCD camera were used to record the chopper position and rate as a function of temperature. Both in the thermovac chamber and in flight the chopper responded as it had in tests on the ground at 300 K. Stability of the amplitude was measured to be within 40" on both short and long time scales.
4.3.6 Main Beam Beammaps

The optical system is aligned by adjusting the position of the radiometer with respect to the fixed primary and secondary mirrors. The mechanical interface between the radiometer and the gondola allows translation in the vertical plane and rotation about a horizontal axis in front of the dewar. The horizontal direction is explored by a final rotation about the dewar axis. Initially, the radiometer is mounted on the telescope with the secondary and chopper removed from their mounts. In the far field of the radiometer (\(\sim 50 \text{ m}\)), 90 and 150 GHz Gunn diodes are mounted and the cryostat is moved until the response from the two optical systems is roughly maximized\(^{14}\). Once alignment is made, a pen laser is mounted onto the radiometer snout and is aligned so that it illuminates the feed horns of the two sources. The secondary is then replaced and the radiometer is repositioned until the laser bounces off the center of the secondary and the center of the primary mirror. In this manner, the microwave beam can be centered on each mirror to within 0.5 cm.

Beammaps for the MSAM II telescope have been made a number of times on the ground and once, during a raster on Jupiter, in flight. In flight there seems to have been a problem with the optical system which, as of yet, is not understood. The beamsizes are both a strong function of frequency and chopper position. Neither of these effects were witnessed in any of the ground-based measurements. We are currently conducting tests to root out the nature of the problem. For the purpose of this work, I will present both the ground based and in-flight measurements and then offer some speculation as to what might have caused the differences.

\(^{14}\)Since there is a relative misalignment of the high and low frequency optical systems of 4\(^{\prime}\), peaking-up of both beams simultaneously is impossible.
Ground Based Measurements

Measurements of the beam response were made from the ground during the ‘97 campaign in Palestine, Texas. In flight configuration, the gondola was hung near the South bay doors in the NSBF highbay with the front wheels removed to provide sufficient elevation motion to do the beammap. A jiffy jack was mounted underneath the structure holding the secondary to provide the elevation adjustment. Gondola azimuth was fixed by resting the front wheel bases on lead bricks. At the end of the driveway (100±3 m along the ground), Tiny Tim — a very large machine used by NSBF for balloon launches — was parked with his forks fully extended and raised by ~10°. Two Gunn Oscillators and some support electronics were mounted directly on a platform behind the forks 8 m above the ground. The sources were a 91.5 GHz tunable Gunn diode from Epsilon-Lambda [115] (50-70 mW - hereafter the ch2 source) and the 150 GHz Gunn (30 mW - hereafter ch4 source). Both sources were chopped with external function generators and power supplies. The ch2 source was chopped at 69 Hz and the ch4 source was chopped at 73 Hz. It was later found that the chop frequency for the ch2 source varied during the test by as much as 1 Hz on short time scales. In order to achieve linearity in the detectors, each source had to be attenuated by both an Aerowave [116] variable attenuator as well as rotary vane attenuators. Ecosorb shielding on the back side of the attenuators was required to avoid source leakage from behind the attenuators. Figure 4.20 shows the view of the microwave sources as seen by the star camera.

Inside the highbay the gondola was moved until the beam was peaked on the sources. After finding the peak, the linearity of the detectors was tested by iterations of adding and subtracting 3 dB of attenuation in the two attenuators. The beammap was then made by jacking the gondola elevation stage up and down in 1° scans. A full scan from top to bottom to top again took approximately 4
Figure 4.20  A view of the microwave sources on the forks of “Tiny Tim” during the ground-based beam mapping of the MSAM II telescope. This photo was taken with the flight-star camera near dusk on the first day of the measurements. A 20′ radius circle is drawn around the sources to give the reader an idea of the scale of the 20′ diameter beam.
minutes. Steps in azimuth were made by offsetting the chopper. As long as the coma of the telescope is small, this is equivalent to moving the entire gondola in azimuth.

The raw time stream was binned in time bins of 0.5 s. Each bin was then assigned an inclinometer reading and a value of the chopper offset. The data in the bin was Fourier transformed and the region around the peak was summed (±1 Hz) to give the signal size. Finally, a piecewise linear drift in time was removed from each of the scans in elevation since it was found that the peak signal size decreased by 30% over the course of the measurement\(^\text{15}\). In order to get a higher S/N on the bin position, the gyro data was fit to the inclinometer to take out sky rotation and used as the relative position sensor.

Table 4.9 lists the best-fit parameters when each beam map is fit to a two dimensional Gaussian with the amplitude, peak position, and widths in azimuth and elevation as free parameters. All elements of the table are in arcminutes. Ch2 and Ch4 left(right) correspond to measurements of the beams with the chopper to the right(left) by \(\sim 60\text{'}\).

<table>
<thead>
<tr>
<th>Data Set</th>
<th>El-center [']</th>
<th>Az-center [']</th>
<th>FWHM in el [']</th>
<th>FWHM in az [']</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ch2 center</td>
<td>171.0</td>
<td>-2.2</td>
<td>20.96</td>
<td>19.25</td>
</tr>
<tr>
<td>Ch4 center</td>
<td>173.2</td>
<td>0.6</td>
<td>19.06</td>
<td>19.25</td>
</tr>
<tr>
<td>Ch2 left</td>
<td>175.7</td>
<td>61.8</td>
<td>20.52</td>
<td>19.60</td>
</tr>
<tr>
<td>Ch4 left</td>
<td>178.8</td>
<td>64.6</td>
<td>19.21</td>
<td>20.89</td>
</tr>
<tr>
<td>Ch2 right</td>
<td>176.8</td>
<td>-56.8</td>
<td>20.08</td>
<td>20.15</td>
</tr>
<tr>
<td>Ch4 right</td>
<td>179.2</td>
<td>-55.3</td>
<td>20.37</td>
<td>20.37</td>
</tr>
</tbody>
</table>

From Table 4.9 it is clear that regardless of chopper position the beam size of the

\(^{15}\)This drift in the signal size was not totally unexpected since the ambient temperature fell at dusk by \(\sim 10\text{°}\) C and the output power of the sources is temperature sensitive.
instrument is within 10% of the design goal of 20'. We can now ask the question, how reasonable would it be to assume Gaussianity of the beams? Fortunately, we have already developed the mathematical tools to answer this in section 4.1.2. Here, we would like to compare the measured beam map, $B_m$, with the best-fit Gaussian beam map, $B_g$. Recall that to make a comparison we must choose what we believe to be a “reasonable” correlation function, $C(k)$. Here, we choose the correlation function resulting from the “standard cold dark matter cosmology” (SCDM) parameter values\(^{16}\).

From section 4.1.2, our beam error is defined as

$$\langle \delta^2 \rangle = \int dk |B_m(k) - \alpha B_g(k)|^2 C(k)$$

(4.38)

where I have introduced the dimensionless parameter $\alpha$ as a factor which can be varied to maximize the “similarity” of the beams. A value of $\alpha$ near 1 implies that the beams are already quite similar. Minimizing $\langle \delta^2 \rangle$ with respect to $\alpha$ then gives

$$\alpha_0 = \frac{\int dk \text{Re}(B_m^* B_g) C(k)}{\int dk |B_g|^2 C(k)}.$$  

(4.39)

Re-inserting $\alpha_0$ into equation 4.38 then gives the minimum value for the “beam match error”

$$\langle \delta^2 \rangle = 1 - \frac{(\int dk \text{Re}(B_m^* B_g) C(k))^2}{\int dk |B_m|^2 C(k) \int dk |B_g|^2 C(k)}.$$  

(4.40)

Table 4.10 shows the calculated values of $\alpha_0$ and the beam match error, $\delta$, for this set of measurements.

Since the temperature fluctuations on the sky are a random variable, the beam-match values given above are a measure of added noise to the data due to not knowing the features of the beam. Whether this is acceptable or not is determined by the level of signal-to-noise of the measurement we make. The more accurately

\(^{16}\)See Chapter 1 for a description of the SCDM parameters.
Table 4.10 Values of $\alpha_0$ and the beam match error for the ground based main beammap measurements.

<table>
<thead>
<tr>
<th>Beam</th>
<th>$\alpha_0$</th>
<th>Beam Match Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ch 2 Center</td>
<td>1.01</td>
<td>11.8%</td>
</tr>
<tr>
<td>Ch 4 Center</td>
<td>1.01</td>
<td>6.1%</td>
</tr>
<tr>
<td>Ch 2 Left</td>
<td>0.97</td>
<td>11.1%</td>
</tr>
<tr>
<td>Ch 4 Left</td>
<td>0.99</td>
<td>12.9%</td>
</tr>
<tr>
<td>Ch 2 Right</td>
<td>0.96</td>
<td>8.4%</td>
</tr>
<tr>
<td>Ch 4 Right</td>
<td>0.95</td>
<td>12.7%</td>
</tr>
</tbody>
</table>

we measure the signal, the more we fool ourselves if we adopt the best-fit Gaussian as our beam model. If, however, we achieve a S/N level of 2 or 3, this effect will remain buried in the detector noise. It should be mentioned that this was not a very high S/N measurement of the beam. Since the beam match error was found to depend sharply on the amount of high frequency power in the beammaps, it is conceivable that it could be further reduced by doing a better job in measuring the beam.

