The Millimeter-wave Bolometric Interferometer

by

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THE MILLIMETER-WAVE BOLOMETRIC INTERFEROMETER

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The Millimeter-wave Bolometric Interferometer (MBI) is a novel instrument for measuring signals from the cosmic microwave background (CMB) radiation. MBI is a proof-of-concept designed to control systematic effects with the use of bolometers and interferometry. This scheme extends radio astronomy techniques of spatial interferometry, which rely on coherent receivers, to a system using incoherent detectors.

In this thesis we outline the principles upon which MBI works and provide the reader with an understanding of both the particulars involved in the design and operation of MBI as well as the analysis of the resulting data.

MBI observes the sky directly with 4 corrugated horn antennas in a band centered on $\lambda = 3\, mm$. A quasi-optical beam combiner forms interference fringes on an array of bolometers cooled to 300 mK. Phase modulation of the signals modulates the fringe patterns on the array and allows decoding of the visibilities formed by each pair of antennas. An altitude-azimuth mounting structure allows the horns to observe any point on the sky; rotation about the boresite extends the $u - v$ coverage of the interferometer and allows for systematics checks and measurements of the Stokes parameters.
MBI was deployed at the Pine Bluff Observatory near UW - Madison in winter 2008 for its first test observations of astronomical and artificial sources. Interference fringes were seen from a microwave generator located in the far-field, verifying our basic model of bolometric interferometry. Further analysis is needed to measure the scattering matrix of the instrument and to compare it against simulations.

Peter Timbie (Adviser)
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Chapter 1

Introduction

1.1 A Brief History of the CMB

The last century has seen vast changes in the field of Cosmology. With the advent of Hubble’s Law the era of the static universe came to an end, dooming Einstein to his “greatest blunder.” Einstein’s Theory of General Relativity naturally produced a dynamic universe through the interaction of mass, energy and space-time. To reconcile this with his belief in a static universe Einstein introduced a Cosmological Constant, \( \Lambda \), to balance the equations and produce a static universe. Later, after coming to terms with the reality of a dynamic universe, Einstein referred to his introduction of the Cosmological Constant as his greatest blunder. The new mobility of the Universe was found to reside in expansion. Hubble’s Law states that objects have a recessional velocity from us that is proportional to their distance from us. Run time backwards and everything comes to a point at a single instant in time. This is the Big Bang theory.

The Cosmological Principle, the assumption that on large scales the universe is homogeneous and isotropic, precludes having the Earth at the center of the universe.
This problem is solved by using General Relativity and proposing that space-time is expanding everywhere. This produces objects that are locally at rest, but moving with respect to one another, with the speed of that movement proportional to the amount of space-time between them.

As the universe expands after the Big Bang it cools, the fundamental forces separate into the ones we are familiar with, and the soup of high mass, exotic particles and anti-particles gives way to lower mass and less exotic particles. Photons, neutrinos and a plasma of protons and electrons remain. Eventually the plasma cools to the point where neutral hydrogen forms. As a result, all the remaining photons begin to travel in straight lines because there are no free charged particles to scatter them. This transition happened about 360,000 years after the Big Bang and those photons have continued to travel unimpeded. The stretching of space-time has red-shifted them until their originally high energy spectrum now peaks at microwave wavelengths and they now make up the Cosmic Microwave Background (CMB).

The light from what is called the last scattering surface or LSS is what makes up the CMB. The LSS was formed at the moment of hydrogen formation, which is called “last scattering” as it is the last time that CMB photons scattered off of matter. After last scattering an observer would see a glow coming from all directions. If $t_{ls}$ is the time at which last scattering occurred and $t_0$ is the current time, then the photons would have been traveling for $t = (t_0 - t_{ls})$ and would have traveled a distance $ct$. An observer will see a glowing surface which is a distance $\sim ct$ from them. This spherically symmetric surface is the LSS. The photons from the LSS started out at $\sim 3000$ K, but they have been red-shifted to a temperature of $2.73$ K by the expansion of the
universe. The temperatures quoted here correspond to the black body spectrum produced. The light from the LSS is what makes up the CMB and it has encoded in it information about the universe at the time of last scattering.

1.2 Problems

Now we shall take a brief look at the problems with the original Big Bang model.

1.2.1 Structure Formation

The Cosmological Principle implies that all points in the universe are equivalent and in a universe filled with plasma that means the density is the same everywhere and the temperature very uniform. Early CMB experiments found this to be the case. However, this condition would not allow any structure formation. We see structure in galaxies, galactic cluster and super clusters, but there is no mechanism to form them.

1.2.2 Horizon

Another problem is that the temperature of the CMB is too uniform. Areas of the universe that are out of causal contact, or outside of each other’s horizon, have no way of coming to thermal equilibrium. Therefore we should expect that areas of the sky that are of order the horizon size at last scattering should be the same temperature. Areas of the sky that are separated by more than a horizon size should have uncorrelated temperatures. What is found is that the entire sky, spanning many horizons, is at a uniform temperature to a high degree of accuracy.
1.2.3 Flatness

The problem of flatness is along the same lines. Flatness is a measure of the geometry of the universe itself, but since space-time is warped by energy it is also a measure of the energy density of the universe. A flat universe is one where the coordinates of space-time are laid out in a grid, just like we are used to. Two parallel lines will never cross and will always have the same separation. Curvature can also be positive, also known as a Closed geometry, or negative, also known as an Open geometry. In a closed universe two parallel lines will come together and cross. In a closed universe composed entirely of matter and radiation the universe will eventually re-collapse. In an open universe two parallel lines will diverge and never cross and the universe will expand forever. A way to visualize these types of universes is to imagine as an analogy a two-dimensional universe. A flat universe is one that exists on a flat sheet of paper, a closed universe is one that exists on the surface of a sphere and an open universe exists on a saddle. Saddle points look like Pringle (c) chips. In general it is very difficult to get a flat Universe, which is troublesome because we live in a universe that is flat to high precision.[4] [5] It requires some very fine tuning of the energy balance to get a flat universe. If $R$ represents the curvature of a universe then $R > 0$ means the universe is closed, $R < 0$ means it is open, and $R = 0$ means it is flat. There are an infinite number of values for $R$ that lead to an open universe and an equally infinite number that lead to a closed universe. On the other hand there is only one value that will give you a flat universe. It is infinitely unlikely that a randomly chosen $R$ will give you a flat universe.

$$P(\text{flat}) = \frac{1}{\infty + \infty + 1} = 0$$ (1.1)
Our measurements show that the universe is flat to a high precision. So how did we end up with a flat universe if it is so unlikely?

1.3 Inflation to the Rescue

These are some pretty serious problems to try gloss over with the Big Bang theory. Thankfully inflation theory solves them all. Inflation states that at some time very early in the history of the universe space-time expanded incredibly fast. We are now in the realm of General Relativity.

With an expanding space-time it is easier to deal with a “co-moving coordinate”, \( \vec{r} \), and a scale factor \( a(t) \). \( \vec{r} \) doesn’t change with time and represents the proper distance between objects right now. \( a \) represents how much space-time has stretched between now and the time of interest. How the scale factor changes with time depends on the energy density of the universe and is described by the Friedmann equation:

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left( \rho(t) + \frac{\rho_{cr} - \rho_0}{a^2} \right)
\]  

where \( \rho(t) \) is the energy density of the universe as a function of time, \( \rho_{cr} \) is the critical density required to make the geometry of the universe flat now and \( \rho_0 \) is the energy density now.

Typically the energy density is diluted when space-time expands. For example, if space-time were a box and energy were marbles in that box, then expanding space-time would correspond to making the box larger. The energy density would then go down as the same amount of energy filled a larger box. If, however, the energy density is not diluted as the universe expands then the scale factor would look like this:

\[
a(t) = a_0 e^{H(t-t_0)}
\]
This is a very strong dependance on $t$. Dependencies for matter-dominated and radiation-dominated universes are $a \propto t^{2/3}$ and $t^{1/2}$ respectively.\[1\]

During inflation space-time expanded faster than the speed of light. In less than a second the Universe expanded by a factor of at least $10^{26}$, or $e^{60}$. This scaling is described as 60 $e$-folds, where an $e$-fold is a change in scale by $e$. Two $e$-folds means the scale has increased by $e^2$. Let us now look at how this relatively simple assumption can solve the aforementioned problems.

### 1.3.1 Structure Formation

By structure we mean stars and galaxies, not buildings, but then without the former there wouldn’t be the latter. The connection to inflation comes if we take a little leap of faith and assume that quantum theory has general consequences for everything. In General Relativity this means that fluctuations show up in the space-time metric. These perturbations can be of scalar, vector or tensor types. Scalar perturbations lead to particle under- and over-densities, while tensor perturbations are what we call gravitational waves. Vector perturbations represent shear flow in the metric and are expected to have zero amplitude.

We would like to understand how the perturbations from inflation evolve. We will mention a few highlights along the way to reaching a set of equations which will govern this evolution. A more detailed procedure is the realm of textbooks such as Chapter 5 of [1]. These perturbations to the space-time metric $\Phi$ and $\Psi$ are the scalar perturbations to the metric representing Newtonian and space-time curvature.
contributions, respectively, and show up in the following way:

\[
g_{ij} = \begin{pmatrix}
-1 - 2\Psi(\vec{x}, t) & 0 & 0 & 0 \\
0 & a^2(1 + 2\Phi(\vec{x}, t)) & 0 & 0 \\
0 & 0 & a^2(1 + 2\Phi(\vec{x}, t)) & 0 \\
0 & 0 & 0 & a^2(1 + 2\Phi(\vec{x}, t))
\end{pmatrix}
\]

We will be calculating the Einstein equations.

\[
G_{\mu\nu} = 8\pi G T_{\mu\nu}
\]  

(1.4)

To do this we must compute the Ricci tensor, \(R_{\mu\nu}\), and Ricci scalar, \(R\), with the perturbed metric and use \(G_{\mu\nu} \equiv R_{\mu\nu} - g_{\mu\nu}R/2\) to obtain the left hand side of the equations. The right hand side of the equations, the energy-momentum tensor, is the sum of energy densities. We do not need all the equations which this method produces as there are only two independent equations. At the end of the calculation we are left with the Einstein equations:

\[
k^2\Phi + 3\frac{\dot{a}}{a}(\Phi - \Psi \frac{\dot{a}}{a}) = 4\pi Ga^2(\rho_m\delta_m + 4\rho_r\Theta_{r,0})
\]

(1.5)

\[
k^2(\Phi + \Psi) = -32\pi Ga^2\rho_r\Theta_{r,2}
\]

(1.6)

Here \(k\) is the wave number for the perturbations, \(\rho_m\) and \(\rho_r\) are the energy densities of all matter and radiation respectively, \(\delta_m\) is a measure of the matter over densities, and \(\Theta_{r,0}\) and \(\Theta_{r,2}\) are the radiation monopole and quadrupole, respectively. When inflation occurred those quantum fluctuations were expanded. Slight over- and under-densities in particle fields were made larger and separated so that they persisted. These over- and under-densities eventually led to the galaxies and galactic clusters we see today.

We must treat the tensor perturbations separately from the scalar ones. Again I will present a few highlights and not delve deeply into derivations. The metric terms
for tensor perturbations are given by $g_{00} = -1$, zero space-time components, $g_{0i} = 0$, and spatial elements
\[ g_{ij} = a^2 \begin{pmatrix} 1 + h_+ & h_x & 0 \\ h_x & 1 - h_+ & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]

Here $h_+$ and $h_x$ are two components of a divergenceless, traceless, symmetric tensor.

Since tensors do not perturb the Ricci scalar we drop those terms from the perturbed first-order Einstein tensor leaving:
\[ \delta G^i_{\ j} = \delta R^i_{\ j} \]

(1.7)

Solving this matrix equation yields the following equation, which governs the tensor perturbations to the metric:
\[ \dot{h}_\alpha + 2\frac{\dot{a}}{a} h_\alpha + k^2 h_\alpha = 0 \]

(1.8)

where $\alpha = +, \times$, the orthogonal tensor modes.

1.3.2 Horizon Size

Our second problem was one of horizon size. Inflation comes to the rescue here by producing a time in which everything was moving much faster than it is now. If we move time backward we reach a point when everything comes together very quickly. This means that before inflation everything we can see in the sky, including the LSS, was close together. For the universe to thermally equilibrate a photon needs enough time to move from one side to the other. If we assume that inflation occurred at $10^{-34}$ seconds then the size of the visible universe would have to be $(3 \cdot 10^8 \text{ m/s}) \cdot (10^{-34} \text{s}) = 3 \cdot 10^{-26} \text{m}$. Inflation means that the Hubble radius, the farthest distance from which information can currently reach us, is smaller than the
horizon, the farthest distance from which information has ever been able to reach us. Before inflation the tiny volume which the visible universe then occupied could reach thermal equilibrium. When everything was expanded all of the universe would be the same temperature. This explains why the CMB is of nearly uniform temperature across the sky.

1.3.3 Flatness

When we described inflation terms like “60 e-folds” were bandied about. We might wonder where such a number comes from. It comes from the solution of our last problem. Let us again look at our model 2D universe. If we were to start with a universe on a sphere, inflation would mean increasing the radius of the sphere. We cannot look at the entire universe; we are only aware of the current horizon, the patch that we can see. This is like looking at a little patch on the surface of the sphere. The catch is that the patch we can see is a circle with a radius corresponding to our horizon. As the radius of the sphere gets bigger it becomes more and more difficult to detect the curvature of the patch we can see. At some point the patch we can see looks flat and may as well be flat for all intents and purposes. The same reasoning with the saddle universe leads to the same conclusion, which is that a sufficiently large re-scaling will produce a universe that is flat. The factor of 60 e-folds is the minimum amount of re-scaling needed to produce a geometry that is as close to flat as ours is. We are still left with the questions of why and how inflation occurred, but those answers are for another thesis.
1.4 Acoustic Oscillations

The quantum mechanical fluctuations that were present at the beginning of the universe are expanded during inflation. This means that they are stretched out to large scales and also their amplitude is magnified (i.e. the amplitude of the fluctuations grows as inflation progresses.). That means that an over-density of matter at the quantum mechanical level gets stretched to a very large size and the amplitude of the over-density increases. One is then left with patches of matter which cause the surrounding matter to fall into the gravitational well. In the early universe, before last scattering, this causes acoustic oscillations. During this time matter and radiation are tightly coupled; there are strong and frequent interactions between photons and baryonic matter. As a result there is energy transfer between the two and they thermally equilibrate. Before any oscillation happens the power spectrum of the density perturbations produced by inflation is scale-invariant, but as length scales enter the horizon oscillations start to occur on those scales. These oscillations are produced in the following way.