In-Flight Measurements of the Beamsize

During the '97 flight, the telescope beam was measured by rastering the beam over Jupiter. Jupiter, with an angular diameter of 42'', is an unresolved point source to the telescope which makes it a nearly perfect far-field delta-function emitter for measuring the beam.

Approximately 6.5 hours after launch and after an initial set of CMB scans to the North, the telescope was turned to the location of Jupiter. Since at this point we did not know the exact location of the microwave beam with respect to the star camera center, we performed an elevation scan of 2° amplitude at 2.35 's and subsequently peaked-up on the planet. After peaking we began the raster of Jupiter. Figure 4.21 shows the raster pattern in gyro coordinates. The raster fills a rectangle which is 240' in azimuth and 84' in elevation with 29 steps in
elevation of 3'. The raster width is sized to sample the beam pattern in each of the 32 positions of the chopper as it moves the beam across the primary mirror. The height is chosen to allow sampling of the undemodulated beam out to points which are less than -20 dB of the peak response.

The motion of the gondola during the raster is as follows. Beginning with the telescope peaked on Jupiter, the gondola offsets in azimuth by -120' and by 42' in elevation. Holding constant elevation, the telescope moves the beam center across the sky by 4° at a rate of 7.5'/s. Once the 4° strip has been traversed, the elevation is stepped down by 3' and the gondola heads in the opposite direction in azimuth. This process is repeated until the entire rectangle has been covered — at which point the telescope returns to Jupiter.

As mentioned above, there was a problem with the optical setup during the
flight. At this time, we do not know the cause of the problem but there is speculation that it could be in-part due to a shifting of an optical component during the launch. Perhaps related to this problem we measured a larger than expected optical loading on the bolometers which degraded their sensitivity by a factor of 4 over the expected value. Since the Jupiter raster was designed for a system with a factor of 2 better sensitivity (we had already included a factor of 2 in preparation of the flight), the signal-to-noise of the resulting beammaps is poor. Regardless, it is possible to glean information about the optical system from the raster.

The data from the raster are analyzed as follows: Since the chopper and detectors are sampled simultaneously during the raster, the detector data are binned in chopper coordinates. Both the chopper and detectors are sampled 32 times each time the chopper moves the beam from one side of the primary mirror to the other (160' on the sky) so we make 32 separate total-power beam maps from the raster data for each detector. Each beam map represents a different chopper position. For each map, there are three vectors; the beam azimuth, the beam elevation, and the bolometer signal. To parameterize the beams, these three values are fit to a two dimensional Gaussian with the amplitude, the width in azimuth, the width in elevation, the position, and a linear offset in elevation as free parameters.

As mentioned above, the resulting best-fit Gaussians are not the same as the ground-based measurements. Table 4.11 shows the measured beam widths in azimuth for a few of the chopper positions. For all channels, the beam is wider in azimuth at the ends of the chopper throw than in the center. This broadening is quite symmetric about the center — suggesting that the problem is not from misalignment of the beams to one side or the other of the secondary mirror. In addition, all channels have wider central beams than the 20' size derived from the ground-based measurements. Finally, there is a frequency dependence to the size of the beam which was not evident in ground based measurements.
**Table 4.11**  The azimuthal beamsize as measured in flight for MSAM II. Listed are the best-fit Gaussian FWHM values for azimuth in the extreme left and right chopper positions as well as the center chopper position. The statistical errors on the beamsizes clustered around 1' except for channel 5 which had too little signal-to-noise to make a measurement.

<table>
<thead>
<tr>
<th>Channel</th>
<th>FWHM (left)</th>
<th>FWHM (center)</th>
<th>FWHM (right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ch 1</td>
<td>34.3'</td>
<td>25.5'</td>
<td>34'</td>
</tr>
<tr>
<td>Ch 2</td>
<td>29.5'</td>
<td>23'</td>
<td>28'</td>
</tr>
<tr>
<td>Ch 3</td>
<td>24'</td>
<td>21.5'</td>
<td>24'</td>
</tr>
<tr>
<td>Ch 4</td>
<td>29'</td>
<td>21'</td>
<td>28'</td>
</tr>
<tr>
<td>Ch 5</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

What could cause this behavior of the optical system? This question is currently under examination and must be answered before MSAM II can be flown again. The symmetry of the broadening in the beam suggests that the beam was well centered on the primary in the horizontal direction — suggesting that an offset in position in the vertical direction may be the cause of the problem. The frequency dependence of the effect also works with this argument since the amount of beam overlapping the edge of the primary would decrease as a function of frequency for channels 1–3. Finally, the increased optical loading of the bolometers might also be explained by the 250 K gondola structure being illuminated by the part of the beam missing the mirror. This is just speculation at the moment, however, and the real causes of these effects will not be known before the investigation is complete.

### 4.3.7 Far Sidelobe Response

As was discussed in section 4.1.3, the sidelobe response of a CMB telescope must be low enough to ensure rejection of the earth, moon, and other foreground sources. The far sidelobes of the MSAM II telescope were measured in three different ways during the '97 campaign in Palestine. The first test was a measure-
ment of a cut through the beam map in elevation which extended several degrees out from the peak. The second measurement was a “spritz” test\(^\text{17}\) done from below the gondola’s ground screening — designed to measure potential pickup from the earth. The final test was a similar “spritz” test performed from above the gondola. The three tests and their results are described below.

**The Far Sidelobe Map**

Two Gunn diodes, one broadcasting at 90 GHz and one at 150 GHz, were mounted at the focal plane of a 30 cm parabolic reflector (a satellite TV antenna covered with aluminum tape) and served as the source for the measurement. Each Gunn diode fed a calibrated rotary vane attenuator which was set to ensure detector linearity. The two sources were chopped by square waves at 37 Hz and 50 Hz for the 90 GHz and 150 GHz sources respectively.

The source telescope was mounted on a tripod with a manually adjusted elevation mount which was mounted, in-turn, to a rotary bearing to facilitate the azimuth adjustment. The entire apparatus was placed on the roof of the staging building at the NSBF and pointed towards the East.

The gondola was placed 0.77 km away from the source at the far edge of an unused launch pad. The elevation angle of the source was 0.8° above the gondola’s horizon. Both telescopes were manually moved to aim at each other until signal was seen in the detectors. Once this rough peaking was accomplished, the source telescope was moved in azimuth and in elevation until the maximum signal was received. The same was then done with the gondola. Since the detectors have a small dynamic range, linearity was periodically checked by increasing the attenuation at the sources by 3 dB and looking for a drop in the bolometer signal.

\(^\text{17}\) Spritz tests are made by probing the antenna response of the telescope from discrete locations around the system — that is to say, it is not a spatially continuous measurement.
Figure 4.22 The total-power sidelobe response of the MSAM II telescope. Shown is the normalized response of channels 2 and 4 for the far sidelobe test. The variations in the signal a couple of degrees from the peak are statistically significant and show real structure in the sidelobe response. The region to the left of the peak (in the downward direction) is contaminated by reflections from the asphalt between the source and the telescope.

by a factor of 2. A radio modem transmitted the gondola signals back to the staging building where signal readout and conditioning was done with the flight ground station.

Far field maps were made for channels 2 and 4 by manually changing the elevation of the telescope and scanning until all the dynamic range in the attenuators was used. To do this, a small jack was mounted underneath the chopper and the elevation was raised or lowered until the signal became noise limited. At this point, the attenuation was lowered until the detector approached its non-linear regime. This process was repeated a number of times until the detector signal had
stopped decreasing. Figure 4.22 shows the total power response of the telescope in channels 2 and 4 in the far sidelobes. Within a few degrees of the peak, the signal has dropped to a relatively constant value between -60 and -70 dB of the peak amplitude. The error bars on the points are a bit smaller than the plotting symbols so it is evident that there is real structure in the far sidelobe response of the telescope. The plots for the two channels have been separately normalized and shifted to have the same peak location.

The Earth and Moon Spritz Tests

Figure 4.23 shows the gondola and source positions for the two spritz tests. In both cases, the gondola was wheeled into the parking lot in front of the staging building at the NSBF and pointed towards the south-east. The gondola elevation was set (as read by the inclinometer) to 46.5°. The sources used were the same Gunn diodes used in the far sidelobe map test and were chopped at the same frequencies. The chopping frequencies were stable to within 0.5 Hz over the course of the measurement.

To determine the sidelobe response from the earth by the gondola, the sources were mounted on a camera tripod 6.1 m from the center of the gondola and 1.3 m off the ground. Measurements were made at 8 locations in gondola azimuth beginning with gondola North (the direction of primary illumination at the center chop position) and covering points at 45° intervals.