When matter falls into a gravitational well it heats up, exchanging gravitational potential energy for kinetic energy. That kinetic energy is then transferred to photons that are present. These photons are not bound to the gravitational well and will try to stream out, only to be held back by their coupling to the in-falling ionized matter. The increase in photon energy translates into a photon pressure on the matter that is in-falling. As the in-fall continues the temperature rises and the photon pressure increases until it is strong enough to push the matter back out of the potential well. Everything cools off and the photon pressure decreases to the point that it can no
longer hold the matter out of the potential well. The matter then starts to in-fall again and the process starts all over. These are acoustic oscillations. The frequency of these oscillations depends on the size of the local matter over-density. The over-densities come from the quantum mechanical fluctuations at the time of inflation. Their sizes correspond to an angular size on the sky, just like the size of the horizon at last scattering corresponds to an angular size on the sky. These oscillations are not free to evolve as you might think. For an oscillation to take place the different parts of the wave must be able to “talk” to one another. We will discuss how the evolution of these oscillations can be used to glean information about the universe a little later, but in Figure 1.1 we can see how the oscillations evolve over time. We can see that the oscillation labeled ‘Second Peak’ has gone through about 3/4 of a period. Oscillations at smaller scales will have gone through more periods while the ‘Super-Horizon’ oscillation has barely changed at all.

Also, the presence of Dark Matter will affect the oscillations. It will fall into the gravity well due to the over densities, but it does not interact electromagnetically so Dark Matter feels no photon pressure. Without the photon pressure to hold it back and then push it back out the Dark Matter just falls in. This creates an offset in the forces that cause the acoustic oscillations. The Dark Matter attracts the baryonic matter and serves as a force constantly pulling the matter back into the gravity well.

1.5 What the CMB tells us

What can the CMB tell us? To answer that, we need to know what we can measure from the CMB. A light wave has two properties: energy and polarization. The energy of a photon is the easiest to measure. By comparing the spectrum of
photons from different parts of the sky we can measure the difference in the CMB temperature, $\Delta T_{\text{CMB}}$. Polarization can also be measured, but it is a significantly more difficult enterprise. So let’s start with the temperature of the CMB. We say temperature because we are dealing with a spectrum of energies when we look at the CMB.

### 1.5.1 Temperature Differences

The temperature of the CMB is determined by measuring its spectrum, which is a black body spectrum. The CMB is in fact the most ideal black body spectrum ever measured. An ideal black body spectrum assumes that the emitting body is in prefect thermal equilibrium with all incident radiation. As we mentioned earlier, the conditions just prior to last scattering are that the matter in the visible universe was
in good thermal contact with the radiation. Such good coupling over such a large volume has not been seen since and we should expect that the CMB will have a good black body spectrum.

The differences in temperature that we will be talking about are very small, 1 part in 100,000. For example, the average temperature of the CMB is $2.725 \pm 0.001$ K.[6] The temperature fluctuations that are produced are at the tens of $\mu$K level. And those are the maximal temperature variations. We have talked about how the CMB should all be roughly the same temperature, but it should have some differences which correspond to what has become galaxies and clusters of galaxies today. The dense, hot places in the early plasma have higher energy photons coming from them, but they are necessarily in a gravitational well, which means that they become red-shifted as they climb out of that well. The relative contributions from the higher temperature and red shifting work out such that the red shifting is stronger than the extra heating from the in-falling matter. Thus, photons coming from a hot spot at the time of last scattering have a lower energy than the average. This is known as the Sachs-Wolfe effect.[7] Conversely photons coming from a region of under density are cooler, but are blue-shifted to a sufficiently high temperature as they move out. That is, photons coming from a low density region of the LSS display a spectrum with a higher than average temperature. Detecting these hot and cold spots tells us exactly what the density profile of the plasma was across the surface of last scattering at the time of last scattering. That in turn tells us a lot about the density of the universe.

As mentioned earlier there were acoustic oscillations going on in the early plasma of the Universe. The time it takes for an oscillation to occur depends on the wavelength
of the oscillation. Oscillations on small scales will experience damping from photon
diffusion, referred to as Silk Damping [1], which decreases the amplitude as time goes
on. Oscillations whose size is larger than the mean free path of photons in the plasma
do not suffer from photon diffusion and do not suffer as much damping.

Wavelengths of a size such that the oscillation only has time to collapse once
between the end of inflation and last scattering will reach a maximum compression;
let us call $\lambda_m$ the wavelength of maximal compression. Oscillations with longer wave-
lengths will not have collapsed all the way and will not produce as strong a difference
in temperature as the maximally compressed wave. Waves with wavelengths longer
than the horizon size will not oscillate at all because communication is not possible at
that scale. So if we look at a large patch of the sky, and by large we mean large enough
that the distance on the LSS is larger than the horizon size at that time, and measure
its average temperature and then do the same for another patch we should find that
they are very close to the same temperature. The temperature differences generated
by inflation remain relatively small. Wavelengths which have had time to collapse to
a maximum compression will produce larger temperature differences, but at angular
scales greater than 1 degree wavelengths have not entered the horizon and have not
had time to oscillate.[1] On angular scales larger than a degree temperature differences
come solely from inflation’s quantum fluctuations. If we look at scales that are smaller
than the horizon size at last scattering we should see that some are hotter and some
are cooler because acoustic oscillations have had time to collapse etc. How drastic
the difference those temperatures are from the average CMB temperature depends on
how close the wavelength is to $\lambda_m$. We can calculate half a period of oscillation for
\( \lambda_m \) then we have found the time between the end of inflation and last scattering.

### 1.5.2 Power Spectrum

To understand what else the CMB temperature differences, or anisotropy, can tell us about the universe we need to understand what the angular power spectrum is. We said that the time for an oscillation scales with the wavelength of the oscillation. So if there were no Silk Damping we would expect to see large temperature differences at an angular scale corresponding to a single collapse after inflation and scales that are harmonics of it. Here we will only refer to angular scales and it is implied that these angular scales refer to the physical scales at the time of last scattering. What we do is compute a two point correlation function, \( \langle \Delta T^2 \rangle \), of the temperature anisotropy for the entire sky.[1]

\[
\langle \Delta T^2 \rangle = \frac{1}{4\pi} \sum_l (2l + 1)C_l F_l
\]

Here \( l \) denotes a multipole moment, \( C_l \) is the amplitude of a given multipole and \( F_l \) a window function which describes the beam of an experiment or the processing of the \( l \)-modes during detection.

At an angular separation corresponding to the single collapse and its harmonics we should see maxima, meaning that at that angular separation the two points have positively correlated temperatures, both hot or both cold. At the half harmonics we should see minima, meaning negative correlation with one point hot and one cold. If we plot the amount of power we see on the y-axis and the angular separation on the x-axis we get what is called a power spectrum. For ease of plotting it is more common to plot the inverse of the angular separation or the spherical harmonic \( l \)-modes on the x-axis. For a clear discussion of \( l \)-modes and spherical harmonics David Griffiths' book...
Figure 1.2: WMAP 3 year power spectrum. Notice how the fundamental peak is significantly larger than the two harmonics.[2] This is due to Silk Damping.[1]

is recommended.[8] If there were no Silk Damping we would expect to see an oscillating power spectrum. If we were to plot in $l$ space and have a logarithmic scale it would be a sinusoid and all the peaks would be the same height. If we have photon diffusion then we expect the peaks of the harmonics (high $l$) to be lower than the peak of the fundamental (lowest $l$ peak) because the diffusion bleeds energy from the oscillations with short wavelengths. This effect can be seen in the power spectrum measured by the NASA’s Wilkinson Microwave Anisotropy Probe (WMAP) team. In Figure 1.2 the first and second peaks are clearly visible and the third strongly suggested.

Dark Matter’s affect on the oscillations will also show up in the power spectrum. The Dark Matter attracts the baryonic matter gravitationally and serves as a force
constantly pulling the matter back into the gravity well. This means that the baryonic matter will compact farther and will not expand as far as it would otherwise. The odd-numbered peaks in the power spectrum are from the compression phases of the oscillations and even-numbered peaks are from the rarefactions. We can think of the power spectrum as the square of a density power spectrum, where the density is the offset from the average density of the Universe. Rarefactions then have a negative density. With Dark Matter present the rarefactions have a smaller negative amplitude and therefore a smaller square, while the compressions have a larger amplitude and square. If there is Dark Matter present then we expect that odd numbered peaks, 1,3,5 etc., will have an enhanced amplitude while even peaks have a smaller amplitude. The ratio of those amplitudes, accounting for Silk Damping, tells us the ratio of Dark Matter to baryonic matter in the Universe. If there is very little Dark Matter then the ratio of odd to even peak amplitude should be close to 1. If there is a lot of Dark Matter then the ratio should be significantly less than 1.

We can also learn about the geometry of the Universe. $\lambda_m$ corresponds to a particular physical size, but the angular size on the sky depends on the geometry of the Universe. Open and closed geometries will produce smaller and larger angular scales than a flat geometry, respectively. This will shift the $l$ in the power spectrum at which the first peak appears. WMAP produced full sky temperature maps and computed a power spectrum for the temperature anisotropy, Figure 1.2. The temperature power spectrum along with a temperature and polarization cross-power spectrum, which we will talk about shortly, WMAP was able to constrain many cosmological parameters of interest to high precision. Figure 1.3 shows the best fit parameters for an inflationary
Figure 1.3: WMAP’s 3 year parameters. $\Omega_b h^2$ and $\Omega_m h^2$ are baryon density and matter density respectively. $h$ is a unitless parametrization of $H_0$. $n_s$ is the spectral index, the slope of the scalar perturbation spectrum. $dn_s/d\ln k$ is the running of the spectral index or how it changes with wave number. $r$ is the tensor to scalar ratio, the relative power in the fluctuations. $\tau$ is the optical depth to reionization. $\sigma_8$ is the rms mass fluctuation amplitude in spheres of size $8h^{-1}$ Mpc, and measures the normalization of the matter power spectrum.[2]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Lambda$CDM + Tensor</th>
<th>$\Lambda$CDM + Running + Tensors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$</td>
<td>$0.0233 \pm 0.0010$</td>
<td>$0.0219 \pm 0.0012$</td>
</tr>
<tr>
<td>$\Omega_m h^2$</td>
<td>$0.1195^{+0.0004}_{-0.0003}$</td>
<td>$0.128 \pm 0.011$</td>
</tr>
<tr>
<td>$h$</td>
<td>$0.787 \pm 0.052$</td>
<td>$0.731 \pm 0.055$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$0.984^{+0.029}_{-0.028}$</td>
<td>$1.16 \pm 0.10$</td>
</tr>
<tr>
<td>$dn_s/d\ln k$</td>
<td>set to 0</td>
<td>$-0.085 \pm 0.043$</td>
</tr>
<tr>
<td>$r$</td>
<td>$&lt; 0.65$ (95% CL)</td>
<td>$&lt; 1.1$ (95% CL)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.090 \pm 0.031$</td>
<td>$0.108^{+0.034}_{-0.033}$</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>$0.702 \pm 0.062$</td>
<td>$0.712 \pm 0.056$</td>
</tr>
</tbody>
</table>

model using only WMAP data. For an exhaustive discussion of WMAP’s parameter estimation please see [2].

1.5.3 Polarization

1.5.3.1 E-Modes

The other property of light that can be measured is the polarization. To have polarized light means that there is some kind of asymmetry in the system. The environment in which it is emitted cannot be homogeneous; otherwise there is no mechanism
to select an angle of polarization. The early plasma was very homogeneous with small density perturbations, $\Phi$, which led to acoustic oscillations and also to polarization. It is difficult to polarize light in free space. We can only produce polarized light if the electric fields scattering off an electron in free space have a quadrupolar distribution.\[9\]

The question then becomes how density perturbations lead to quadrupole in the electric field. The answer is that as an object falls into a gravity well along a geodesic its relation to objects around it changes. In the reference frame of infalling matter, emission from above and below is red-shifted, while emission from the same depth is blue-shifted, thus creating a quadrupole. This red-shifting and blue-shifting is a small effect and therefore the amount of polarization is small. In temperature units, the polarization is at the level of about one tenth of the quadrupole component of the CMB temperature anisotropy as seen at that point on the sky. This polarization mechanism produces polarization that either points towards an over-density or 90 degrees to an under-density. If we were to make a map of the polarization produced by the density perturbations we would find that there was only a gradient component to it. This type of polarization pattern is named E-mode for its similarity to the electric field. As mentioned above, E-mode polarization anisotropies carry roughly one tenth the amplitude of the temperature anisotropies and are thus more difficult to detect.

The Degree Angular Scale Interferometer (DASI) experiment[10], located at the South Pole, made the first direct detection of the E-mode power spectrum and has since been joined by the CBI[11], WMAP[12], BOOMERanG[13] and CAPMAP[14] experiments. The WMAP 1st year team made a TE cross-power spectrum; a two point correlation between an E-mode map and a temperature map. There is significantly
more power in such a map than in an EE power spectrum alone, making it easier to detect. We will explore these experiments and detections in the next chapter.

1.5.3.2 B-Modes

There is also a polarization pattern similar to the magnetic field, having only curl components and no gradient; it is called B-mode. B-mode polarization can be produced in the CMB, but the process involves some exotic things, which is all the more reason to be interested in it. As a gravitational wave passes through space it distorts it by stretching one direction perpendicular to its direction of travel and compressing the other. Also the peak and trough stretch and compress in different directions. The nice thing is that as the peak moves past a point there is still a direction that is stretched. If we were to draw a circle on flat space that circle would
Figure 1.5: Shown is a schematic of a gravitational wave passing through a sphere corresponding to the LSS. An observer sits at the center of the sphere and the orientation of the space-time distortions are shown as ovals. be distorted into an ellipse when the peak of a gravitational wave reached it. As the relative position of the ellipse moves from peak to trough the orientation of the major axis will rotate through 90 degrees. The ellipse continues to rotate until it is again oriented as it was when the peak first came by and now we have reached the 2nd peak. If we were to allow a planar gravitational wave to pass through the spherical surface of last scattering then we would see that these orientations would show up and our “ellipses” would seem to rotate in one direction as we moved around the surface. This stretching and contracting of space-time illuminates an electron at the LSS with
red-shifted light oriented along the long axis and a blue shift oriented along the short axis. As a result the scattered radiation is polarized with an orientation that rotates as a function of the position on the sphere, producing a B-mode pattern. These are necessarily very long $\lambda$ gravitational waves and there is nothing in the universe today that could produce such a wave as the energy involved would be unimaginable. Luckily there was something that could produce such waves at early times: inflation. If we assume a quantized theory of gravity then gravitational waves were popping in and out of existence at the quantum level during inflation. When inflation stretched and magnified everything it transformed these quantum fluctuations into horizon-spanning gravitational waves. That would mean that there were gravitational waves spanning the last scattering surface during decoupling. These gravitational waves would leave B-mode imprints on the entire sky. The amount of power in the BB power spectrum is unknown, but the maximum possible would be $1/10$ of the EE power spectrum or $1/100$ of the TT power spectrum. That would correspond to a polarization power spectrum at the level of one part in $10^7$ of the CMB temperature, or $\sim 0.1\mu$K.