At each azimuthal position, the sources were pointed at the three portions of the gondola illustrated on the left in figure 4.23. In the low position, the sources point at the momentum wheel structure. Here we are looking for reflections that get in around the dewar snout or in overlooked cracks in the ground shields. At the medium position the sources illuminate the ground screening directly. In the high position, the sources are pointed at the aluminum suspension structure above
Figure 4.23  The relative positions of the gondola and source during the two spritz tests. For the earth spritz test (shown to the left), the sources were placed underneath the ground screening of the gondola. For the moon spritz test (shown to the right), the sources were lifted above the gondola and pointed down towards the radiometer window.
the primary mirror. Both sources have an opening angle of \( \sim 20^\circ \) which translates to a \( 7^\circ \) diameter circle (FWHM) on the gondola.

To test for contamination from the moon we asked the NSBF staff to lift the two sources (and an experimentalist) above the gondola in the NSBF “cherry picker”. This is a truck with a long arm and bucket. The cherry picker can reach heights as high as 18.3 m while maintaining a large enough distance from the gondola to avoid worries about scattering off the truck’s arm. Figure 4.24 shows a picture of the experimental setup for the moon spritz test.

As illustrated in figure 4.23, we tested a variety of possible moon positions. These tests were repeated for \( \frac{1}{2} \) of the circle in azimuth beginning at gondola South and moving towards gondola North in the counter-clockwise direction. The symmetry of the system suggests that the other half of the circle would yield similar results. The different source angles tested were meant to illustrate both the different possible positions of the moon during flight as well as (in the extreme angle cases) the pickup from the balloon itself.

The calibration for these two tests was performed twice. Each time the sources were placed on top of a large aluminum ladder and hoisted approximately 15° off the ground by a forklift. The assembly was parked at the end of the driveway to the staging building 89 m away. With the chopper stationary, the gondola was peaked up on the sources and attenuation was set to ensure linearity in the detectors. The two calibrations (taken almost a week apart) agreed to 20% which is quite good given the intrinsic errors associated with atmospheric and pointing variations.

To analyze the data for the two spritz tests, at each location and illumination angle of the source a power spectrum of the signal was made. At the source modulation frequency and each of the first 8 harmonics of the chopper cycle, the signal was integrated over a small frequency window and an estimate of the
Figure 4.24  The author and microwave sources hanging precariously above the MSAM II telescope while spritz testing. Shown is the measurement at the highest elevation and largest angle. This is equivalent to the angle of the radiation coming from the balloon.
residual noise in the region around the peak was removed. This number was then compared to the calibration to determine the amount of rejection in that particular demodulation.

Justification for this method is as follows. We are interested in what portion of the signal from the ground and/or moon can contaminate the data. Since the data are demodulated, we would like to compare the signal here with the same demodulations. As noted in section 4.1.1, a reasonable approximation to the optimal demodulations is the set of sine waves. Taking the power spectrum actually lumps cosine demodulations in with the sine demodulations. This would not be a concern if the chopper and data stream were perfectly in phase since the cosine demodulations would contain no power, however, since there is an undetermined phase-lag between the two, some power from the cosine demodulations adds to the signal. This effect introduces a 0.75 dB error into the measurements.

Tables 4.12 and 4.13 show the result of the earth spritz test for channels 2 and 4. As naively expected from diffraction and edge-illumination arguments, channel 2 does not reject the earth nearly as well as channel 4. A little more surprising is that there is a particularly weak spot for both channels in the gondola north/high position. By placing a piece of bumpy Eccosorb over the center of the gondola’s cross bar above the primary, the contaminating signal can be reduced by as much as 20 dB. This level of offset is consistent with the predicted amount of edge illumination at the bar assuming the beam is Gaussian. Of course, this leads to the age-long debate of whether to coat the bar with Eccosorb (increasing the total loading) or to leave it as a shiny reflector of the earth. For the ‘97 flight we chose the latter.

Figures 4.25 and 4.26 show the entire data set from the earth spritz test measurements. Plotted are the sidelobe rejections at the various source positions as a function of the demodulation number (or harmonic of the chopper frequency).
Table 4.12  Data from the earth spritz test for channel 2. Shown is the rejection in dB of channel 2 for the fundamental (total-power), first and eighth demodulations.

<table>
<thead>
<tr>
<th>Azimuth</th>
<th>Source Position</th>
<th>fundamental</th>
<th>1st Har.</th>
<th>8th Har.</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>low</td>
<td>80</td>
<td>97</td>
<td>101</td>
</tr>
<tr>
<td>North</td>
<td>medium</td>
<td>70</td>
<td>86</td>
<td>93</td>
</tr>
<tr>
<td>North</td>
<td>high</td>
<td>59</td>
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<td>87</td>
</tr>
<tr>
<td>NW</td>
<td>low</td>
<td>83</td>
<td>103</td>
<td>105</td>
</tr>
<tr>
<td>NW</td>
<td>medium</td>
<td>81</td>
<td>102</td>
<td>102</td>
</tr>
<tr>
<td>NW</td>
<td>high</td>
<td>75</td>
<td>97</td>
<td>97</td>
</tr>
<tr>
<td>West</td>
<td>low</td>
<td>86</td>
<td>104</td>
<td>105</td>
</tr>
<tr>
<td>West</td>
<td>medium</td>
<td>86</td>
<td>101</td>
<td>112</td>
</tr>
<tr>
<td>West</td>
<td>high</td>
<td>71</td>
<td>86</td>
<td>86</td>
</tr>
<tr>
<td>SW</td>
<td>low</td>
<td>89</td>
<td>105</td>
<td>103</td>
</tr>
<tr>
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<td>medium</td>
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<td>107</td>
<td>99</td>
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<tr>
<td>NE</td>
<td>high</td>
<td>67</td>
<td>84</td>
<td>94</td>
</tr>
</tbody>
</table>
Table 4.13  Data from the earth spritz test for channel 4. Shown is the rejection in dB of channel 4 for the fundamental (total-power), first and eighth demodulations.

<table>
<thead>
<tr>
<th>Gondola Az</th>
<th>Source Position</th>
<th>fundamental</th>
<th>1st Har.</th>
<th>8th Har.</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>low</td>
<td>109</td>
<td>110</td>
<td>109</td>
</tr>
<tr>
<td>North</td>
<td>medium</td>
<td>100</td>
<td>110</td>
<td>109</td>
</tr>
<tr>
<td>North</td>
<td>high</td>
<td>89</td>
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<td>99</td>
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<td>low</td>
<td>110</td>
<td>110</td>
<td>109</td>
</tr>
<tr>
<td>NW</td>
<td>medium</td>
<td>101</td>
<td>109</td>
<td>106</td>
</tr>
<tr>
<td>NW</td>
<td>high</td>
<td>103</td>
<td>110</td>
<td>109</td>
</tr>
<tr>
<td>West</td>
<td>low</td>
<td>108</td>
<td>110</td>
<td>109</td>
</tr>
<tr>
<td>West</td>
<td>medium</td>
<td>97</td>
<td>110</td>
<td>109</td>
</tr>
<tr>
<td>West</td>
<td>high</td>
<td>98</td>
<td>109</td>
<td>110</td>
</tr>
<tr>
<td>SW</td>
<td>low</td>
<td>109</td>
<td>109</td>
<td>109</td>
</tr>
<tr>
<td>SW</td>
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<td>103</td>
<td>109</td>
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</tr>
<tr>
<td>SW</td>
<td>high</td>
<td>100</td>
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<tr>
<td>South</td>
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<td>95</td>
<td>108</td>
<td>107</td>
</tr>
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<td>South</td>
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<td>94</td>
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<tr>
<td>NE</td>
<td>low</td>
<td>109</td>
<td>110</td>
<td>110</td>
</tr>
<tr>
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<td>medium</td>
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<td>109</td>
</tr>
<tr>
<td>NE</td>
<td>high</td>
<td>96</td>
<td>109</td>
<td>110</td>
</tr>
</tbody>
</table>
Figure 4.25  Earth spritz test results for channel 2. Plotted are the data from the spritz test for each of the demodulations. A demodulation number of 0 corresponds to the unchopped rejection of the system. Each line represents a separate position of the source with respect to the gondola. Refer to table 4.12 to determine which line represents which position of the source.

From the two plots we can see that for most of the demodulations, the chopped sidelobe response of the instrument is “roughly” scale invariant and reduced from the unchopped response by a factor of 100. For a few source positions there are significant differences in the response from the different demodulations. For example, demodulation 6 in channel 4 has a response which is nearly 10 dB greater than the average response for a number of source positions.

The moon spritz test data for channels 2 and 4 are given in tables 4.14 and 4.15. If we take the moon as a 300 K source with 100% emissivity, a rejection of 74 dB would result in an offset of 10 μK. Included in the moon spritz test data are measurements of the sidelobe response from angles which include the balloon limb (angles greater than 60°). With the following assumptions:
Figure 4.26  Earth spritz test results for channel 4. Plotted are the data from the spritz test for each of the demodulations. A demodulation number of 0 corresponds to the unchopped rejection of the system. Each line represents a separate position of the source with respect to the gondola. Refer to table 4.13 to determine which line represents which position of the source.
• The emissivity of balloon = 10%.

• There is 10% contrast on 1.5° scales.

• There is a constant 65 dB response to balloon (this is the average response for our measurements in channel 2).

• There is a random temperature distribution on the 250 K balloon.

• The balloon emissivity is the dominant contributor to the balloon signal.

we find an offset due to the balloon of 12 μK in all demodulations for channel 2. As long as the reflectivity of the balloon material is less than 10% then the assumption that the emissivity dominates is valid.

The results from the moon spritz-test lead to limitations on the possible flight times for the instrument. Clearly, having the moon at angles greater than 20 degrees above the plane of the ground screens leads to an offset in the data which is comparable to the signal size in some demodulations. What is worse is that this offset is not stable over long times (since the demodulated sidelobe response of the telescope at these angles is not a smooth function of any angle) and is fixed with respect to the sky like the CMB. For this reason, we limit our possible flight dates to nights when the moon is less than 20° above the plane of the ground screen for the course of the observations.

In conclusion, it should be noted that, while the level of sidelobe response of MSAM II is not unexpected, if the problem in the optics described in section 4.3.6 occurred before the launch of the telescope, it is possible that this same problem may have contaminated these measurements. In any case, the measured sidelobe response of the telescope is consistently lower than measurements made by the MSAM I telescope (at a much higher frequency!) which suggests that like the MSAM I telescope, MSAM II should be immune to contamination from sidelobe pickup.
Table 4.14 Data from the moon spritz test for channel 2. Shown is the rejection in dB of channel 2 for the fundamental (total-power), first and eighth demodulations. The source angle is the angle of the microwave source with respect to the plane made by the top of the ground shielding.

<table>
<thead>
<tr>
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<th>source angle</th>
<th>fundamental</th>
<th>1st har.</th>
<th>8th har.</th>
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Table 4.15  Data from the moon spritz test for channel 4. Shown is the rejection in dB of channel 4 for the fundamental (total-power), first and eighth demodulations. The source angle is the angle of the microwave source with respect to the plane made by the top of the ground shielding.

<table>
<thead>
<tr>
<th>position</th>
<th>source angle</th>
<th>fundamental</th>
<th>1st har.</th>
<th>8th har.</th>
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</thead>
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<tr>
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<td>South</td>
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<tr>
<td>South</td>
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</tbody>
</table>
Finally, the desire for frequency independence of the far field beam tells us that the beam radius on the primary mirror must go approximately like $1/\lambda$. Thus, for a given edge-taper requirement of the telescope, the beamsize on the sky is determined by the lowest frequency of interest. This is true for all of the optical surfaces of the telescope. For MSAM II, channel 1 completely drives the optical design so if it should be true that there is no foreground contamination of the MSAM II signals by synchrotron, free-free, or dust emission, a future version of the experiment could operate with a frequency dependent beamsize with the channel 4 and 5 beams as small as 12'. For the moment, however, until we have a better survey of the galactic foreground sources we must be conservative with our efforts in order to have confidence in our results.
Chapter 5

Cosmological Parameter Estimation

5.1 Motivation

As shown in Chapter 1, the power spectrum of the CMB contains a wealth of information about the physical properties of our universe. Like never before, we are now on the verge of exploiting this radiation to learn the fundamental character of our surroundings. On the theoretical side, we now understand how to calculate and predict the effects on the CMB of changing the initial conditions of the universe for a variety of cosmological models. On the experimental side, the possibility of measuring a large patch of sky with 20 μK sensitivity and 20′ resolution is within our reach today. By 2002, the MAP satellite will have made a full sky map with ∼18′ resolution [117]. One of the principal tasks which remains is to develop a robust method for combining the information from these anisotropy measurements with the theoretical predictions in order to make definitive statements about the cosmological parameters of our universe.

Two recent developments in CMB physics have made this feasible. First of all, we now have actual measurements of anisotropy from a host of experiments and at a variety of angular scales [7, 8, 9, 10, 11, 16, 2]. While, with the exception of the COBE and FIRS measurements, the sky coverage from these experiments is small, the data is representative of the data soon to come from the next generation of experiments (that is, the noise is real instrument noise,
the data are contaminated by drifts, the beam patterns are not truly Gaussian, etc.) and so serves as an ideal testbed for the development of computational techniques. The second accomplishment which allows us to place bounds on cosmological parameters is the advent of a fast program, CMBFAST, to calculate accurate correlation functions for variants of the Cold Dark Matter (CDM) cosmology [25]. CMBFAST uses a line of sight integration technique to calculate the correlation function of the fluctuations given the set of 11 input parameters $\tilde{\theta} = [\Omega_b h^2, \Omega_{CDM} h^2, \Omega_c h^2, \Omega_{\Lambda} h^2, H_0, T_{cmb}, Y_{He}, z_{re}, x_e, n, T/S]$ which are: the density in baryons, the density in cold dark matter, the density in hot dark matter, the density associated with a cosmological constant, the Hubble constant, the temperature of the CMB, the helium mass fraction, the redshift of reionization, the ionization fraction, the spectral index of fluctuations, and the ratio of tensor to scalar mode contributions. For example, CMBFAST requires a few minutes on a Pentium-Pro 200 MHz PC to calculate a correlation function out to $l = 1000$. This is over two orders of magnitude faster than previous correlation function generation codes [25]. Unfortunately, CMBFAST is limited to considering the subset of viable cosmologies whose fluctuations are described by Gaussian statistics. As a result, for the remainder of this chapter I only consider correlation functions from inflationary cosmologies.

The following section gives an overview of a maximum likelihood-based method for performing the parameter estimation calculation. Section 5.3 describes computational techniques for increasing the efficiency of this calculation followed by a description of a program I have written called THRESH which implements the techniques previously covered. This is done in section 5.4. Finally, section 5.6 gives a working example of THRESH in terms of the combined three years of MSAM I flights and discusses the implications for future balloon-borne and satellite missions.

THRESH is the result of work by the author and David Cottingham of Global
5.2 An Overview of the Calculation

Once the data reduction, analysis, and general decontaminating has occurred, we can think of the results of a CMB experiment in a simple manner. A CMB experiment measures the temperature fluctuations on the sky with a beam response \( B(\vec{x}) \) and derives a signal, \( t \), given by the integral over the beam pattern on the sky,

\[
t = \int_{\Omega} B(\vec{x})T(\vec{x})d\vec{x},
\]

where \( T(\vec{x}) \) is the temperature distribution on the sky and \( \vec{x} \) is the direction of the observation. If an experiment measures \( n \) separate directions on the sky with \( m \) demodulations of the beam, the net result of the experiment is \( n \times m \) measures of the temperature distribution \( T(\vec{x}) \).

It is important to note the importance of the orientation of the beam pattern with respect to a fixed coordinate system on the sky. Since, in general, the idea of a synthesized beam pattern is to eliminate atmospheric effects, the beam pattern is oriented so that the differencing is taking place parallel to the local horizon at the time of observation. As a result, as the sky rotates above the instrument, the orientation of the beam pattern on the sky changes. The asymmetry of the beam then makes knowing the relative orientation important in being able to interpret the measurements. As an example, figure 5.1 shows the relative orientation and spacing of the measurement points for the third flight of the MSAM I instrument. For orientation, the chop direction is parallel to the local tangent to the curve — implying that the points furthest to the right are rotated by approximately 40° with respect to the points furthest to the left of the figure. The axes are the sky positions as given by the data sets.
Figure 5.1  The relative orientation of the MSAM I measurements for the ‘95 flight.

As discussed in Chapter 1, $T(\vec{x})$ is a random variable whose expectation value, when averaged over the sky, is the correlation function of the sky. For observations of multiple spots on the sky, we can construct a theoretical covariance matrix of the measured points

$$[C_T]_{i,j} \equiv \langle t_i t_j \rangle = \int_{\Omega_1} \int_{\Omega_2} B_i(\vec{x}_1)B_j(\vec{x}_2) \langle T(\vec{x}_1)T(\vec{x}_2) \rangle d\vec{x}_1 d\vec{x}_2. \quad (5.2)$$

The expectation value $\langle T(\vec{x}_1)T(\vec{x}_2) \rangle$ is the postulated correlation function of the sky fluctuations and, due to isotropy in the mean, only depends on the distance between the measured points, that is, $\langle T(\vec{x}_1)T(\vec{x}_2) \rangle = C(|\vec{x}_1 - \vec{x}_2|)$. Since this correlation function depends implicitly on the set of cosmological parameters, $\vec{\theta}$, by comparing the CMB experiment data with an ensemble of calculated covariance matrices, we can limit the viable space of parameters to those consistent with our observations.
CHAPTER 5. COSMOLOGICAL PARAMETER ESTIMATION

Note that $C_T$ in equation 5.2 is not to be confused with the noise covariance matrix, $C_n$, which is constructed from the noise in the observations. The diagonal elements of $C_n$ are the squares of the noise in each sky observation (i.e., the variance of each measurement) whereas the off-diagonal components are the correlations in noise between different pixels. These correlations come from the time constant of the detector, removal of drifts in the instrument, and filtering in the signal chain. In general, different demodulations of the chopper throw of the experiment are designed such that the noise is uncorrelated between the different demodulations. This is known as making the demodulations “orthogonal” in the noise.