The most interesting part is that gravitational waves are the only way to produce B-mode polarization at large angular scales. As a result B-mode polarization has become a holy grail of sorts in cosmology. Detection of B-modes at low $l$-modes would provide strong proof for inflation and Quantum Gravity. The other mechanism for producing B-modes is gravitational lensing. Gravitational lensing is a process which mixes E- and B-mode polarization into each other. At small angular scales, corresponding to the size of galactic clusters, an excess of B-mode polarization points to E-modes being converted into B by lensing. It would also allow us to conduct
1.5.4 Link to Inflation

The energy level at which inflation occurs depends on the potential of the inflaton field. We won’t go into the field theory of how this works and merely state that the inflaton field rolls down from a high potential to a lower one assumed to be the ground state. This change in the potential energy of the field is what fuels inflation. The potential difference between these two states tells us a lot about the underlying physics of the universe. The amplitude of scalar (matter) and tensor (gravitational) perturbations is tied to the initial potential of the inflaton. A larger initial potential delivers more energy during inflation. This energy produces larger amplitudes in the scaler and tensor perturbations. Together the tensor and scalar perturbations provide a link back to inflationary parameters.

We will leave a detailed discussion of inflation to Dodelson [1] and Padmanabhan [15]. We will content ourselves with an outline of slow roll parameters. The following reasoning can be found in Padmanabhan. Theorists have shown that the inflaton must roll down its potential slowly for inflation work properly. We therefore define parameters to quantify the rate of the roll, $\epsilon$ and $\eta$. They are defined in the following way

$$\epsilon = \frac{m_p^2}{16\pi} \left( \frac{V'}{V} \right)^2, \quad \eta = \frac{m_p^2}{8\pi} \left( \frac{V''}{V} \right).$$

(1.10)

$m_p$ is the Planck mass. To link CMB measurements back to these parameters we need to know how the magnitudes of the tensor and scalar perturbations and tilt $(d/d\ln k)$ relate to $V$ and $m_p$. Again we will quote these relations without working through the
\[ \Delta^2_T \sim \left( \frac{V}{m_p^4} \right), \quad (1.11) \]
\[ \Delta^2_S \sim \left( \frac{V^3}{m_p^6 V'^2} \right), \quad (1.12) \]
\[ \frac{d}{d \ln k} = -\frac{m_p^2 V'}{8\pi V} \frac{d}{d\phi}, \quad (1.13) \]

where the subscript \(T\) denotes tensors and \(S\) scalars. What we find here is that
\[ (\Delta_T/\Delta_S)^2 \simeq m_p^2 (V'/V)^2 \simeq 16\pi \epsilon. \] So the tensor to scalar ratio, \(r\), provides a link back to \(\epsilon\). (A more precise factor is 12.4\(\epsilon\).)

The next link is made using the tilt of the scalar perturbations, \(-d \ln \Delta^2_S / d \log k = (1 - n)\). Padmanabhan evaluates this to \((1 - n) = 6\epsilon - 2\eta\). Making use of \(|\epsilon| \approx |\eta|\) we find \((1 - n) \approx 4\epsilon\). This combined with \((\Delta_T/\Delta_S)^2 \approx 12\epsilon\) yields \((\Delta_T/\Delta_S)^2 \approx \mathcal{O}(3)(1 - n)\), relating three observable quantities. This serves as a check on proposed inflationary models.
Chapter 2

Current Measurements of the CMB

2.1 What and Who

The CMB provides significant opportunities and challenges for those who wish to glean information from it. These challenges come from the small amplitudes of the signals involved. The average temperature of the CMB is 2.73 Kelvin. On top of that average temperature are variations that are tens of micro Kelvin. The majority of the signal from the CMB is unpolarized, but there is a slight imbalance between the amounts of average power in each polarization direction. This results in a polarized signal on top of the unpolarized signal which has an amplitude $10^6$ times smaller than the unpolarized signal. These measurements are analogous to measuring the depth of the ocean and from those measurements trying to tell the height of the waves on the surface. Clever researchers have come up with many ways to get around this problem, and all of them rely on some sort of differencing measurement. The WMAP experiment is a giant in the field, having made very accurate temperature measurements and polarization measurements[2][12]. The Degree Angular Scale Interferometer (DASI)[16] experimenters made a name for themselves by being one of the few experiments to
not be scooped by the WMAP team. DASI was the first experiment to detect the polarization signal in the CMB. WMAP’s first year results also detected the polarization signal of the CMB and those two measurements have since been confirmed by the teams working on the Cosmic Background Imager (CBI), BOOMERanG and Cosmic Anisotropy Polarization Mapper (CAPMAP) experiments. They use many different approaches to measuring the CMB, but there is at least one thing that all the CMB experiments have in common: they all use differencing measurements.

In this chapter we will explore some of the equipment and techniques used by each of these experiments as well as a few others of note.

2.2 Detectors vs. Receivers

There are two main types of instrument for recording CMB signals. Coherent receivers, which maintain the phase information of a light wave, and detectors, which do not and are also known as incoherent detectors.

2.2.0.1 HEMTs

Coherent receivers detect the amplitude and phase of the incoming signal. For this reason they are often used in interferometric systems so that individual signals can be interfered or correlated. Most of the coherent receivers used in CMB observations use High Electron Mobility Transistors or HEMTs. To faithfully capture the phase information for the CMB HEMTs must be very fast. This need for speed is the main drawback of HEMTs; their sensitivity decreases at \( \sim 100 \) GHz or above.
2.2.0.2 Bolometers

Bolometer comes from the Greek βoλη meaning 'beam of light' and μετρον meaning measure. Literally, something that measures a beam of light. A bolometric measurement has come to mean a measurement of the total power from a signal and, as their name suggests, bolometers make this kind of measurement. The temperature of a bolometer changes based on the total power incident on the detector and all information about phase, frequency or spectrum is lost. A bolometer is usually made from a semiconductor or superconductor and consists of three parts.

The first part is the absorber. The majority of the bolometer’s mass is made up of the absorber and true to its name it absorbs the incoming light. A reasonable match between the size of the absorber and the wavelength of the incoming light is required for good absorption. As incident radiation deposits power on the absorber it warms up. The change in temperature is then detected by the thermometer.

The thermometer makes up the second part of the bolometer. Though we have described it as a separate piece, the thermometer need not be a separate component from the absorber. A simple bolometer may have wires connected to the absorber to measure its resistance, and therefore its temperature. In this case absorber plays dual role. Some bolometers have more sophisticated thermometry, such as Transition Edge Sensors (TESs) that are very sensitive to small temperature changes. Regardless how it is accomplished the temperature must be measured.

The last essential part of a bolometer is a thermal link to a cold reservoir. This is needed to drain heat from the absorber. Without the link the absorber will continue to heat up ad infinitum. Too strong of a link and the bolometer will not heat up at all.
This means that the relation between the weak thermal link to the reservoir and heat capacity of the absorber must be tuned based on the expected power from the incident radiation and desired speed of the measurement. Once the heat capacity and thermal conductivity are known one can calculate the power the bolometer is receiving based on the temperature.

In principle bolometers can work at any temperature, however they can be exquisitely sensitive if cooled to cryogenic temperatures and so cryogenic bolometers are desirable for CMB observations where detector sensitivity is important.

Incoherent detectors measure the power ($\propto A^2$) of the incoming signal, so all information about the phase is lost upon detection. To recover phase information experimenters must rely on physical spacing between detectors and/or signal modulation of some sort. The benefit provided by bolometers is that they can allow for high frequency, broad band detections and high sensitivity.

### 2.3 Optics

#### 2.3.1 Reflecting and Non-Reflecting

Reflecting optics allow for a large collecting area while using a small amount of material, but introduce systematic effects to polarization signals. When unpolarized light reflects off a surface some power is lost in the polarization basis normal to the plane of reflection. This introduces polarized artifacts into measurements. These types of artifacts can be avoided by using non-reflecting optics.

Non-reflecting optics, such as feed horns, couple waves in free space directly into wave guides with little selective loss of polarization power. However, non-reflecting
optics require substantially more material to achieve the same collecting area.

2.3.2 Interferometry

Astronomical observations can be arbitrarily divided many different ways. We will make the distinction between the following two groups: Single-dish and Interferometric observations. Single dish observations deal with the image or data from a single telescope, while interferometers correlate, or interfere, the signals from two or more telescopes.

Most interferometers can be described as wavefront-splitting or amplitude-splitting. [17] Interferometers used in radio astronomy, like the VLA, are examples of wavefront-splitting, where the individual telescopes, or elements, each observe a different part of a single wavefront. Perhaps the best well known example of amplitude-splitting interferometry is the Michelson-Morely experiment, which signaled the end of the Aether theory.

Interferometers are commonly used in radio astronomy and most often they are paired with receivers that result in a multiplying interferometer.[18] In such an arrangement the electric fields from two elements are multiplied together. If $E_1$ is the electric field from one element and $E_2$ the field from the other element, then the resulting output is $E_1^* E_2 + E_1 E_2^*$. This output is called the visibility. We will talk more about visibilities shortly, for now it will suffice to say that they are the quantities of interest when looking at the data from an interferometer.

An interferometer can also be constructed so that the two electric fields are added and then squared (an “adding” interferometer). This produces an output that
Figure 2.1: Notice that the modulation shows up in the phase term and not in the total power term.

looks like:

$$(E_1 + E_2)(E_1^* + E_2^*) = |E_1|^2 + |E_2|^2 + E_1E_2^* + E_1^*E_2$$

(2.1)

We can see that the first two terms constitute a total power measurement and large offset, while the second two terms are the visibility we described above. This, of course, raises the question of how we can separate the total power terms from the visibility. As figure 2.1 shows, in an adding interferometer the introduction of a modulation into the phase of the signal from one element produces a modulation in the visibility terms, but not in the total power terms. This would allow for the separation of the visibility from the total power using a lock-in amplifier. This can be more clearly seen if we rewrite the electric fields as:

$E_1 = E_1 e^{i\phi_1 + \Delta \phi}$

(2.2)

$E_1 = E_2 e^{i\phi_2}$

(2.3)
Here $\phi_1$ and $\phi_2$ are the phases of the electric fields at the two elements and $\Delta \phi$ is as phase difference introduced to the first element. Now rewriting the output we find:

$$|E_1|^2 + |E_2|^2 + E_1 E_2 e^{i[\phi_1 - \phi_2] + \Delta \phi} + E_1 E_2 e^{-i[\phi_1 - \phi_2] + \Delta \phi}$$

As $\Delta \phi$ is modulated the visibility terms will also be modulated, while the total power terms will be unaffected.

It can be easily shown [19] that the Fourier transform of an image on the sky is the visibility that a baseline measures. This is because the physical result of passing plane waves through an aperture is a Fourier transform. This also means that the units in which the image is mapped on the sky are transformed. They move from $\theta$ and $\phi$ (or $x$ and $y$ for a flat sky approximation) to $u$ and $v$. In the same way that an image has a certain value at each point in $\theta$ and $\phi$ on the sky, that image has a corresponding value for the visibility at each point in $u$ and $v$ on the $u - v$ plane.

The position of a point on the $u - v$ plane is determined by the baseline, $\vec{B}$, i.e. the separation of the two elements, and the orientation of baseline w.r.t. the sky. We have glossed over the mathematical gymnastics required to get from describing the image on the sky in relative coordinates, to absolute coordinates and from there to the $u - v$ plane, however the $u - v$ plane is a result of these machinations and we will discuss it unencumbered by the intervening steps. Malu provides a good example of these missing steps.[19] Let us say that

$$\mathbf{u} \equiv u\hat{\mathbf{u}} + v\hat{\mathbf{v}}$$

What we find is that

$$|\mathbf{u}| = \frac{B}{\lambda}.$$
So a single baseline provides a measurement of a single point in the $u - v$ plane at a distance $\frac{B}{\lambda}$ from the center. If we are observing directly over head, then rotating the baseline around the center point will trace out a circle in the $u - v$ plane. From these points in the $u - v$ plane a map can be reconstructed. If a higher fidelity map is desired then more points in the $u - v$ plane must be recovered.

The physical distance from each telescope, or observing element, to an astronomical source is fixed for a relative position on the sky. This physical difference translates into a phase difference for each point on the sky.

Let us take a point on the sky which has a $2n\pi$ phase difference between the two elements. The electric fields will constructively interfere, producing a strong response. Moving a little away from this point we may find another point where the phase difference is $(2n + 1)\pi$. The electric fields will interfere destructively, producing a null in the response. These effects continue across the FOV of the interferometer, creating a fringe in the interferometer’s response to the sky. Most often this is exploited to increase the angular resolution of a measurement beyond what a single dish could achieve.

Therefore, the beam response of an interferometer produces a better angular resolution than a single dish of the same size. For a filled dish system to have comparable resolution the dish diameter, $d$, must equal the separation between the interferometer elements, $B$. If one is building a system to look at small angular scales it becomes much more cost effective to build an interferometer than a filled dish system. Physically, the basic element of an interferometer is an antenna. Two antennas define a baseline, $B$, in an interferometer. Together the antenna properties and baseline properties define
the quantities of interest for a given interferometer.

The fringe in the FOV of an interferometer makes a baseline sensitive to a particular $l$-mode of the sky’s spherical harmonics. (More precisely, to a small range of $l$-modes based on the band width of the measurement, but we will leave out that small complexity.) This feature explains two things. First, it is another reason why interferometry is well suited for CMB observations with their focus on power spectrum analysis. It also explains why you need measurements from many baselines to reconstruct an image of the sky: the true image is the sum of all $l$-modes and each baseline is only sensitive to one of them.

2.3.2.1 N Inputs, N(N-1)/2 Outputs

In this type of interferometer the signal from each input is split into multiple optical paths and individually combined. For example if we had $N$ inputs each input would be split $N-1$ ways and then interfered with the signals from the other $N-1$ inputs, one at a time. This makes very clear which signals are from which baselines. This approach, however, becomes cumbersome with large $N$. $N(N-1)$ optical paths are needed to recover information from the $\frac{N(N-1)}{2}$ detectors, one for each baseline produced. This $N^2$ scaling quickly reduces the signal to noise ratio even for modest numbers of inputs because the signal level is reduced by $\sim \frac{1}{N^2}$. The noise level from the system remains the same. This is not as bad as it may seem. If photon noise dominates then we can say that it scales like

$$\Delta n = \sqrt{n^2 + n}. \quad (2.6)$$
Here $n$ is the photon occupation number,

$$n \equiv \frac{\eta}{e^{\hbar \nu / kT} - 1}$$

and $\eta$ is the optical efficiency. In the limit of $n \ll 1$ (shot-noise limit), $\Delta n = \sqrt{n}$. For the CMB, however, $n \gg 1$ so $\Delta n \sim n$. In this case the noise on each of the $N(N-1)/2$ detectors decreases as $\sim N^2$. So the signal to noise ratio is not affected by dividing the signal over many detectors.