**Maximum Likelihood Estimation**

The set of observations of a CMB experiment is a set of random numbers. This is so both because the fluctuations on the sky are random themselves and because the instrument noise which contaminates the observations is random as well. Conversely, the cosmological parameters which describe our universe are not random. In fact, there is just one set of these parameters which are the “correct” values. The question we would like to ask, then, is: given a set of $n \times m$ observations $[t_k]$, made with the set of $m$ demodulations $[B_j]$, and with corresponding noise covariance matrix $C_n$, what sets of cosmological parameter values, $\tilde{\theta}$, are consistent with the observations?

For uncorrelated data, the probability of the data set is the product of probabilities of each data point. To quantify this, we define the likelihood of an uncorrelated data set given a set of parameters, $L(t; \tilde{\theta})$, as

$$L(t; \tilde{\theta}) = \prod_{k=1}^{N} f(t_k; \tilde{\theta})$$  \hspace{1cm} (5.3)
where $f(t_k; \tilde{\theta})$ is the probability of the $t_k$th observation. Semantically, $L(t; \tilde{\theta})$ is the likelihood of the observations, $t$, given the set of parameters, $\tilde{\theta}$. For example, if the $N$ data points in the experiment were uncorrelated with constant variance, $\sigma^2$, and obeyed a Gaussian distribution, the likelihood becomes

$$L(t; \tilde{\theta}) = \left( \frac{1}{\sqrt{2\pi\sigma}} \right)^N \prod_{i=1}^{N} \exp \left( -\frac{1}{2} \left( \frac{t_i - T(\bar{x}_i)}{\sigma} \right)^2 \right)$$  \hspace{1cm} (5.4)$$

where $T(\bar{x}_i)$ is the true temperature at position $\bar{x}_i$.

For the case of actual CMB observations, the signal and noise are correlated. Assuming that they are Gaussian distributed as well\footnote{We only consider models which predict Gaussian distributed signals.}, the likelihood is [118]

$$L(t; \tilde{\theta}) = \frac{e^{-\frac{1}{2}tM^{-1}\tilde{\theta}t^T}}{(2\pi)^{N/2}\sqrt{\det M}}$$  \hspace{1cm} (5.5)$$

where $t$ is the $N$-element vector of observations and $M(\tilde{\theta}) = [C_T(\tilde{\theta}) + C_n]$ is the $(N \times N)$ covariance matrix of the observations.

There is a divergence in philosophy when it comes to the definition of the likelihood in equation 5.5. Bayesians make a leap of faith by stating that the probability of the data given the parameters is the same as the probability of the parameters given the data. While this makes finding the confidence intervals on parameters straightforward, it requires a willingness to believe that a set of parameters can have a probability. On the other side of the fence sit the frequentists. Frequentists are unwilling to take this leap of faith and, instead, define a new set of random variables which are “estimates” of the original parameters. Multiple realizations of the experiments are then used to determine the probability distribution (and therefore the confidence intervals) of the estimate [119].

Since our goal is to make an estimate of a subset of the true values of the
parameters, \( \hat{\theta} \), we define a vector of random variables, \( \hat{\theta} \), to be an estimator of the subset of the parameters we wish to bound. Those parameters we do not wish to bound are held fixed at predetermined values. The Maximum Likelihood Estimator (MLE) is the value of the estimator which maximizes the likelihood in equation 5.5 for a given set of observations, \( t \). Since MLEs are functions of \( t \), they are random variables themselves, and we can derive their probability distribution. Thus, in order to derive confidence limits on a set of parameters from a set of data (for example, to be able to say that \(.01 \leq \Omega_B \leq .1 \) with 90% confidence), we need to find the probability distribution of the MLE. Note that we choose the MLE as our estimator because maximum likelihood estimators have the minimum variance in the limit of large samples [118].

By maximizing the likelihood in equation 5.5 for a large number of “simulated” data sets created from a sky representative of our MLE, we sample the probability distribution of the MLE. Another way to think about this is as follows. First, we find the most likely value of the estimator given our data. As a one-dimensional example, if we are interested in estimating the value of \( \Omega_B \) we would first find the correlation function, \( C(\Omega_B) \), which maximizes the likelihood given our data while holding all other parameters fixed. For consistency of notation let us call \( \hat{\Omega}_B \) the value of \( \Omega_B \) which creates this correlation function. To learn about the probability distribution of our estimator, \( \hat{\Omega}_B \), we then ask: If we measure the sky built from the correlation function \( C(\hat{\Omega}_B) \) with our instrument and then perform this same analysis, how will the results compare to our original estimate of \( \Omega_B \)? In fact, doing this a large number of times gives us the actual probability distribution of \( \hat{\Omega}_B \).
5.3 Making the Calculation Feasible

While the method described above is a “correct” method for estimating the parameters, in its simplest form it is computationally infeasible due to the length of time required to do the calculation. Even for experiments with relatively small datasets like MSAM I, the covariance matrix can be as large as $700 \times 700$. Not only is this time consuming to invert but the calculation of each matrix element (see equation 5.2) is a four dimensional integral. In this section I outline a number of techniques and approximations which make the calculation computationally feasible. Beginning with a re-writing of the covariance matrix in a more convenient form, I then give a description of the use of signal-to-noise eigenvalue decomposition to reduce the total number of significant elements of the covariance matrix. Finally, a prescription for generating simulated observations is discussed.

5.3.1 Calculating the Covariance Matrix

The theoretical covariance matrix calculation is the most time consuming process in the maximization. Recall from equation 5.2 that each element of the covariance matrix is given by

$$[C_T]_{i,j} = \langle t_i t_j \rangle = \int_{\Omega_1} \int_{\Omega_2} B_i(\vec{x}_1) B_j(\vec{x}_2) \langle T(\vec{x}_1) T(\vec{x}_2) \rangle \, d\vec{x}_1 \, d\vec{x}_2 \quad (5.6)$$

where $B_i$ and $B_j$ are beam response functions (synthesized beam patterns) of the experiment. $B_i$ and $B_j$ may be the same or different demodulated beam patterns. If we define $\vec{a}_{ij}$ as the separation vector on the sky between the measurements and $A$ as the rotation matrix which re-aligns the axis\(^2\) of $B_i$ and $B_j$, we can re-write

\(^2\)Note that knowing the orientation of the beam pattern (“twist”) with respect to some fixed axis for each data point is crucial for properly interpreting the signal.
equation 5.6 so that the two beam maps share a common coordinate system

\[ [C_T]_{i,j} = \int_{\Omega} \int_{\Omega} B_i(A\tilde{x}_1) B_j(\tilde{x}_2 + \tilde{a}_{ij}) C(|\tilde{x}_1 - \tilde{x}_2|) d\tilde{x}_1 d\tilde{x}_2 \quad (5.7) \]

where we have also made the substitution \( C(|\tilde{x}_1 - \tilde{x}_2|) = \langle T(\tilde{x}_1) \rangle \langle T(\tilde{x}_2) \rangle \).

Since most CMB observations take place over relatively small portions of sky, we can simplify the calculation of the integral by replacing the beam patterns and the correlation function with their Fourier transform integrals. (Appendix A justifies the use of the flat-space approximation.)

\[ [C_T]_{i,j} = \int_{\Omega} d\tilde{x}_1 \int_{\Omega} d\tilde{x}_2 \int d\tilde{k}_1 B_i(A\tilde{k}_1) e^{-i\tilde{k}_1 \cdot \tilde{x}_1} \int d\tilde{k}_2 B_j(\tilde{k}_2) e^{-i\tilde{k}_2 \cdot \tilde{x}_2} e^{i\tilde{a}_{ij}} \int d\tilde{k} C(\tilde{k}) e^{i\tilde{k} \cdot (\tilde{x}_1 - \tilde{x}_2)}. \quad (5.8) \]

A small amount of calculus transforms this to the two dimensional integral

\[ [C_T]_{i,j} = \int d\tilde{k} C(\tilde{k}) B_i(A\tilde{k}) B_j^*(\tilde{k}) e^{i\tilde{k} \cdot \tilde{a}_{ij}} \quad (5.9) \]

and the four dimensional integral of equation 5.2 has been reduced to a two dimensional Fourier transform.

### 5.3.2 Calculating the Likelihood — Signal-to-Noise Eigenvalue Decomposition

Once the covariance matrix is calculated, the MLE is found by maximizing the log of the likelihood

\[ \ln L = -\frac{1}{2} \sum_{k,l} t_k M_{kl}^{-1} t_l - \frac{N}{2} \ln (2\pi) - \frac{1}{2} \ln \det M. \quad (5.10) \]

What makes this calculation non-trivial is the inversion of the full \( N \times N \) covariance matrix \( M \). For \( N \sim 700 \), such as for the combined data sets of MSAM
I, the direct inversion of the covariance matrix requires nearly 50% of the total calculation time. For data sets with larger sky coverage and more demodulations, such as MSAM II, direct inversion of the covariance matrix becomes prohibitively time consuming.