These concerns can be sidestepped by recording the amplitude and phase of each element and then correlating the digitally. We would now have HEMT receivers and a correlator like the VLA’s. While this have been very successful for the VLA it does not address the challenges that are particular to the MBI so we will not address this approach.

### 2.3.2.2 N Inputs, N Outputs

A Butler Combiner is a type of interferometer where a signal can be split fewer ways, thus increasing the S/N. A Butler Combiner is a way of interfering signals using guided wave structures such as wave guides or microstrips. $N$ inputs are split $N$ ways and recombined with set phase relations. $N$ outputs are produced each with a unique phase relation between the $N$ inputs. This means that a portion of the signal from input $i$ can be recovered from any output $j$. This method of combination was introduced by its namesake, Jesse Butler, and is also referred to as a Fourier combination, because the signals at the outputs can be thought of as Fourier Transforms of the inputs.\cite{20}\cite{21}
2.3.2.3 Stellar Michelson Interferometer

The MBI uses a Stellar Michelson Interferometer [22], which is an optical analogue to the Butler. The previous two interferometer types require waveguides to direct the fields together for interference. In a Stellar Michelson approach the waves are collected via sky horns or mirrors and then re-emitted into a telescope optical system and allowed to interfere with each other on a focal plane. The MBI also conditions the signal using phase modulators before re-emitting. This greatly reduces the amount of material required as there are no wave guides or microstrips needed. Those considerations are exchanged for two others. First, the focal plane must be populated densely enough with detectors to reconstruct the interference pattern produced by the interferometer elements. Second, some of the power which enters the sky horns doesn’t make it to the detectors. The interference pattern can be bigger than the focal plane. Also, unless the entire focal plane is populated with detectors, there will be spaces between detectors which produce no signal.

2.4 Past and Present Experiments

The small amplitude of CMB temperature and polarization anisotropies provide the experimentalist with numerous challenges including systematic effects from the atmosphere, astrophysical foregrounds, emission from the Earth and Sun etc. A variety of measurement techniques have overcome these challenges. Here we will describe a few. This will by no means be an exhaustive list.
2.4.1 WMAP

WMAP is a NASA funded satellite experiment. It orbits the Sun at the second Lagrange point and observes the entire sky every six months. WMAP uses differential radiometers; rather than “chopping” a single beam on the sky WMAP measures the difference in temperature between two points on the sky separated by 140 degrees. WMAP’s radiometers process the RF signal with a series of orthomode transducers, hybrid T’s, HEMT amplifiers and phase shifters. For each pair of horns the polarization components are separated, processed, and amplified. The signals recorded by a pair of detectors are a combination of a single polarization from both of the horns. Differencing the two detector signals then produces a result that is only proportional to the difference in polarization between the two horns. From these measurements the WMAP team can reconstruct the amplitude and orientation of the CMB’s polarized signal at each point on the sky. For a more complete discussion of how WMAP measures polarization anisotropy please refer to: [2][12]. With their 1st and 3rd year results release the WMAP team has provided a laundry list of cosmological parameters that have been constrained based on their data and chosen cosmological model. (Figure 1.2) The WMAP experiment was designed to most effectively measure the temperature anisotropy of the CMB between, 2 < l < 900, which it has done with unparalleled success. WMAP’s 5 year results provide some of the following as limits on popular cosmological parameters [23]: Baryonic matter density of $\Omega_b h^2 = 0.02273 \pm 0.00063$, the Dark Energy density at $\Omega_\Lambda = 0.742 \pm 0.030$, the scalar spectral index at $n_s = 0.963^{+0.014}_{-0.015}$, an optical depth of $\tau = 0.087 \pm 0.017$, $\sigma_8 = 0.796 \pm 0.036$ and a red-shift of instantaneous reionization of $z_{reion} = 11.0 \pm 1.4$ with 68% confidence.
2.4.2 DASI

The Degree Angular Scale Interferometer is a 13 element co-planar interferometer array that observed from the South Pole. It used HEMT amplifiers in the 30 GHz atmospheric window specifically from 26-36 GHz with the frequencies broken into ten, 1 GHz wide bands.[16] DASI uses right and left circular polarizers to separate polarizations. This is in contrast with the WMAP experiment which uses linear polarizers. While there is no fundamental difference between decomposing polarization into linear or circular components there are some significant differences in how systematic errors propagate and how difficult they are to separate from desired signals.[24] DASI focused on $140 < l < 900$. DASI found a $6.3 \sigma$ significant detection of the EE power spectrum and a $2.9 \sigma$ detection of the TE cross correlation power spectrum[25]. DASI’s data, like WMAP’s data, shows no evidence of a B-mode power spectrum and unfortunately DASI will not be able to detect the B-mode power spectrum because DASI can not accommodate the large detector array required.

2.4.3 CBI

The Cosmic Background Imager is another thirteen-element interferometer array located at an altitude of 5000 meters in the Atacama Desert in Northern Chile.[26] It used similar techniques to DASI and, like DASI, CBI uses HEMTs to amplify the signal from each antenna. The signal is then heterodyned from 26-36 GHz to 2-12 GHz, filtered into ten 1 GHz wide bands, down-converted further to 1-2 GHz and finally split into the number of baselines, 78. The signals from each pair of antennas in a band are correlated using a complex correlator. The entire system following the feed horns is cooled to reduce the system temperature. Like MBI, CBI has a three-axis
rotational mount with one axis rotating around the line of sight. This provides better
coverage of the $u - v$ plane and also helps to identify false signals which are generated
by the receiving electronics. These false signals will not be affected when CBI rotates
around the line of sight. CBI has recorded an $11.7 \sigma$ significance detection of the
EE power spectrum, a $4.2 \sigma$ significance detection of the TE power spectrum and no
evidence of B-mode power spectrum.[10] The CBI team has placed an upper limit on
the power of the B-mode spectrum of $3.8 \mu K^2$ with at 95% confidence. The CBI data
is completely consistent with a simple inflation-based cosmological model.

2.4.4 BOOMERanG

The first measurement of the TE cross-correlation using bolometric detectors
was made by the BOOMERanG experiment in 2003. Using four pairs of polarization
sensitive bolometric detectors at 145 GHz, too high a frequency for HEMTs, and 200
hours of observing time over 22% of the sky from a balloon gondola the BOOMRanG
experiment made a $> 3.5\sigma$ significance detection of the TE cross correlation in the
$l$ range of $50 < l < 950$.[27] Boomerang also measured a $4.8\sigma$ significance detection
of the EE power spectrum in the $100 < l < 1000$ range. The team also placed
upper limits on the B-mode power spectrum, $8.6\mu K^2$ with $2\sigma$, and on the EB cross
correlation, $7.0\mu K^2$ with $2\sigma$, which is expected to be $0\mu K^2$.[13]

2.4.5 CAPMAP

The Cosmic Anisotropy Polarization Mapper (CAPMAP) is located in Craw-
ford Hill, New Jersey and observers the sky with four antennas, each with its own
polarimeter attached to measure the polarization in a single orientation. The individ-
ual signals are then correlated to produce a measurement of the linear polarization. All the antennas operated in the 90 GHz atmospheric window and with 433 hours of live time on a 2°2 patch of the North Celestial Pole the CAPMAP team has placed limits on the E-mode and B-mode power spectra. Within the \( l = 940^{+330}_{-300} \) the EE power spectrum is measured at \( 66^{+53}_{-39} \mu K^2 \). They do not make a positive detection of the B-mode power spectrum, but limit its power to a maximum of \( 38 \mu K^2 \) in the \( l = 1050^{+590}_{-520} \) range with 95% confidence.

### 2.5 Future Experiments

#### 2.5.1 Planck

Planck, the European Space Agency’s answer to WMAP, is a space-based imaging experiment. Designed around a single optical system, instead of WMAP’s back-to-back differencing design, Planck will orbit at the second Lagrange Point. Planck has a 1.5m off-axis parabolic primary and an ellipsoidal secondary. Its two instruments are the High Frequency Instrument \([28]\) (HFI) and the Low Frequency Instrument \([29]\) (LFI). The HFI, which spans a frequency range from 100 to 857 GHz, employs arrays of both standard and polarization sensitive bolometers as detectors. The LFI, which covers 30 to 70 GHz, employs small arrays of standard HEMT radio receivers. Planck will map out individual pixels of the sky, covering it in its entirety every 6 months.

#### 2.5.2 QUIET

The Q and U Imaging ExperimenT (QUIET) was originally a collaboration between the CAPMAP and CBI experimental teams. More collaborators have joined
to help produce this imaging experiment. Phase I of QUIET will use a 1m side-fed Cassegrain telescope with two arrays of densely packed HEMT receivers produced by JPL. The HEMT arrays are sensitive to 40 and 90 GHz. This optical system will be located at the CBI observing site and will use CBI’s mount. Phase II of QUIET will have two more arrays for both 40 and 90 GHz and each of those arrays will have an order of magnitude more receivers in the array. Phase II will again use CBI’s mount but will have three 2m optical systems installed as well.

2.5.3 EBEX

The E and B EXperiment is an upcoming imaging balloon borne experiment which utilizes a 1.5m Dragone-type telescope. EBEX will use a magnetically levitated rotating achromatic half-wave plate to modulate the polarized signal and then lock-in to it. The telescope is expected to provide 8 arc minute resolution over four focal planes each with a 4 degree diffraction-limited field of view at frequencies up to 450 GHz. EBEX will use a focal plane of bolometers readout with TESs and frequency-multiplexed SQUID arrays.[30]

2.5.4 Spider

Spider is a planned balloon-borne imaging experiment. Its name comes from the original design which called for eight optical systems which were suspended from the gondola by a long cable. The current design only requires six optical systems, all cooled to 4K by a common LN/LHe cryostat.[31] Each of the six optical systems observes at a different frequency with all six spanning 80 to 275 GHz. Spider is designed to make a 30 day mid-latitude flight, based from Australia, which will map
50% of the sky each night of observing. It should greatly expand the characterization of the interstellar dust foreground and reduce the viable parameter space for $n_s$, $\tau$, and $r$ by over a factor of 50.

2.5.5 Polarbear

Polar Bear is a ground-based imaging telescope now under construction. It would utilize a large off-axis primary and secondary design with lock-in detection of the polarization signal achieved with the use of a rotating half-wave plate. Polarbear will use the same back end electronics for bolometer readout as EBEX.

2.5.6 $C_\ell$OVER

$C_\ell$OVER is an imaging experiment which uses TES bolometer detectors. Clover has three frequency bands: 90, 150 and 220 GHz. Each is defined by a separate telescope scaled to the right size for the frequency of interest. Each telescope has four optical imaging systems on the same mount with its own focal plane. Signals from corresponding pixels on the focal planes are incoherently added and then detected using a TES bolometer. The incoherent addition makes these telescopes imagers and not interferometers. The Clover team expects to be able to detect the B-mode signal from gravitational waves.[32]

2.6 MBI

The Millimeter-wave Bolometric Interferometer will receive a more in depth discussion. Details of the instrumentation will follow in chapter 3.

Bolometers do not retain phase information, but they are so very sensitive that
one would like to harness them for CMB observations. Interferometry’s natural sensi-
tivity to a small range in \( l \)-space also recommends it for CMB observations and their
focus on power spectra. This raises the question of how to use an incoherent detector
with an observation technique that depends on knowing both the amplitude and phase
of two electric fields. The MBI is one realization of an answer to this question.

The key to using bolometers in interferometry is to encode the phase information
in something other than the response of a single bolometer. The MBI does this by
using a non-standard beam combiner, a stellar Michelson interferometer approach.[22]
Michelson’s approach covers the aperture of a telescope and then allows light through
two portions of the aperture. The result produces a set of interference fringes on the
focal plane of the telescope. The MBI differs from Michelson’s experiment in that it
has four elements instead of two; the number of apertures can be extended without
limit. The complication which this produces will be addressed later. For now we will
discuss the two-element case. The interference fringes, produced by the relative phases
of the electric fields, hold the phase information which the bolometers lose. If there
is no phase difference between the fields entering the elements then there will be a
peak in the fringe located exactly between a projection of the two apertures onto the
focal plane. Any shift of the peak away from this position is due to a phase difference
between the elements. The position of the fringe will also tell you how much relative
phase has been accumulated. \( \pi \) radians of phase will move the maxima to the previous
location of the minima and vice versa.

The first level of complexity for the MBI comes from the background noise in
which the detection is taking place. Anything that will change the bolometer response
will mask these fringes to some extent. Therefore, we need a way to distinguish this
signal from temperature drifts in the cryogenics or signals from light that may have
leaked around our optics and made it to the focal plane via a different path.

To do this we modulate the relative phase of the elements. The simplest way to
do this is to place a phase modulator in one element and switch it between 0 and \( \pi \)
 radians of phase. At a bolometer located at a peak in the fringe, w.r.t. 0 phase, this
will produce modulations in the response that are, let’s say, in phase with the driver
signal for the phase modulator. A bolometer at a null will show a modulation that is
\( \pi \) radians out of phase. As a result, if we apply lock-in detection to each bolometer
signal, those located at peaks will have a large positive response, while those at nulls
will have a large negative response. In this manner we can recover the fringe on he
focal plane from one baseline.

For the MBI we have the more demanding task of separating the fringes from
6 baselines at the same time. We use the same general principle of relative phase
shifts, however now we modulate the phases with specially chosen Walsh functions so
that each pair of elements (each baseline) has a unique and orthogonal relative phase
pattern. By using a digital lock-in to select the signal from one baseline at a time we
can recover just the signal of interest. The particulars of how this is done are discussed
in Chapter 5.
Figure 2.2: Frequency and $l$ coverage for the experiments discussed
Chapter 3

Instrumentation

3.1 Introduction

MBI is a pathfinder for the technique of using bolometers for interferometry. We have chosen this method because of the maturity of both these technologies in their own rights. Bolometers have been shown to achieve sensitivity near the background limit from the ground and during balloon flights, while interferometry has a rich and detailed history of success in radio astronomy. In MBI we combine the strengths of both these methods to show that we can produce extremely sensitive measurements of the CMB.

3.2 Interference

We will first discuss the operation of MBI’s stellar Michelson interferometer as a whole and then in the next section the individual parts.

The four sky horns are each sensitive to a Gaussian beam on the sky. Radiation from the sky is funneled through the horns and into circular wave guides. These sky horns form the basis of MBI’s phase front interferometry. The physical process of
passing radiation through the sky horns is the same as taking a Fourier Transform of the spatial distribution of the signal on the sky. The sky signal is expressed in terms of spherical harmonics, $Y_{l,m}$’s. The spacing of the sky horns and their size make a baseline sensitive to only a small range in $l$ and their orientation determines the $m$. As the number of baselines with unique lengths and orientations goes up, more information can be recovered about the image on the sky.

The radiation is then reemitted by horns identical to the sky horns. These internal horns are the beginning of MBI’s quasi-optical beam combiner. The internal horns illuminate an on-axis Cassegrain telescope. The light from the internal horns is imaged on a focal plane of bolometers. For a given point on the focal plane the stellar Michelson interferometer, which acts as a beam combiner, introduces a unique relative phase difference between each of the four internal signals. This reemission can be though of as an inverse Fourier Transform of the sky signal. Therefore the image formed on the focal plane of MBI is an image of the CMB sky. The catch is that by processing the signal through our 6 baselines we are only sensitive to 6 $l$-modes. Therefore our picture will have a low fidelity to the original sky. This process is the same as Young’s classic two-slit interference experiment; as laser light hits the two slits the first transform is done and as it is emitted from the far side of the slits the second transform is done. What shows up on the screen is a single Fourier mode of the incoming signal.

The phase shifts introduced by the phase shifters are constantly altering the image that is formed from each of the 6 baselines. By using a digital lock-in detection we can isolate the signal from a single baseline. The signal will be a fringe pattern
just like a two slit interference pattern. By recovering all six fringes and adding them together we can reconstruct a picture of the sky. We can also take an individual fringe and fit it with the function of a Gaussian damped sine wave. Then the amplitude and phase of the fringe will allow us to calculate the visibility associated with that baseline. Combining these visibilities using aperture synthesis allows an alternate means for recovering the sky image. In this manner MBI is both an imaging telescope and an interferometer.

3.3 Optical system

MBI’s optical system was constrained by some considerations. Our 20% bandwidth constrains the longest baseline to no greater that five times the length of the shortest baseline. While the sensitivity of a receiver to broadband signals increases as the square root of the band-width, for interferometers, the bandwidth restricts the
angular range, $\theta$, over which fringes are detected.[33][34] If we assume the path lengths for a source at the center of the FOV are equal, then the path length difference for a source at an angle $\theta$ from the center along the baseline axis is $\theta B$, where B is the baseline distance. If this path length difference is small compared to the coherence length of the light, $\lambda^2/\Delta \lambda$, then the fringe contrast is not affected. Thus the FOV is determined by $\theta_{\text{FOV}} \leq (\lambda/\Delta \lambda)(\lambda/B)$. This equation indicates that for angles of the order of the product of the spectral resolution times the angular resolution, the fringe smearing is important. This relation imposes restrictions on the ratio between the maximum baseline achievable by the interferometer and the spectral bandwidth of the receiver. For MBI, the bandwidth of 20%, sets the maximum baseline to about 4 times the diameter of each antenna, $\sim 5$ cm, which is the minimum baseline length.

The arrangement of the sky horns was determined using a simulated annealing process to produce optimal coverage of points in the $u - v$ plane. This assumed that for each pointing the instrument would rotate around the line of sight through 18 steps of 20°.[35] The result of the annealing process is the cross shape of MBI’s horn positions in Figure 3.2.

Figure 3.3 provides a schematic of MBI’s optical chain. The chain begins with vacuum windows for each horn, followed by filters. There are two filters, one at the 77K shield and one at the 4K shield. Light then enters the sky horns.

The sky horns are circular corrugated feed horns. The corrugations help to produce a beam response on the sky that is close to ideal by producing destructive interference at angles where there would otherwise be sidelobes. The beam response of the sky horns is a Gaussian to high accuracy that is systematic in the $E$ and $H$
Figure 3.2: MBI’s sky horns are the four circles with horizontal or vertical lines inset. This cross pattern produces uniform coverage in the $u-v$ plane. The inset lines designate the polarization orientation which is allowed to propagate through the horn.
Figure 3.3: A schematic representation of MBI’s optical path.
planes with a FWHM of $7.5^\circ$. The output of the sky horns is a circular waveguide for a wavelength of 3mm.

Following the optical path through the sky horns we come to a circular to rectangular waveguide transition. This serves to select a single polarization of the light. The relative alignment of these outputs is critical as we need to select perpendicular polarizations to as great an accuracy as possible. The alignment is set by eye and then fine tuned using micro-screws. Two sky horns are set to select an $x$ polarization while the other two are set at $90^\circ$ to select the $y$ polarization. This will allow us to interfere $E_x$ with $E_x$, $E_y$ with $E_y$, and $E_x$ with $E_y$. This allows us to recover Stokes’ $Q$ and $U$ measurements, the importance of which is discussed in Chapter 4.

A $45^\circ$ twist then rotates the polarization either $45^\circ$ clockwise or counterclockwise. This is done in order to align the $x$ and $y$ polarizations so that they will be able to interfere with each other later in the focal plane.

Following the $45^\circ$ twists are the ferrite phase modulators. Developed by Brian Keating at the University of California San Diego these phase shifters use the Faraday Effect to rotate linear polarization that passes through them. This rotation is caused by differential propagation speed of left and right circularly polarizations of light (LCP, RCP) in the ferrite. This produces a phase difference between the LCP and RCP components of the incident linear polarization. The end result of the LCP and RCP phase differences is a phase shift in the linear polarization which accompanies the rotation. The angle of rotation is also equal to the shift in phase. These phase modulators are operated so that they shift the polarization either $+$ or $-$ $90^\circ$ from its original orientation. This also produces a rotation by $90^\circ$. By controlling the
direction of rotation, the phase of the light at the output is then changed between two states which have a 180° phase difference. Because the light has been rotated 90°, the rectangular wave guide that leaves the phase shifter is rotated 90° with respect to the one that enters the phase shifter. A useful feature of the ferrite modulators is that they can be turned “off” by sending zero current through them. In this mode, RF signals entering the sky horns are blocked from entering the beam combiner.

The sequence of these applied phase shifts is discussed in Chapter 5. Each of the four sky horns has a different sequence of phase shifts applied to it. The result is that each baseline (pair of horns) has a unique pattern of relative phase shifts between them over the course of the sequence. This relative phase sequence is in the form of a Walsh function. A Walsh function is a periodic function which takes only the values of 1 or -1 and will average to 0 over a period.[33] The four applied phase shifts are picked so that the six relative phase shifts are each unique. This will be used later to extract the signal from a baseline of interest.

Next the light is reemitted into the cryostat by horns which are identical to the sky horns.

The light is reflected off the primary and secondary mirrors designed by our collaborators, J. Anthony Murphy, Cridhe OSullivan, Marcin Gradziel, and Gareth Curran at the National University of Ireland, Maynooth. These serve to focus the radiation onto our focal plane. The position and curvatures have been optimized for the internal geometry of the cryostat.

The entire focal plane is covered by a metal mesh filter provided by collaborators at Cardiff University, (Figure 3.4). The focal plane is a block of copper to which
Figure 3.4: Shown here is the cold metal mesh filter provided by Cardiff University in place over the focal plane.

electroformed conical, smooth walled feed horns are attached. Excess material is removed to reduce the thermal mass, (Figure 3.5). A corrugated polyethylene lens attached to each aperture to flatten the phase of the incoming radiation across the aperture, (Figure 3.6).

The focal plane feed horns then terminate with the bolometer assembly, which holds the spiderweb bolometers, (Figure 3.7).

3.4 Detectors

MBI’s bolometers were designed for the ACBAR experiment. They are not polarization sensitive. The bolometers are in a resistance bridge and voltage biased. The resistance of MBI’s bolometers increases as they are cooled. If the bolometer’s resistance changes then the voltage drop across it will change as the percentage it represents of the total resistance changes. The voltage across the bolometer is measured and we work backwards to find a bolometer temperature. The voltage bias of the
Figure 3.5: Shown here are the bolometer PC board mounts, the machined bolometer feed horns, the intermediate heat sinking stage (copper encircling the focal plane), heat sinking for the signal wires (two large PC boards) and the thermal connections to the focal plane and intermediate stage.

Figure 3.6: Polyethylene lenses to flatten the phase of incoming radiation. Two bolometers are blanked off using conductive Al tape as a ‘dark’ check. The cold filter goes directly over the lenses.
Figure 3.7: A picture of a bolometer in its resonator cavity behind the PC board. On the left is the connection used to apply the bias and read out the voltage.

Bolometers is modulated in the form of a sine wave with frequency of 208 Hz. This frequency is faster than the expected rate of temperature change in the bolometers. Let us assume that the temperature is constant for practical purposes. The result of the voltage bias is that the voltage will vary in a sinusoidal manner and its amplitude will be proportional to the temperature of the bolometer. We use a lock-in amplifier to measure this signal, which is inversely proportional to the power received by the bolometer. If, after many cycles, the temperature of the bolometer goes up then the amplitude of the voltage oscillation will go down and the signal from the lock-in amplifier will go down as well. Random thermal or electrical noise can be integrated out by averaging the signal after the lock-in. The only signals we are sensitive to are slowly varying ones.
3.5 Cryogenics

MBI’s LHe/LN cryostat was made by A. S. Scientific Products Ltd. and has a hold time of 96 hours without any thermal load from wiring or windows. Due to the geometry of MBI’s optics a large cryostat was needed and the final cryostat is a cylinder 0.7m in diameter and 1m long.

The cryostat has two cryogen tanks, an outer one for liquid nitrogen and an inner one for liquid helium. The final cooling is done using a series of He refrigerators.

The refrigeration unit is a “Helium 10” system made by Simon Chase and is composed of a $^4$He and two $^3$He refrigerators which provide a cold head temperature of 260 mK, (Figure 3.8). This system was chosen based on the cooling requirements for the detectors and the performance that the system has shown. The cold plate of the first $^3$He stage is thermally linked to the bolometer readout wiring at a preliminary stage that is thermally isolated from the focal plane. This is done to intercept heat flowing down the wires from the hot stages before reaching the bolometers. The thermal connection is made using a gold plated copper cold finger and thick copper braid. The first $^3$He stage is used because it has more cooling power and a higher temperature than the final $^3$He stage. The bolometer focal plane is connected to the final $^3$He stage by a similar arrangement. The phase shifters are separately cooled to 4.2 K, because the large operational thermal load of the phase shifters makes it impractical to cool them with the $^4$He and $^3$He refrigerators while they are in use.
3.6 Signal Processing

The signal size makes careful signal processing important. The sinusoidal voltage required to bias the bolometers is produced by the bias board. The bias board, a copy of the BLAST bias board, resides in a crate sitting on the base of the mount. A single bias line is fed into the cryostat and all of the bolometers are biased using the same signal. The signal is a 208 Hz signal which has a digital level control. Twelve digital lines control the amplitude of the bias signal between 0 and 16 Volts peak-to-peak. The frequency for the bias is set using an external reference signal provided by one of two National Instruments Field Programmable Gate Array (FPGA) boards. The boards (NI 7833R) are used later in the acquisition chain to lock-in to the bias signal; this arrangement minimizes spurious signals due to small differences in clocks.

The voltage across the bolometer is amplified by the JFET module. The JFET module is located on the 77K shield of the cryostat, but generally operates at 120 - 130K. The module is partially self-heating and the wires connecting it to the 300K connector have been made such that sufficient heat flows down them to keep the JFET at ∼125K. The JFET module can be used at room temperature, but cooling it reduces the internal noise. There is an optimal operating temperature of 120 K where the JFET noise is minimized.

The out-going signal from the JFET module goes through a shielded cable to the readout board, which sits in the same crate as the bias board. The readout board, a copy of the BLAST preamp board, uses a band-pass filter centered around 208 Hz to filter out noise and then amplifies the signal again by a factor of 196. The amplified signal from the readout board passes through another shielded cable to the analog
inputs of the FPGA boards.

The FPGA boards perform a lock-in detection complete with phase information on the 8 channels each can handle. This is the lock-in detection we discussed with the bolometer bias. The reference frequency is the same signal that produces the bolometer bias. The result is read off the FPGA board by a Labview vi on the data acquisition computer and recorded. Sampling occurs approximately three times a second. The resulting data is a record of the bolometer signal with 208 Hz lock-in already performed.

The Labview vi also records the states of the phase modulators. Another digital lock-in is done later to lock into the phase-modulated signal from a given baseline. This process does not use a sinusoidal multiplier, but instead a Walsh Function and will be discussed in more depth in chapter 5. The second lock-in is not done in real time because it would increase the computational overhead on the FPGA boards, slowing down their performance. Also the FPGA lock-in program relies on a purely sinusoidal reference signal. The Walsh functions used in the phase shifter modulation can not be matched to a purely sinusoidal reference signal and therefore the second lock-in cannot be done by the FPGA, without extensive reprogramming.

3.7 Telescope Mount

MBI’s observing strategy requires that we be able to observe any point on the sky and also be able to rotate the instrument around the line of sight so that we can measure either Stokes’ Q or U. This is accomplished with a standard Az/El design for pointing and with an added rotation axis around the line of sight, which we will refer to as the z-axis.
Figure 3.9: Scale schematic drawing of MBI cryostat and mount structure. The tripod base is shown in black with the attached wheels and jack-screws in green. The driving disc, azimuth bearing and mating plate are shown in green, black and red respectively. The frame and stiffening place are in blue. The cryostat and motors are in red and the z-axis is in green.
3.7.1 Base of the Mount

When reading this section and referring to Figure 3.9 on page 60, the base refers to everything below the frame, drawn in blue. Figure 3.10, though meant to show the upper portions of the mount best, also has some of the base visible at the bottom.

Leveling the entire mount structure is important so the bottom of the mount is a tripod. Each corner of the triangle has a large, hard rubber wheel to allow for easy movement and a jack-screw with a foot plate. Once in place the mount is raised and leveled with the jack-screws.

On top of the tripod is the driving disc. This is a 1” thick, 53” diameter Al disc. The amount of compression needed to drive the azimuthal rotation would deform the Al so 1/8” thick 3-16 stainless steel ring was placed around the outer edge of this disc. The ring forms the driving surface which turns the mount on its azimuthal axis. It was rolled from a length of 1” wide, 1/8” thick stainless and then welded to be slightly smaller in circumference than the driving disc. The driving disc was then immersed in liquid nitrogen until it reached thermal equilibrium. The disc was taken out, the ring placed around it and the assembly allowed to warm up. This method of gearing was also used to good effect on the White Dish experiment.[36]

Secured at the center of the driving disc is a post, to which the Azimuthal encoder attaches.