To help reduce the matrix inversion burden, we use signal-to-noise eigenvalue decomposition to reduce the size of the working data set by disregarding points with low signal-to-noise, $s/n$. This technique, known in image processing as “Karhunen-Loève” eigenvalue decomposition [8], has been suggested by a number of authors as a potential solution to the upcoming deluge of data from the Planck and MAP satellite missions [120, 121]. The idea is to eliminate the observations which do not contribute significantly to the parameter determination effort — that is, those observations with low signal-to-noise. To do this, we find the projection which diagonalizes the theoretical covariance matrix in a basis where each element of the covariance matrix is written in terms of $[s/n]^2$. In this basis, it is clear which elements to keep and which to discard. The covariance matrix can then be sorted in order of decreasing signal-to-noise and trimmed to remove those elements which fall below a pre-determined signal-to-noise cutoff.

The process, as described in [121] is as follows:

1. Calculate the theoretical covariance matrix, $C_T$.

2. Decompose the noise covariance matrix, $C_n$, into eigenvalues to create $C_n^{-\frac{1}{2}}$ such that

\[
C_n \rightarrow C_n^{-\frac{1}{2}} C_n (C_n^{-\frac{1}{2}})^T = I
\]  \hspace{1cm} (5.11)

where $I$ is the identity. This is true if

\[
[C_n]_{ij}^{-\frac{1}{2}} = \left[ \frac{1}{w_i} \right]^{\frac{1}{2}} V_{ij}
\]  \hspace{1cm} (5.12)

where $w_i$ are the eigenvalues and $V$ is the matrix of eigenvectors. We can
think of $C_n^{-\frac{1}{2}}$ as the projection into the basis which “whitens” the noise (that is, where the noise covariance is the identity).

3. Eigenvalue decompose $C_n^{-\frac{1}{2}}C_T(C_n^{-\frac{1}{2}})^T$ to get the matrix $R$ such that

$$R[C_n^{-\frac{1}{2}}C_T(C_n^{-\frac{1}{2}})^T]R^T = \text{diag}(\varepsilon) \quad (5.13)$$

where $\varepsilon$ is the diagonal matrix of eigenvalues of $C_n^{-\frac{1}{2}}C_T(C_n^{-\frac{1}{2}})^T$. $\varepsilon$ is in terms of $\left[\frac{\lambda}{n}\right]^2$, and is an indicator of which elements of the covariance matrix carry the most cosmological information. Note that the Karhunen-Loève projection operator, $P = RC_n^{-\frac{1}{2}}$ also whitens the noise matrix $C_n$.

4. Transform the vector of signals, $t$, by $P$.

$$\xi \equiv RC_n^{-\frac{1}{2}}t \quad (5.14)$$

which has units of $[s/n]$.

5. Sort the eigenvalues, $\varepsilon$, in decreasing order (simultaneously re-sorting the covariance matrices and the vector of signals), keep only the $m$ eigenvalues which fall above a threshold value of signal-to-noise.

6. Construct the new $(m \times m)$ covariance matrix

$$\langle \xi_i \xi_j \rangle = [1 + \varepsilon_{ij}] \quad (5.15)$$

The steps outlined above are only followed the first time the theoretical covariance matrix is calculated and some care must be taken to select a correlation function which results in a “fair” amount of signal, to avoid over (or under) cutting the data. On subsequent calculations of the covariance matrix, the full matrix is
CHAPTER 5. COSMOLOGICAL PARAMETER ESTIMATION

operated on by $P$

$$C_T \rightarrow C'_T = PC_T$$ (5.16)

and the resulting matrix (which is no longer diagonal in this basis) is sorted in order of decreasing $s/n$, trimmed at a pre-defined cut-off in $s/n$, and inverted.

5.4 The THRASH Approach to the Calculation

THRASH is a C++ program written by the author to extract bounds on cosmological parameters using the tools outlined above. In order to calculate these bounds as quickly as possible, THRASH incorporates signal-to-noise eigenvalue decomposition as well as a host of other time-saving techniques which are described below.

Recall from section 5.2 that frequentists define the estimator of a parameter as a random variable which is an estimate of the parameter. In order to bound the estimator, we must find the likelihood of the estimator given the data, $L(\hat{\theta}; t)$ which is the probability distribution of the estimator. Since we know how to calculate the likelihood of the data given the parameters, $L(t; \bar{\theta})$, we can now calculate the likelihood of the estimator given the data $L(\hat{\theta}; t)$. The connection between the two goes as follows: we hypothesize that the correlation function which is constructed from the MLE of the data, $C(\hat{\theta})$, is representative of the probability distribution of the data. Since the estimator, $\hat{\theta}$ is a function of the data, $t$, the probability distribution of the estimator can be derived by calculating the MLE for different simulated values of $t$ — all of which are created with the same probability distribution. This is equivalent to a Monte Carlo integration of $L(\hat{\theta}; t)$. The 90% confidence interval, $[\theta_{\text{low}}, \theta_{\text{high}}]$, of a single dimensional parameter, $\theta$, is given by

$$\int_{\theta_{\text{low}}}^{\theta_{\text{high}}} L(\hat{\theta}; t) d\hat{\theta} = 0.9.$$ (5.17)

The algorithm of the calculation is then:
1. Calculate the MLE, $\hat{\theta}_0$, of the original data set, $t$, by maximizing $L(t; \bar{\theta})$.

2. Hypothesize that the data has a statistical distribution given by the correlation function, $C(\hat{\theta}_0)$, that is, assume that this is the "true" correlation function of the sky. Now, using this correlation function, $C(\hat{\theta}_0)$, generate a new set of observed data, $t'$. By our hypothesis, this data set will be distributed the same way as the original data set.

3. Find the new MLE, $\hat{\theta}'$, of the likelihood, $L(t'; \bar{\theta'})$. This step is a single iteration in the Monte Carlo integration above.

4. Repeat steps 2 and 3 a large number of times — remembering the set of MLE’s found from the simulated data sets.

5. Using the results of step 4, calculate the set of values $L(t; \hat{\theta}')$ where $t$ is the original data set.

6. The 90% confidence interval can be defined as the boundary of constant likelihood, $L(t; \hat{\theta}') = L_0$, where $L_0$ is defined such that 90% of the MLE’s, $\hat{\theta}'$, give $L(t; \hat{\theta}') \geq L_0$.

The last two steps are somewhat arbitrary since any interval containing 90% of the found MLEs would suffice to bound the parameters with 90% confidence. The steps laid out here are analogous to bounding with a surface of constant $\chi^2$. In fact, we choose our bounds with a surface of constant likelihood which also happens to be a surface of constant $\chi^2$.

**How Many Monte Carlo Iterations Must Be Done**

Recall that the process outlined above is actually a Monte Carlo integration of the probability distribution of the estimators. Since each calculation of the likelihood is computationally expensive, we wish to know the minimum number of iterations
which we must do to bound the estimator. In other words, we want to know the uncertainty of the confidence interval as a function of the number of iterations.

A change of variables in the integrand gives us the answer. Since this is actually a counting problem (where we count the number of values of the MLEs which fall between the low and high limit), we can make a change of variables such that the integral in equation 5.17 is constant inside the bounds and zero outside. The constant value inside the bounds is the desired confidence in the resulting bounds, $\gamma$, where in our case $\gamma = 0.9$. The uncertainty on the bound, $\delta \gamma$ is then given by [119]

$$
\delta \gamma = \sqrt{\frac{\gamma - \gamma^2}{N}}
$$

where $N$ is the number of MLEs which must be found. Thus, in order to find the 90%±1% confidence interval, we must do 900 iterations.

**Minimizing the Computation Time**

Each element of the theoretical covariance matrix, $C_T$ (equation 5.9), is simply the two-dimensional Fourier transform of the quantity $C(\vec{k})B_i(A\vec{k})B^*_j(\vec{k})$ evaluated at $\vec{x}_{ij} = \vec{a}_{ij}$. Thus, to calculate the set of elements of the covariance matrix, $C_T$, made from two beammaps which are rotated with respect to each other by the same matrix $A$, we do a single two-dimensional Fourier transform. The total number of Fourier transforms that we must do to compute the entire covariance matrix depends on the number of demodulations (beam patterns) and the range of orientation angles of the points on the sky.

A large savings in efficiency can be achieved by pre-computing as much as possible and by recognizing a priori what elements of the covariance matrix can be calculated with a single two-dimensional Fourier transform. On machines with limited physical memory (or for problems with a very large number of demodulations and rotation angles) two Fourier transforms must be done for each set
of angles making up the covariance matrix. The first moves the unrotated (or rotated) input beam pattern into frequency space while the second Fourier transform is the solution of the integral in equation 5.9. \texttt{THRASH} takes the approach of pre-computing the needed Fourier transforms of the rotated beam patterns — leaving only the second Fourier transform to compute in real-time. While this has the obvious disadvantage of relying on a large amount of system memory or storage space, the resulting time savings is on the order of 30\% of the entire calculation time as is shown below.