Bolted to the driving disc is the azimuth bearing. The bearing consists of 6, 60° arcs of curved track and four carts which run on the track. The track and carts are THK R Guides, part number HCR15A+60/300R.
Atop the azimuth bearing sits the mating plate. This plate is connected to the four carts with 16 small screws and allows the rest of the mount to be connected using 8 much larger screws. When the mount is placed on the base it must be lifted by crane and lowered into place. The mating plate reduces the accuracy with which this procedure needs to be accomplished. The mating plate completes the base of the mount.

3.7.2 Mount Proper

Figure 3.10, page 63, shows most of the structure discussed in this section. The frame of the mount was recycled from the COMPASS experiment.[37][38] It was found during that experiment that the frame was not stiff enough on its own and needed a plate attached to the bottom to keep it from buckling. On MBI the stiffening plate is 3/4” Al and is attached to the frame via many 1/4” diameter screws. The bottom of the frame is open and anything sitting there rests on the stiffening plate.

The stiffening plate is attached to the mating plate with eight 1/2”bolts. It also has a cutout made in one corner to allow for the motor to drive the azimuthal rotation. We use a SimpleServo EZ94 controller, a custom Torque Systems motor, Dojen reducer and a 3-16 stainless steel cylinder to drive the rotation. The motor assembly is secured to a small plate with a channel cut almost all the way across it. The plate is bolted to the stiffening plate and the section with the motor is compressed against the driving disc with a screw. This arrangement can be seen in Figure 3.11 at the bottom of the picture. The steel on steel contact provides a good friction contact between the gears and does not have the danger of cracking that a rubber on steel contact might. We have found that this scheme works very well. The stiffening plate
Figure 3.10: Pictured here is the entire mount. The computer, controllers and power supplies are inside the teal signal processing crate [1]. The blue azimuth [2] and z-axis [3] motors are visible at the top and bottom, while the elevation arm [4] can be seen just above and to the right of center. The green z-axis [5] and elevation [6] encoders are at the top of the picture. The elevation motor and azimuthal encoder are hiding behind or underneath other hardware.
also has a hole in the center allowing for the azimuth encoder to be attached to the post at the center of the driving disc by a flexible helical connector.

Sitting atop the frame is the Z-axis bearing. This is also a set of 60° arcs manufactured by THK, HCR25A+60/500R. These have a larger diameter to accommodate MBI’s cryostat. The Z-axis bearing is sandwiched between two annuli. The upper annulus has a 3-16 steel ring fitted to it in the same manner as the driving disc. Attached to the lower annulus is a plate with the same motor assembly as the azimuth along with an encoder assembly, Figure 3.12. The encoder assembly has a similar steel cylinder sitting in a bearing and attached to a 4-bit encoder. Both of these assemblies are compressed against the steel ring with screws. The encoder assembly is used here because it was not feasible to place an encoder at the top of the instrument, because it would block the antenna beam, or the bottom of the instrument as there are connections leaving the cryostat on axis on the bottom. This arrangement has been as reliable as the azimuthal one.

The lower annulus is also connected to two, horizontal, 3” diameter shafts. These make up the elevation axis. Two “pillow block” bearings, one on each side allow for free rotation. An encoder is bolted to the frame on one side such that its shaft fits snuggly into a machined hole in the shaft. This is how the elevation is read out.

The elevation movement is accomplished with of a Pacific Scientific Powermax II stepper motor and Preferred Power Industries worm gear. An arm, with pivot, is attached to the lower annulus and the worm gear’s extension or contraction varies the angle from 0 to 90 degrees from vertical. Figure 3.10 provides the best picture of this.
Figure 3.11: Visible at the bottom of the picture is the plate which is used to mount the azimuthal motor. Above the motor are the control electronics.
Figure 3.12: In the foreground is the encoder assembly. The driving cylinder for the Z-axis motor has had a bead of welding added and then turned smooth to allow for greater compression.
3.8 Control and Monitoring

Using the same frame, elevation mechanism and encoders as the COMPASS experiment’s meant that we could also reuse the COMPASS control program with minor adjustments for MBI. The LabView control program already had star tracking and smooth motion for the elevation control built in. Modifications were made to accommodate the servo motors used in MBI.

The majority of the time integrating the new hardware with the old program was spent on the communication between LabView and the servo controllers. The methods of communication and motor control we wished to use had never been attempted before. AC Tech - Lenze Motion Control, the company which made the controllers, was very generous with their time working on the project.

The LabView program used to control the cycling of the fridge was written by Dr. Andrei Korotkov. Please contact him for documentation on that program.

The programs for control and data acquisition run on separate computers. The mount control computer is located in the teal signal processing box at the base of the mount, (Figure 3.11). The data acquisition computer is located in a crate on the front of the mount. This was done to facilitate the development of each. With the mount being developed in Madison, Wisconsin and the instrument in Providence, Rhode Island having two separate computers eliminates the need for frequent communication between programmers or difficulties in testing programs, at two locations, on a single computer.
3.9 Observing Site

The University of Wisconsin - Madison’s Pine Bluff Observatory site was selected for MBI’s deployment because the seeing is reasonable in MBI’s observing window near 3 mm (90GHz), it is a secure site close to Madison and we already have infrastructure in place to accommodate MBI. Also, Wisconsin’s dark skies are also good for sighting guide stars and calibration targets. Figure 3.13 shows MBI deployed during its first observing run at Pine Bluff.
Chapter 4

Systematics

4.1 Introduction

The level of anisotropy signal in the CMB is very small. This has lead to the development of very sensitive experiments. Because experiments are so exquisitely sensitive they also have strong demands on the level to which errors must be controlled. There have been a number of papers looking at the effects of various errors on an experiment. Hu, Hedman and Zaldarriaga set out to give a relatively general treatment how common errors would distort temperature and polarization maps.[39] Their treatment was for single dish imaging experiments. Bunn later extended that formalism to interferometric experiments.[24]

MBI is a proof-of-concept experiment and as such is not intended to be able to observe the B-mode signal from the CMB and observing the E-mode signal may be challenging. However, we would still like an assessment of the design challenges for a future version of MBI with more antennas. In this chapter we will lay out some basic calculations based on the formalism developed by Bunn. It is strongly recommended
that the reader familiarize him or herself with this paper.

4.2 Formalism

Before diving into the effects of various errors we will need to briefly describe the formalism in which we will be discussing them. We will not be covering the formalism in great detail here, but rather providing a “Greatest Hits” version and then discussing how the result applies to MBI.

There are two main descriptors for an experiment used: the Jones Matrix, $J$, and Antenna Pattern, $A(r)$. $J$ describes effects that are purely part of the signal processing of the instrument, while $A(r)$ characterizes effects that occur on the sky or from the observation of the sky. This separation is a bit arbitrary in that one could be subsumed into the other, but they are convenient conceptual distinction.

The total response of an instrument, $R$ would be the multiplication of these two matrices, $R(r) = JA(r)$.

4.2.1 Jones Matrix

A Jones matrix is a standard way to dealing with the various effects that an instrument, or part of an instrument, have on a signal. We are dealing with two polarizations of light and so will be using a $2 \times 2$ Jones matrix here. In equation 4.1 $i$ denotes a type of polarization and $j$ a specific antenna.

$$J_i^{(j)} = \begin{pmatrix}
1 + g_1^{(j)} & e_1^{(j)} \\
e_2^{(j)} & 1 + g_2^{(j)}
\end{pmatrix} \tag{4.1}$$
For completeness:

\[
\begin{pmatrix}
    x' \\
y'
\end{pmatrix} = \begin{pmatrix}
    1 + g_1^j & \epsilon_1^j \\
    \epsilon_2^j & 1 + g_2^j
\end{pmatrix}\begin{pmatrix}
    x \\
y
\end{pmatrix}
\]
or

\[
x' = (1 + g_1^j)x + (\epsilon_1^j)y \\
y' = (\epsilon_2^j)x + (1 + g_2^j)y
\]  \hspace{1cm} (4.2)

Let us take 1 to be \(x\) polarization and 2 to be \(y\). \(g_i^{(j)}\) represents the deviation from 1 of the gain measured for \(x\) and \(\epsilon_i^{(j)}\) quantifies the amount of \(y\) polarization that is coupled into \(x\). We classify errors from \(g_i^{(j)}\) as Gain Errors and those from \(\epsilon_i^{(j)}\) as Coupling Errors.

4.2.2 Antenna Pattern

Equally important for assessing errors and their effect on power spectra is the antenna pattern. The antenna beam, \(A(\hat{r})\), encodes the antenna’s response to the sky while pointing in a given direction, \(\hat{r}\). The beam is a function of position on the sky centered at \(\hat{r}\) and describes the antenna’s sensitivity to portions of the sky relative to \(\hat{r}\). We can think of the antenna beam has having a Jones’ matrix of the form:

\[
A = \begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}
\]  \hspace{1cm} (4.3)

Here \(A_{11}\) represents the beam on the sky that correctly identifies \(x\) polarization as \(x\). \(A_{12}\) is the beam that spuriously reads \(y\) polarization as \(x\). Similarly \(A_{21}\) is a measure of the beam mixing \(x\) into \(y\) and \(A_{22}\) a measure of the beam correctly reading \(y\) polarization. The equation describing the processing of the sky though the antenna
now looks like:

\[
\begin{pmatrix}
  x \\
  y
\end{pmatrix} =
\begin{pmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{pmatrix}
\begin{pmatrix}
  X \\
  Y
\end{pmatrix}
\]  

(4.4)

Here \( X \) and \( Y \) represent maps of the sky in \( x \) and \( y \) polarizations. The result of the horn processing the sky signal is a single value for \( x \) or \( y \) polarization.

Let us take a step back now and look at the antenna pattern independently of the Jones matrix. The simplest incarnation of \( A \) outside of the Jones’ matrix comes in the output on a single antenna:

\[
\epsilon_{\text{out}} = \int d^2 \hat{r} A(\hat{r}) \cdot \epsilon_{\text{in}}(\hat{r}) e^{i(k \cdot \xi - \omega t)}
\]  

(4.5)

Here \( \epsilon_{\text{in}} \) and \( \epsilon_{\text{out}} \) are the incoming electric field and the electric field that is output by the antenna, respectively. For an interferometer a visibility is the thing of interest

\[
V^{(jk)}_Z \equiv \int d^2 \hat{r} A^{(j)}(\hat{r}) Z(\hat{r}) A^{(k)}(\hat{r})^* e^{-2\pi i u_{jk} \cdot \hat{r}}
\]  

(4.6)

Where \( Z = I, Q, U, V \) is the instrument’s response to the given Stokes parameter and \( j, k \) denote elements of the interferometer.

### 4.3 Effects on the Visibilities

For a linear polarization experiment like MBI sensitivity to Stokes parameters can be represented in a matrix format.

\[
S = \begin{pmatrix}
  I + Q & U + iV \\
  U - iV & I - Q
\end{pmatrix}
\]  

(4.7)

Which we can rewrite as

\[
V = \begin{pmatrix}
  XX & XY \\
  YX & YY
\end{pmatrix}
\]  

(4.8)
where X and Y represent the polarizations being interfered for the visibility. Therefore we see that we get

\[ V_I = \frac{1}{2}(V_{XX} + V_{YY}), \quad (4.9) \]
\[ V_Q = \frac{1}{2}(V_{XX} - V_{YY}), \quad (4.10) \]
\[ V_U = \frac{1}{2}(V_{XY} + V_{YX}), \quad (4.11) \]
\[ V_V = \frac{1}{2i}(V_{XY} - V_{YX}). \quad (4.12) \]

(4.13)

Information about linear polarization comes from Stokes U and we can write down the measured visibility \( V_{U}^{(jk)} \) in terms of the visibility with no systematic errors \( \tilde{V}_{U}^{(jk)} \) and terms from mixing other parameters.

\[ V_{U}^{(jk)} = \tilde{V}_{U}^{(jk)} + \frac{1}{2}\left[ V_{I}^{(jk)}(\epsilon^{(j)}_1 + \epsilon^{(j)}_2 + \epsilon^{(k)*}_1 + \epsilon^{(k)*}_2) \right. \]
\[ + V_{U}^{(jk)}(g^{(j)}_1 + g^{(j)}_2 + g^{(k)*}_1 + g^{(k)*}_2) \]
\[ + V_{Q}^{(jk)}(-\epsilon^{(j)}_1 + \epsilon^{(j)}_2 - \epsilon^{(k)*}_1 + \epsilon^{(k)*}_2) \]
\[ + V_{V}^{(jk)}(g^{(j)}_1 - g^{(j)}_2 - g^{(k)*}_1 + g^{(k)*}_2) \] \quad (4.14)

Errors in the visibilities due to gain errors can be written as: \( \delta V_Q = \gamma_Q V_Q \) and \( \delta V_U = \gamma_U V_U \). Where

\[ \gamma_{Q,U} = \frac{1}{2}(g^{(j)}_1 + g^{(j)}_2 + g^{(k)*}_1 + g^{(k)*}_2), \quad (4.15) \]

and \( g^{(j)}_i \) is evaluated at the time of the Q or U is measured. This equation is determined from equation 4.14 and noting that gain errors produce errors in U by
mixing in extra power from U and V visibilities. Stokes V is expected to be zero for the CMB so we only concern ourselves with the contribution from the $\hat{U}$ term.

Since we use the difference between Q and U measurements, it is the difference between these values which is important to characterize errors.

$$\gamma_2 = \frac{1}{2} (\gamma_Q - \gamma_U)$$

(4.16)

Coupling errors, on the other hand, can be seen to mix power from I into U. This is a very serious problem. We characterize this effect by $\varepsilon$ in the same way Gain errors are characterized by $\gamma_2$.

$$\varepsilon = \frac{1}{2} (\epsilon^{(j)}_1 + \epsilon^{(j)}_2 + \epsilon^{(k)*}_1 + \epsilon^{(k)*}_2)$$

(4.17)

We will be using $\gamma_2$ and $\varepsilon$ to characterize the effect of an experiment’s errors later.

We will now rewrite Jones matrix for the beam, filling in some more explicit forms for the $A_{ij}$ values. We will skip the intervening steps here and just present result.

$$A^i = \left( \begin{array}{cc} A^i_0 + \frac{1}{2} A^i_1 \cos 2\phi & \frac{1}{2} A^i_1 \sin 2\phi \\ \frac{1}{2} A^i_1 \sin 2\phi & A^i_0 - \frac{1}{2} A^i_1 \cos 2\phi \end{array} \right)$$

(4.18)

$A_0$ is the ideal beam shape and $A_1$ is a departure from this that leads to $x$ and $y$ mixing. For an ideal experiment $A_1$ is 0 and $A_0$ is something nice to deal with like a Gaussian.

We shall cover three ways for $A$ introduce errors: pointing errors, beam shape errors and cross-polarization errors.
4.3.0.1 Pointing Error

Pointing errors occur when the sky antennas don’t all point in the same direction. The pointing error for antenna $j$ is of the form:

$$A^{(j)}(\hat{r}) = A^{(j)}(\hat{r} + \delta\hat{r}_j)$$  \hspace{1cm} (4.19)

where $\delta\hat{r}_j$ is the error in pointing for antenna $j$. Conveniently, the product of two Gaussians is another Gaussian centered at the midpoint of the two. This means that the $A \cdot A^*$ term in the visibility can be rewritten as

$$A^{(j)}(\hat{r})A^{(k)}(\hat{r}) \propto e^{-\frac{1}{2}(\delta\hat{r}_j + \delta\hat{r}_k)^2}/\sigma^2}. \hspace{1cm} (4.20)$$

which can in turn be simplified by defining an error parameter for one pair of antennas

$$\delta_{jk} = \frac{(\delta\hat{r}_j + \delta\hat{r}_k)^2}{2\sigma} \hspace{1cm} (4.21)$$

Measurements of power spectra require the differencing of Stokes Q and U visibilities. So the figure of merit is how much $\delta_{jk}$ changes between the two measurements.

We now have

$$\delta_{\pm} = \frac{1}{2}(\delta_Q \pm \delta_U). \hspace{1cm} (4.22)$$

Where $\delta_Q$ is the value of $\delta_{jk}$ for the Q measurement and similarly for the U measurement. Since MBI is a linear polarization experiment it is only $\delta_-$ which will contribute to the error in the power spectrum.
4.3.0.2 Beam Shape Error

The beam shape error is modeled as follows: the beam is assumed to be a Gaussian of width $\sigma$, with an actual shape that is elliptical with beam widths of $\sigma_1^{(j)}$, $\sigma_2^{(j)}$ along the principle axes. This leads to an effective visibility beam pattern

$$A^{(j)}(\hat{r})A^{(k)}(\hat{r}) \propto e^{-\hat{r}^T(1+\Delta_{jk})\hat{r}/(2\sigma^2)} \quad (4.23)$$

Here $\Delta_{jk}$ is a symmetric $2 \times 2$ matrix characterizing the deviation from the ideal Gaussian. The eigenvectors of $\Delta_{jk}$ give the principle axes of the elliptical beam with widths of $\sigma/\sqrt{1+\lambda_i}$ where $\lambda_1$, $\lambda_2$ are the eigenvalues.

Again it is the relation between these values w.r.t. $Q$ and $U$ which is of interest.

$$\Delta_\pm = \frac{1}{2}(\Delta_Q \pm \Delta_U) \quad (4.24)$$

Where, as always, $\Delta_Q$ and $\Delta_U$ are taken when $Q$ and $U$ are measured respectively. $\Delta_+$ characterizes the average beam shape when the two visibilities are measured, and $\Delta_-$ characterizes errors in beam shapes that are different between $V_Q$ and $V_U$. We will be using the following assessment to quantify the effect on power spectra:

$$\zeta_{\pm,i} = \frac{\delta\sigma_{\pm,i}}{\sigma} = -\frac{\lambda_{\pm,i}}{2} \quad (4.25)$$

Where $\zeta_{+,i}$ parameterizes common beam errors and $\zeta_{-,i}$ parameterizes differential errors. For MBI $\zeta_{-,i}$ will be the parameter we are interested in.
4.3.0.3 Cross-polarization

Cross-polarization is an effect that mixes $x$ and $y$ polarizations. This can occur when the beam covers a sufficiently large angle that the flat sky approximation [40][19] is no longer valid. In this case $x$ and $y$ will be mixed away from the center of the beam. An orientation of polarization on the sky that is $x$ will look like some portion of its power is in the $y$ direction to the antenna. Cross-polarization is expected to be small near the center of the beam and get progressively worse towards the edge of the beam.

We will model this effect in the following way

$$A_1^{(i)}(r) = \mu_i \frac{r^2}{\sigma^2} A_0(r)$$

(4.26)

where $A_0$ is assumed to have the usual Gaussian form and $\mu_i$ is the parameter characterizing the size of the error and is itself characterized by $\mu = \frac{1}{2} \sigma^2$.

For a visibility, $V_Q$, $V_U$ we are interested in the average $\mu$ during the measurement.

Table 4.1 on page 78 is a reproduction of Bunn’s catalogue of instrument errors and their effects.

$$\mu_{Q,U} = \frac{1}{2} (\mu_j + \mu_k)$$

(4.27)

4.4 MBI in terms of J

MBI does not have one bolometer per measurement of a visibility, it has 16. This provides a factor of 4 decrease in noise if the noise is uncorrelated. In reality it is more complex than that, with each element illuminating the focal plane differently, thereby weighting certain bolometers more than others. This weighting is completely
<table>
<thead>
<tr>
<th>Experiment type</th>
<th>Measurement</th>
<th>Error source</th>
<th>Primary contribution</th>
<th>Fiducial contribution $\delta \hat{C}/C$</th>
<th>Secondary contribution $\delta \hat{C}/C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>B</td>
<td>Gain error</td>
<td>$\kappa_{B,EE,\gamma_2} = \frac{1}{2} \sqrt{s^2}$</td>
<td>$21 \gamma_2$</td>
<td>$\kappa_{B,EB,\gamma_2} = \frac{1}{2}$</td>
</tr>
<tr>
<td>Circular</td>
<td>B</td>
<td>Gain error</td>
<td>$\kappa_{B,EE,\gamma_2} = \sqrt{2s^2}$</td>
<td>$60 \gamma_2$</td>
<td>$\kappa_{B,EB,\gamma_2} = \sqrt{2}$</td>
</tr>
<tr>
<td>Linear/Circular</td>
<td>B</td>
<td>Coupling</td>
<td>$\kappa_{B,TE,\epsilon} = \sqrt{s^2}$</td>
<td>$730 \epsilon$</td>
<td>$\kappa_{B,EB,\epsilon} = 1$</td>
</tr>
<tr>
<td>Linear/Circular</td>
<td>E</td>
<td>Coupling</td>
<td>$\kappa_{E,TE,\epsilon} = 1$</td>
<td>$17 \epsilon$</td>
<td>—</td>
</tr>
</tbody>
</table>

**Table 4.1:** Effects of instrument errors (above line) and beam errors (below line). See reference [24] for details.

dependent on the baseline of interest. For this discussion, however, we will assume that at an instant in time the bolometer gain is the same for all measurements. There will be gain drifts over time that will show up in the $g^{(j)}_i$. These effects can be mitigated by observing a calibration source, either from an emitter or an astronomical one, often enough to track these gain drifts. In this case we will be interested in how much the gain drifts without our knowledge because these drifts will be the ones that contribute to errors. Known drifts should be accounted for already.

There will also be relative gain and phase differences between the four elements. These are introduced by the phase modulators. To first order, the relative gains should be the same, but the response of each phase modulator is slightly different for a given level of current. This produces a difference in the gain as less of the signal is transmitted through the single mode optics that is of the form $E \sin(\theta)$. Where
E is the initial amplitude of the signal and \( \theta \) is the angle through which the ferrite phase modulator rotated the linear polarization. The phase of the signals will only ever be \( \pm 90^\circ \), but each will have a unique modulation pattern. This is encoded in the complex portion of \( g_i^{(j)} \). Now \( g_i^{(j)} \) will have a time component which accounts for the modulation pattern encoded by phase modulators, \( g_i^{(j)}(t) \). This is somewhat simplified because we use a digital lock-in to identify the signal from each baseline. Since, it is \( \langle g_i^{(j)}(t) \rangle \) that we will dealing with we will drop the angle brackets and just refer to \( g_i^{(j)}(t) \) where the time averaging is implied. Since MBI uses Walsh functions in the modulations the average will be evenly weighted for the two states of the phase modulators.

An interesting thing about MBI is that it does not naturally produce large phase errors. The phase modulators have an output that transmits only a single mode of linear polarization. As a result any under rotated signal will have its amplitude reduced, but a \( +90^\circ \) or \( -90^\circ \) component is always transmitted. The most likely source for phase errors comes from how well the primary and secondary mirrors are known, as these are what produce the fringes on the focal plane.

Mixing power between \( x \) and \( y \) as a result of the instrument is described by \( \epsilon_i^{(j)} \). Here again the NTD bolometers play less of a role than the optics because they are not polarization sensitive and do not distinguish between the two modes. The alignment of the single mode optics is what selects \( x \) and \( y \), just prior to the phase modulators. An error in the alignment by an angle \( \delta \) results in \( \epsilon_1^{(j)} = \delta, \epsilon_2^{(j)} = -\delta \).

The major difference between the Jones matrix we described earlier and MBI’s Jones matrix is that each element of MBI only propagates a single polarization mode
The last thing we need to mention before applying Table 4.1 to MBI is that the relative sizes of the baseline, $u$, and beam width, $\sigma$, determine whether the dominant or secondary contributions to error are the most important. Bunn defines a parameter $s^2$ to assess this relation. $s^2$ can be approximated to a high order by,

$$s^2 = \frac{1}{2\pi^2(u\sigma)^2}.$$  \tag{4.28}

MBI has the following $u\sigma$ for its 6 baselines:

<table>
<thead>
<tr>
<th>baseline</th>
<th>$u\sigma$</th>
<th>$s^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>2.598</td>
<td>0.0075</td>
</tr>
<tr>
<td>13</td>
<td>4.061</td>
<td>0.0031</td>
</tr>
<tr>
<td>14</td>
<td>2.912</td>
<td>0.0059</td>
</tr>
<tr>
<td>23</td>
<td>2.727</td>
<td>0.0098</td>
</tr>
<tr>
<td>24</td>
<td>2.560</td>
<td>0.0077</td>
</tr>
<tr>
<td>34</td>
<td>1.913</td>
<td>0.0138</td>
</tr>
</tbody>
</table>

Table 4.2: Calculations of $s^2$ for MBI’s baselines.

and therefore will look like

$$J^{(j)}_i = \begin{pmatrix} 1 + g_1^{(j)} & \epsilon_{1}^{(j)} \\ 0 & 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & 0 \\ \epsilon_{2}^{(j)} & 1 + g_2^{(j)} \end{pmatrix}.$$

These values mean all of MBI’s baselines are well approximated by considering the dominant contribution. To check this one calculates the numerical value in the Dominant contribution column of table 4.1 and compares it to the value in the Secondary contribution column.

4.4.1 Gain Errors

Because of MBI’s single mode optics in each element we find that $\gamma_2$ only has two terms instead of four, either $g_1$ or $g_2$. 
\[ \gamma_2 = \frac{1}{2} \left( g_{1,Q}^{(1)} + g_{2,Q}^{(2)*} - g_{1,U}^{(1)} - g_{2,U}^{(2)*} \right) \] (4.29)
\[ \frac{1}{2} \left( g_{1,Q}^{(1)} + g_{2,Q}^{(4)*} - g_{1,U}^{(1)} - g_{2,U}^{(4)*} \right) \] (4.30)
\[ \frac{1}{2} \left( g_{2,Q}^{(2)} + g_{1,Q}^{(3)*} - g_{2,U}^{(2)} - g_{1,U}^{(3)*} \right) \] (4.31)
\[ \frac{1}{2} \left( g_{1,Q}^{(3)} + g_{2,Q}^{(4)*} - g_{1,U}^{(3)} - g_{2,U}^{(4)*} \right), \] (4.32)

where we have identified antennas 1 and 3 as \( x \) polarization while 2 and 4 are \( y \) polarization. Also I’ve added a Q and U to the subscripts to g’s for those measurements.

Bunn assumes power scaling for Temperature, E-mode and B-mode to be 300\(^2\) : 300 : 1. From our table of the effects of instrument errors we see that \( \gamma_2 \) mixes EE power into B. If we would like to keep the systematics to the percent level \( 21 \gamma_2 < 0.01 \) or \( \gamma_2 < 5 \times 10^{-4} \).

What this means is that gain fluctuations need to be understood to this level: gain drifts of the bolometer array, gain variation from phase modulators (expected to be small), and phase errors from the optics (also expected to be small).

Currently there are no estimates on the gain stability of MBI’s bolometers, but there is data that should provide an estimate of that. Let us assume that they have a 1\% fluctuation that is not treated properly in our analysis. This leads to \( 21 \cdot 0.01 = 0.21 \) or a 21\% error in the B-mode power spectrum. We should be careful to state here that only gain drifts and fluctuations that are not accounted for lead to this. MBI does not need to limit gain drifts to these levels. We only need to understand the gain drifts at these levels and account for them properly in the analysis.
4.4.2 Coupling Between Polarization Modes

Coupling and Cross-Polarization are the most serious effects in that they mix TE power into E and B. $\varepsilon$ is our figure of merit and we find that MBI provides a small advantage due to its single mode optics because half of the terms in $\varepsilon$ are zero.

$$
\varepsilon_{12} = \frac{1}{2}(\epsilon_1^{(1)} + \epsilon_2^{(2)*}) \quad (4.33)
$$

$$
\varepsilon_{13} = \frac{1}{2}(\epsilon_1^{(1)} + \epsilon_1^{(3)*}) \quad (4.34)
$$

$$
\varepsilon_{14} = \frac{1}{2}(\epsilon_1^{(1)} + \epsilon_2^{(4)*}) \quad (4.35)
$$

$$
\varepsilon_{23} = \frac{1}{2}(\epsilon_2^{(2)} + \epsilon_1^{(3)*}) \quad (4.36)
$$

$$
\varepsilon_{24} = \frac{1}{2}(\epsilon_2^{(2)} + \epsilon_2^{(4)*}) \quad (4.37)
$$

$$
\varepsilon_{34} = \frac{1}{2}(\epsilon_1^{(3)} + \epsilon_2^{(4)*}) \quad (4.38)
$$

We see that $\varepsilon$ is the average unaccounted for angular error in the pair of polarizers.

Again if we wish to keep these errors to the percent level of the B signal then

$$
730\varepsilon = 0.01 \Rightarrow \varepsilon = 1 \times 10^{-5}. \quad (4.39)
$$

This corresponds to $5 \cdot 10^{-2''}$.

The restrictions for mixing TE into E is better

$$
17\varepsilon < 0.01 \Rightarrow \varepsilon < 6 \times 10^{-4}. \quad (4.40)
$$

This corresponds to $2''$.