In order to efficiently pre-compute the large array of rotated beam patterns, \texttt{THRASH} first selects the needed rotations based on two criteria — the binning in twist (that is, the size of the bin which groups beams rotated by an angle less than \( \alpha \)) and the low frequency power in the correlation function. To determine a proper bin size in twist, \( \alpha \), we calculate the error in the measurement between an unrotated beam and a beam rotated by \( \alpha \). That is, we calculate

\[
\langle \delta^2 \rangle = \int d\vec{k} \left| B_1(\vec{k}) - B_2(A\vec{k}) \right|^2 C(\vec{k})
\]

where \( A \) is the rotation matrix which rotates the beammap by \( \alpha \) and \( C(\vec{k}) \) is an arbitrary correlation function representative of the correlation functions under consideration in the likelihood analysis. The square-root of this quantity has units of \( \mu K \) and can be thought of as an added amount of “noise” to the data set\(^3\).

As a diagnostic I define the “Binning Degradation Factor” (\( BDF \)) which is the ratio between the system noise including the binning contribution and the system noise neglecting the binning induced error, \( \sigma \),

\[
BDF = \sqrt{\sigma^2 + \langle \delta^2 \rangle/\sigma}.
\]

\(^3\)The “noise” referred to here is actually a systematic error in the data handling. In the case of the \( BDF \) defined here, however, it is equivalent to extra noise in the data.
In other words, the $BDF$ is the factor that the system noise is increased due to the coarseness of the binning choice. Note that in this case the $BDF$ is completely general and can be defined for any type of binning of the data (for example, spatial binning).

Figure 5.2 is a plot of the $BDF$ as a function of the twist bin size for the MSAM I experiment. The beam patterns for the two demodulations are shown in figures 5.3 and 5.4. Each demodulation responds differently to the amount of twist in the bin, with the single difference demodulation being more sensitive to the twist due to its asymmetry about the rotation axis. The correlation function used in this case was a SCDM variant with $\Omega_b = 0.2$ and $\Omega_{CDM} = 0.8$. From the figure, twist bin sizes greater than $5^\circ$ are rejected on the basis of adding a significant amount of noise to the measurement.

The second criterion for choosing which rotation angles are calculated is whether there is significant signal in points which are separated on the sky by a large amount. Clearly, if the correlation function is getting small at low spatial frequencies (that is, large separation angles on the sky), there is no need to do a Fourier transform. THRASH chooses an arbitrary cut-off of $3^\circ$ for the separation of beams on the sky. Points which are separated by more than $3^\circ$ are set to zero in the covariance matrix.

**Gridding of the Beam Patterns**

The correlation functions of interest to us have a variety of shapes and features. As a result, we must take care in choosing how coarsely we do the sampling of the beam pattern in frequency space in order to not under-sample the correlation function. For example, for experiments with beamsizes larger than $\sim 30^\prime$, the sensitivity is constrained to spatial frequencies where nearly all the correlation functions are smooth and monotonic over values of $\Delta l \leq 50$. For smaller scale
Figure 5.2  The $BDF$ for the MSAM I demodulations. The “twist” between the beams represents the bin size. In the plot, “2beam” refers to the single difference demodulation and “3beam” is the double difference demodulation described below. The correlation function used to create this plot is a SCDM variant with $\Omega_B = 0.2$ and $\Omega_{CDM} = 0.8$. Twist bin sizes of greater than 5° are rejected on the basis of adding a significant amount to the total system noise.
experiments which probe past the first acoustic peak, finer frequency resolution is needed. There is a trade-off to be made here, however. The more finely we sample the beam pattern, the larger the resulting matrix and the more costly the calculation of the covariance matrix. On the other hand, we must ensure that we have sampled the beam pattern sufficiently to accurately probe the correlation function of interest.

In the case of THRASH the Fourier transforms of the beam patterns are pre-calculated before considering the correlation function. The size of the Fourier transform (and therefore the size of the bins in frequency space) is fixed for the entire calculation by an estimate of the finest frequency resolution needed for a particular class of correlation functions. While this wastes some time compared to a program which interactively chooses the size of the Fourier transform based on each particular correlation function, this time is more than made up for by the ability to pre-compute the initial round of Fourier transforms rather than doing two separate sets of Fourier transforms for each calculation of the covariance matrix.

The Correlation Functions

The correlation functions, \( C_\ell(\tilde{\theta}) \), are pre-computed using the routine CMBFAST. While CMBFAST is indeed the fastest code available for calculating CMB correlation functions, the few minutes required to calculate each set of multipoles is prohibitively long to incorporate CMBFAST into the THRASH calculation. Instead, THRASH requires an input file of pre-computed values of \( C(\tilde{\theta}) \) over the interesting range of parameters \( \tilde{\theta} \). Correlation functions for values of \( \tilde{\theta} \) which fall between the gridded values which make up the input file are derived by a multi-dimensional cubic spline interpolation. The grid size in \( \tilde{\theta} \) of the input file is determined \textit{a priori} by testing the accuracy of the interpolation routine on a grid which is twice
as coarse as the final grid.

Since the correlation functions in the input file are gridded in \( l \)-space much more finely than the beammats, \textsc{Thrash} re-grids them a final time using a one-dimensional cubic spline interpolation. The correlation function is then packed into a matrix in the coordinates of the beammat while zeroing the highest frequency components.

\section*{Generating Simulated Data Sets}

Once the MLE of the actual data set has been found, confidence limits are generated by performing the same maximization on a large number of “simulated” data sets which are derived from the correlation function stemming from the MLE — \( C(\hat{\theta}_0) \). This simulated data is contaminated by the actual noise in the experiment through the noise covariance matrix, \( C_n \). Here I list the recipe used for generating the simulated data:

1. Decompose \( M = [C_T + C_n] \), where \( C_T \) is the theoretical covariance matrix constructed from the correlation function \( C(\hat{\theta}) \), into eigenvalues to get

\[
M = O\lambda O^T \tag{5.21}
\]

where \( O \) is the matrix of eigenvectors and \( \lambda \) is the diagonal matrix of eigenvectors.

2. Define the operator \( A \) as

\[
A = O\lambda^{\frac{1}{2}} \tag{5.22}
\]

so that \( AA^T = M \).

3. Create a vector, \( x \), of Gaussian random variables with mean zero and unity variance.
4. Simulated data sets can then be calculated by

\[ t' = \sum I A_{kl} x_l. \]  \hspace{1cm} (5.23)

5.5 Testing THRASH

By far the most difficult portion of the THRASH calculation is the calculation of the theoretical covariance matrix. As mentioned above, in order for this to be done efficiently, a large amount of pre-computing must be done and a few approximations must be made. The THRASH covariance matrix calculation has been tested against two programs written by others that calculate the theoretical covariance matrix as well as another program written by the author. Each of these three programs uses a different technique to calculate the theoretical covariance matrix and each agrees with the THRASH calculation to within 6%. This level of agreement is actually quite amazing given the variations which come in the choices of how the beam pattern is gridded, how the correlation function is interpolated, etc.

Once the theoretical covariance matrix is known to be calculated correctly, all the other calculations in the program are trivial to check using test cases. The entire algorithm can be tested by feeding the code data sets which are derived from a known correlation function. This has been done in a number of cases for THRASH. This is a fairly vague way to test the code, however, in the sense that the output of the program is the 90% confidence interval which in most cases is fair fraction of parameter space.

Finally, THRASH has been tested by reproducing the calculations of \( \Delta T/T \) for the Gaussian Autocorrelation Functions which have been historically used in the past for the MSAM I data sets. Again, this test is slightly ambiguous since the choice of a confidence interval is completely arbitrary. However, it has been shown
Figure 5.3  The single difference demodulation for MSAM I. Shown is the beam pattern from the single difference demodulation (the right beam subtracted from the left beam). The demodulation is normalized such that the peak height is 1.

that in no case are the results of the calculations inconsistent with the previous conclusions.

5.6  THRASH Results - MSAM I

THRASH has been used to derive limits on a few cosmological parameters from the combined three-years of data from the MSAM I instrument [7, 41, 42]. MSAM I is a five channel instrument with a 30’ FWHM beamsizes [47]. The chopper chops in a three-position square wave with a stopping point in the middle and at ±40’ to either side. This chopping strategy results in two nearly-orthogonal demodulations: the left beam subtracted from the right beam, and the middle beam subtracted from 1/2 the sum of the outside beams. Figures 5.3 and 5.4 show normalized plots of the two beammaps as a function of azimuth and elevation.
Figure 5.4  The double difference demodulation for MSAM I. Shown is the beam pattern from the double difference demodulation (center-1/2(left+right)). This demodulation is normalized such that the middle beam has a peak height of 1.
In 1992, the MSAM I instrument observed two strips of sky located at a
declination of $82^\circ$ [7]. In all, 195 correlated spots on the sky were measured.
In 1994 an attempt was made to re-measure the same spots on the sky however
a late launch caused a number of early spots to be missed [41]. In the end, 49
spots were measured in the first strip and 95 more in the second. The ’92 and
’94 observations are combined and shown to be consistent in [122]. Finally, in
1995 the MSAM I instrument flew its last flight — making observations in a new
section of sky and covering an additional 166 spots.

The combination of the ’92 and ’94 data sets reduces the total number of data
points for the three years from 1010 (505 points for each demodulation) to 704.
Using a signal-to-noise eigenvalue decomposition cut-off of $\frac{\lambda}{\sigma} \geq .01$ results in a
further cut in the covariance matrix size to 414 points.

We have re-gridded the beam pattern measurements for each of the flights to
lay on a grid with 2.75’ spacing. The beam patterns are zero-padded to a size of
$256 \times 256$ resulting in an $l$-space resolution of 31 units. For the low-$l$ portions of
the correlation function which MSAM I is sensitive, this resolution is more than
adequate to sample the correlation functions of interest.

**Offsets in the Data Sets**

One feature of the MSAM I data sets (which is common to most CMB experi-
ments) is the removal of an arbitrary offset. The lack of knowledge of this offset
is a result of taking differences on the sky and is a reminder that experiments
like MSAM I are not total power experiments. The offset removal is done by
subtracting the value of the last data point from all the preceding measurements.
The corresponding row and column of the covariance matrix are then zeroed to
manually zero the infinite eigenvalue introduced due to the removal of one of the
degrees of freedom in the data\(^4\). This step must be taken for each independent data set.

The impact of this offset removal is that the calculation of the eigenvalues, \(w\), in equation \(5.12\) must be done using singular value decomposition. Thus, to calculate \(C^{-\frac{1}{2}}_n\), the zeroed eigenvalue must explicitly be made infinite to avoid a singularity in the projection matrix. Once this is done, the problem is solved. For the remainder of the code execution, the Karhunen-Loève projection operator will zero the appropriate signal-to-noise eigenvector which will subsequently be thrown out with the other small values of \(\varepsilon\).

### 5.6.1 THRASH Timing for the MSAM I data sets

Table 5.1 shows the timing budget for running THRASH on the full three years of MSAM I data on a Pentium Pro 200 MHz PC with 128 MB of RAM. Listed in the table are the processes which dominate the timing of the calculation of a single value of the likelihood. The sum of the processes not listed take less than 1\% of the total calculation time.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Average Duration [s]</th>
<th>% of Total Job</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial setup and pre-computing</td>
<td>240</td>
<td>–</td>
</tr>
<tr>
<td>Fourier Transforms</td>
<td>34</td>
<td>71</td>
</tr>
<tr>
<td>K.L. Projection</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>LU Inversion</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Misc. (sorting, various matrix multiplica-</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>tions, etc.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With this setup, THRASH requires 48 s to compute the Likelihood for one value of the parameters \(\vec{\theta}\). A nearly equivalent amount of time is needed to calculate each partial derivative of the Likelihood when maximizations over multiple dimensions

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\(^4\)The infinite eigenvalue is manually set to zero so that it is flagged for later care.
are required, however only a small additional amount of computation time is needed to calculate the very useful second partial derivatives. Since the calculation of each MLE requires \(\sim 10\) evaluations of the likelihood and we would like to find \(\sim 1000\) MLE’s, keeping the likelihood calculation fast is of paramount importance.

### 5.6.2 THRASH Bounds on Single Parameters for MSAM I

I have used THRASH to place bounds on \(\Omega_b\) and \(H_0\) while holding all other parameters at their SCDM values\(^5\). For historical reasons, I have also derived limits on \(\delta T/T\) for a Gaussian Autocorrelation Function (GAF) for each demodulation. For each parameter, 2500 MLE’s were found in order to bound the 90\% confidence region. Table 5.2 gives the bounds placed on each of the parameters by the MSAM I data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta T/T) (single difference)</td>
<td>(13.2 \times 10^{-6} \leq \Delta T/T \leq 19.6 \times 10^{-6})</td>
</tr>
<tr>
<td>(\Delta T/T) (double difference)</td>
<td>(31.9 \times 10^{-6} \leq \Delta T/T \leq 46.5 \times 10^{-6})</td>
</tr>
<tr>
<td>(\Omega_B) (SCDM values for other parameters)</td>
<td>(\Omega_B \leq 0.0752)</td>
</tr>
<tr>
<td>(H_0) (SCDM values + (\Omega_B h^2 = 0.0125))</td>
<td>(39.62 \leq H_0 \leq 80.06)</td>
</tr>
</tbody>
</table>

### 5.7 Final Comments on THRASH

While I have only given results for a single parameter, THRASH is capable of finding bounds on multiple parameters at once. The Monte Carlo method for finding the bounds on the parameters generalizes to the multi-dimensional case and the principal differences in the code come in the calculation of the derivatives of the likelihood during the maximization. A run of THRASH on the combined MSAM I

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\(^5\)See Chapter 1 for a description of the SCDM parameters.
data sets is currently under way to set bounds on the spectral index, $n$, and the Hubble constant, $H_0$.

THRASH is a versatile program which is capable of setting limits on any set of cosmological parameters for which correlation functions can be calculated. While THRASH is a suitable choice for data sets such as the MSAM I and MSAM II data sets, larger amounts of data (and more demodulations), will require a far larger amount of computing power than I currently dedicate to the job. However, the Monte Carlo approach which THRASH uses to set the parameter limits is ideal for breaking a particular job up among a number of computers. As long as the random number generator which creates the simulated data sets is started with a different seed, various computers can be used concurrently and their results can be simply combined in the end. Furthermore, since relatively inexpensive PC’s now support 200 MHz (and greater) processors, it is not unrealistic to imagine a bank of PC’s running THRASH to process the data from a large CMB experiment.
Chapter 6

Conclusion

The MSAM II telescope promises to make an unambiguous detection of the first acoustical peak of the CMB power spectrum. The instrument flew for the first time in the summer of 1997 and took over four hours of CMB data. Unfortunately, a problem with the optical system which had never been seen in ground-based measurements led to a frequency and chopper-position dependent beam. In a possibly related problem, the detectors ran much warmer than expected - decreasing the overall sensitivity of the instrument. During the flight we became aware of the state of the detectors and subsequently adjusted our observing strategy to compensate for the reduced sensitivity. The data from this flight is now being analyzed.

Before flying the instrument again, we must find the source(s) of these two problems. Given the nature of the beamsize of the telescope in flight - the beam is wider at the ends of the chopper throw than in the middle and has a frequency dependent width - it is the author’s speculation that the alignment between the secondary and primary mirrors was off in the vertical direction during the flight. Since the edge taper on the primary mirror is a very strong function of frequency, as the chopper moves the beam from side to side on the primary mirror, more of the channel 1 beam would fall off the edge of the mirror than in the case of the channel 5 beam. This may lead to an effect on the beamsize which would be consistent with the flight data. Furthermore, if the beam is spilling onto the
relatively hot gondola structure, this could explain the increased optical loading of the detectors. Unfortunately, since the telescope was disassembled after the flight, this is now nearly impossible to check. In the spring of 1998, a team will return to Palestine with the MSAM II instrument, work to debug these problems and fly the instrument again.

Once this new MSAM II data is taken, we will use the THRASH program to place limits on cosmological parameters in an inflationary cosmology. THRASH has now been run on the combined three years of data taken by the MSAM I instrument in order to bound the baryon content of the universe as well as the Hubble constant. While these bounds are not revolutionary, their calculation demonstrates the utility of the code and points out the strengths and drawbacks of the algorithm. THRASH is general enough to be used for any CMB experiment with any number of synthesized beam patterns or data points.

It is truly a great time to be involved in CMB research. In the last five years we have gone from a drought of anisotropy data to being on the verge of a computational crisis from the huge data sets soon to come from the satellite and long duration ballooning experiments. The MSAM II instrument has benefited from the many lessons learned by the first stage of successful instruments and, despite the problems of its first flight, promises to deliver new information about the power spectrum of fluctuations. Soon after, a variety of long duration ballooning flights will take place - covering from 5% to 25% of the sky. Finally, in the beginning of the next century, two satellite missions will map the entire sky with unprecedented angular resolution. While extracting the cosmological information from the data these experiments return promises to be challenging, so too does the CMB promise to open new doors of understanding about our universe.
Appendix A

Justification of the Flat-Space Approximation

At this point we will justify the use of Fourier transforms in the calculations in Chapter 5 rather than spherical harmonics. Recall from Chapter 1 that the Legendre polynomial expansion of the correlation function \( C(\phi) \) is

\[
C(\phi) = \sum_l \frac{2l+1}{4\pi} C_l P_l(\cos \phi).
\] (A.1)

If instead of expanding in Legendre polynomials we expand in Fourier modes with coefficients, \( b_k \), where \( C_k = \langle |b_k|^2 \rangle \) we have

\[
C(\phi) = \frac{1}{2\pi} \int dk C_k J_0(k\phi)
\] (A.2)

where \( J_0(k\phi) \) is the zero-order Bessel function. From equations A.1 and A.2 we find

\[
C_k = \sum_l \frac{2l+1}{2} C_l \int_0^\infty J_0(k\phi) P_l(\cos \phi) \phi d\phi.
\] (A.3)

For observations which are separated on the sky by small angles \( (\phi < \sim 10^\circ) \) and for large \( k \), \( J_0(k\phi) \approx P_k(\cos \phi) \) and \( \phi d\phi \approx d(\cos \phi) \). Putting these approximations into equation A.3 we find \( C_k \approx C_l \) [123].

Of course, this argument works for the beam patterns of Chapter 5 in an analogous fashion.
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