The alignment of MBI’s polarizers is trusted to 1° or 0.017 radians. This corresponds to 1300% error for B-mode power spectra and 30% error for E-mode power spectra.
4.5 MBI in terms of A

Pointing errors with MBI arise from pointing errors when the sky horns are placed in the cryostat and bolted down. Since the optics are all bolted together these errors are maintained when MBI points at the sky. With careful beam scans of the four elements it should be possible to measure the different pointing orientations. The accuracy of these measurements is most likely to be limited by the noise on the bolometers signals making it difficult to find the maximum of the response. MBI has 16 bit encoders on the Azimuthal and Elevation axes. This corresponds to an angular resolution of $\sim 20^\prime$, however the current operating noise on MBI's bolometers introduces greater uncertainty in the measurement.

Beam shape errors for MBI come from imprecisely knowing the beam shape for a sky horn and the polarization it is sensitive to. This can be minimized by making raster scans with a single sky horn and mapping out its response carefully. Similarly to pointing errors, beam shape errors are ultimately limited by the noise in the bolometers. In practice time constraints limited the number and types of scans we were able to do, leaving MBI’s beams only minimally mapped.

A reasonable estimation of MBI’s cross-polarization has been left to others. The most practical approach to measuring MBI’s cross-polarization seems to be the observation of well characterized, strong polarized point sources, such as Tau A. A strong source is required so that it will be the dominant signal when observing on the edge of the antenna beam. Another approach would be to simulate MBI’s optical processing using actual beam maps to fine tune the simulation to match the actual response.
4.5.1 Pointing Errors

Pointing errors mix EE into B so to keep error down to the percent level

\[ 30\delta_- < 0.01 \Rightarrow \delta_- = 3 \cdot 10^{-4} \]  \hspace{1cm} (4.41)

This corresponds to 1\(^\prime\prime\).

There is no measurement of the relative alignment of MBI’s sky horns, however MBI’s sky horns are all bolted to the same ridged structure so the relative pointing error should not change by much between measurements of U and Q. This means that \( \delta_- = 1/2(\delta_Q - \delta_U) \) should be ideally be zero. Let us estimate that they are aligned to within 0.5\(^\circ\) or 0.009 radians. MBI has a FWHM of 7\(^\circ\) or 0.1222 radians therefore, if we assume a 1% change in the average pointing error between measurements equations 4.21, 4.22 and our table yields 0.05% error in the B-mode power spectrum.

4.5.2 Beam Shape Mismatches

\( \zeta_- \) is the fractional error in the beam shapes and it mixes EE into B. To keep the level we wish

\[ 42\zeta_- < 0.01 \Rightarrow \zeta_- = 2 \cdot 10^{-4}. \]  \hspace{1cm} (4.42)

So we would like 0.02% error in the beam shapes.

The beam maps for MBI’s individual sky horns show that there is a 1% difference between the beam shapes in the \( x \) and \( y \) orientations. Since we do not account for this it will produce 42% error in our B-mode power spectra.

4.5.3 Cross-Polarization

As mentioned earlier cross-polarization is a very dangerous effect. We are adopting the same simple form as Bunn. A careful treatment of propagating polarization
Table 4.3: Sources and estimated levels of error in power spectra for MBI.

<table>
<thead>
<tr>
<th>Error Source</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain(B)</td>
<td>21%</td>
</tr>
<tr>
<td>Coupling(B)</td>
<td>1300%</td>
</tr>
<tr>
<td>Coupling(E)</td>
<td>30%</td>
</tr>
<tr>
<td>Pointing(B)</td>
<td>0.05%</td>
</tr>
<tr>
<td>Beam Shape(B)</td>
<td>42%</td>
</tr>
<tr>
<td>Cross-Pol(B)</td>
<td>??</td>
</tr>
<tr>
<td>Cross-Pol(E)</td>
<td>??</td>
</tr>
</tbody>
</table>

from a curved sky and through the measured beam pattern should lead to a more exact formula for the cross-polarization.

For B-modes and our standard demands on the errors we find

\[ 730\mu = 0.01 \Rightarrow \mu = 1 \times 10^{-5}. \]  

(4.43)

For E-modes we find

\[ 17\mu < 0.01 \Rightarrow \mu < 6 \times 10^{-4}. \]  

(4.44)

4.6 Summary

Table 4.3 shows the percent error for the types and levels of uncertainty we laid out above. We can see that the alignment of the single mode optics for polarization selection is by far the most critical step for an experiment like MBI. Alignment should be checked before any other observations are made.

Gain and beam shape errors have approximately equal weight for our estimates of MBI. The gain uncertainty is currently unmeasured, though there is data from
which the gain fluctuations can be calculated and the level reduced. The beam shape likely have less contribution to the error. Before taking any data the beams of MBI’s sky horns were mapped. These maps are taken so that peculiarities of individual horns can be accounted for.

Unsurprisingly, MBI’s ridged optical system provides the smallest estimated error contribution. It should be noted that all of these error estimates represent unknown or unaccounted for errors. The demands of this formalism and analysis are only that the departures from the ideal are properly treated and accounted for to the specified level. MBI does not need to have perfect gains, polarization separation, pointing or beam shapes. We only need to know when and where MBI departs from these ideals and be mindful of them during analysis to the level specified.

4.7 Next Steps

We would like to have better estimates for MBI’s figures of merit. One source of information about them are beam maps of the elements. Chapter 6 covers both ideal case for mapping MBI’s beams, the critical step in determining its parameters for error estimates, and reality of the beam mapping for MBI’s first data run. In this section we will give a brief overview of the ideal procedure.

The time constraint for MBI’s observing run limited what could be done with respect to data taking and ultimately analysis. In an ideally funded world the next steps for MBI would be:

1. Take three rasters at different power levels and one orientation of each element.

2. Take one raster of each element at 90° to the first rasters.
3. Take one raster of each element on a polarized astronomical source, concentrating on the edges of the beams.

All measurements should be made with phase modulators running. From the different bias level rasters gains for the bolometers can be calculated. The scans taken at 90° to each other provide a measurement of coupling between polarizations. Beam maps for individual elements will give a best estimates for both pointing offsets and beam shape distortions. The final raster of the astronomical source will provide a best estimate for the cross-polarization of MBI’s beams.
Chapter 5

Phase Modulation

5.1 The Whys and Wherefores

Modulating the inputs of an interferometer helps to reduce noise and isolate the signals of interest. By using a $\pm 90^\circ$ phase modulation MBI is able to compare two signals while both are in phase and $180^\circ$ out of phase. This allows us to remove the total power and offset components of the signal. By operating these modulations at a sufficiently high frequency and using lock-in detection we are able to discard the $1/f$ noise in the atmosphere and instrument. Finally by using a modulation pattern that is more complex than a simple periodic modulation we can isolate visibilities from each of the $N(N - 1)/2$ baselines. In this chapter we will look at the last effect of modulation.

For the MBI the phase modulations are Walsh functions.\[33]\ The discussion here will be more general, allowing for multiple phase states, where Walsh functions require two states.
5.2 Describing Phase Modulation

The interferometry for MBI could be performed with either Butler or a stellar Michelson interference. For the MBI we ultimately chose a stellar Michelson approach. Both of these methods introduce fixed phase relations for each input at each detector. These phase relations are used to recover the full, complex, visibility for a given baseline as was discussed in Chapter 6. In this chapter we will focus specifically on how to identify baselines of interest. In addition to these fixed phase relations the MBI utilizes a time-dependent phase modulation of each input signal. For the stellar Michelson approach the fixed phase relations manifest as fringes on the focal plane. We will be using digital lock-in detection and so we will be sensitive to the changing phase modulation and not the fixed phase component.

Say we have \( n \) inputs \( E_1, \ldots, E_n \) entering our interferometer. We modulate them with time-dependent phase shifts \( \phi_1(t), \ldots, \phi_n(t) \), so that the output incident on a particular detector after the interferometer is

\[
E_{\text{out}} = E_1 e^{i\phi_1(t)} + E_2 e^{i\phi_2(t)} + \ldots + E_n e^{i\phi_n(t)}.
\] (5.1)

We change the phase shifts \( \phi_j \) in discrete time steps, and the cycle of phase shifts repeats itself after \( m \) steps. So instead of \( \phi_j(t) \) we can write \( \phi_{jt} \), with \( t = 1, 2, \ldots, m \). Also, let’s assume that the phase shifts are all multiples of 90°, so there are only four possible values for \( \phi_{jt} \):

\[
\phi_{jt} = 0, \pi/2, \pi, 3\pi/2,
\] (5.2)

or

\[
e^{i\phi_{jt}} = 1, i, -1, -i.
\] (5.3)
We will denote these as [0123] for 0, π/2, π, 3π/2 or 1, i, −1, −i.

The relationship between the number of phase steps in 360 degrees, $N$ (in this case 4) and the number of shifts, $m$, in the modulation sequence can be important. In a later section we will see how restricting the relationship to $m = N^x$, where $x$ is an integer, can be very handy.

For an interferometer which mixes all incoming signals, such as a Butler combiner or the stellar Michelson combiner, the (non-squared) beam combiner output incident on a detector is

$$E_{\text{out}}^a(t) = \sum_{j=1}^{n} E_j e^{i\phi_j t} e^{i\delta_j a}. \quad (5.4)$$

The first exponential is the phase shift we apply to the inputs, and the second is the fixed phase shift that each input incurs while traveling to detector $a$. Since the second phase shift is constant it can be accounted for post detection and will be ignored for the rest of this discussion.

The signal we actually measure at the detectors is the absolute square:

$$\langle |E_{\text{out}}^a|^2 \rangle = \sum_{j,k} \langle E_j E_{k*}^* \rangle e^{i(\phi_j t - \phi_k t) + i\delta_{a(j-k)}}$$

$$= \sum_j \langle |E_j|^2 \rangle + \sum_{j<k} \text{Re} \left( \langle E_j E_{k*}^* \rangle e^{i(\phi_j t - \phi_k t) + i\delta_{a(j-k)}} \right). \quad (5.5)$$

So the visibility associated with horns $j$ and $k$ shows up in the $a$th output with two phase factors: the time-dependent phase difference $\phi_j t - \phi_k t$, and the time-independent term $\delta_{a(j-k)}$. Since the second phase shift is constant it can be accounted for post detection and will be ignored for the rest of this discussion.

For an interferometer the quantities of interest are visibilities, which arise from interfering the signals from two inputs. Since we are interfering two signals the modulations of the inputs are not the quantities we are ultimately interested in. We would
like to know the relative phase difference between two inputs. If two inputs both have their phases shifted by $90^\circ$ then the relative phase is $0^\circ$. It is the $0^\circ$ which is important to keep track of for interference and not the $90^\circ$. We will refer to this as the modulation interference.

Since we will be dealing with these modulation sequences quite a bit and some of them can be very long we will introduce a short-hand notation for the patterns. As an illustrative example we choose a modulation pattern with 4 possible states and a sequence of $m = 16$ shifts. We modulate the first input by stepping through the phases as slowly as possible within the $m$ steps. The second input is modulated 4 times as quickly.

$$\vec{\phi}_1 = [0000111122223333], \quad (5.7)$$
$$\vec{\phi}_2 = [0123012301230123]. \quad (5.8)$$

With our requirement that $m = N^x$ we will always be able to step through the phases with the same number of points dedicated to each available state. It also means that we can construct $x$ modulations, each cycling through the available phases $N$ times quicker than the last. In the above example $16 = 4^2$, so a modulation 16 steps long will let us create 2 modulations, one of which cycles 4 times faster than the other. These particular modulation patterns are of interest because of how we construct our short-hand notation.

We shall be constructing a number in modulo $N$. A modulo $N$ system is similar to a base $N$ system. Let us take $N = 10$ as an example. In base 10, when counting past 10 in the one’s digit we add a count to the next digit, reset to 0 and continue:
6 + 6 = 12. In modulo 10 each digit is independent. When the count in the one’s digit passes 10 it resets to 0 and we continue, but we do not add a count to the next digit: 6 + 6 = 2.

To differentiate between base ten numbers and our modulo $N$ notation we will write them in angle brackets ⟨⟩ and separate decimal places with a comma. In the same way that $12 = 10 + 1 + 1$, the modulation ⟨1, 2⟩ = ⟨1, 0⟩ + ⟨0, 1⟩ + ⟨0, 1⟩. Modulations ⟨0, 1⟩ and ⟨1, 0⟩ are the basis for a two digit modulation. There are 4 possible states for this modulation so we will be working in modulo 4. $\vec{\phi}_1$ steps through the 4 states once during the modulation. This will be the lowest basis ⟨0, 1⟩ = $4^0$. $\vec{\phi}_2$ steps through the 4 states 4 times during the modulation and is the next basis ⟨1, 0⟩ = $4^1$. The bases are easily identifiable since each makes $N^x$ oscillations during the modulations. However, ⟨0, 2⟩ is not the modulation that makes two oscillations. ⟨0, 1⟩ + ⟨0, 1⟩ = ⟨0, 2⟩, but in this case that looks like

$$\langle 0, 1 \rangle + \langle 0, 1 \rangle = \langle 0, 2 \rangle$$

$$[000011122223333] + [000011122223333] = [0000222200002222].$$

Since we are adding in modulo 4 and have started counting at 0, 2 + 2 = 4 = 0 and each digit is independent. i.e. 3 + 3 = 2 ≠ 12. A related consequence of requiring that $m = N^x$ is that there are $x$ of these basis modulations. The modulo $N$ nature of the phase shifts also carries over to the modulations so that

$$\langle 3, 3 \rangle + \langle 2, 2 \rangle = \langle 1, 1 \rangle$$

$$[0321321021031032] + [0202202002022020] = [0123123023013012].$$
This is important for the phase shifts because they are a modulo $2\pi$ system, meaning that if you add $3\pi$ of phase you end up with only $\pi$ of phase difference.

### 5.3 Selecting Input Modulations

Now suppose we have more than two horns ($h > 2$). We’ll have a total of $h(h-1)/2$ visibilities. For Butler and stellar Michelson systems the visibility associated with inputs $j$ and $k$ shows up in the output modulated in phase by

$$\Delta \vec{\phi}_{jk} = \vec{\phi}_j - \vec{\phi}_k. \quad (5.9)$$

Since these are all mutually orthogonal, they form a maximal set of orthogonal vectors of the $m$-dimensional space. We want all of our phase shifts to be orthogonal to each other, so we should try pulling all of our phase shifts from this set.

Let’s see how this works out in the case $x = 2$ first. We might as well let our first vector be the one with no phase shifts at all:

$$\vec{\phi}_0 = [0000000000000000] = \langle 0, 0 \rangle. \quad (5.10)$$

We can get two more vectors by choosing the two we’ve already mentioned.

$$\vec{\phi}_1 = \vec{\alpha}_1 = [0000111122223333] = \langle 0, 1 \rangle, \quad (5.11)$$

$$\vec{\phi}_2 = \vec{\alpha}_2 = [0123012301230123] = \langle 1, 0 \rangle. \quad (5.12)$$

Can we get a fourth vector out? Here’s how to find out. The three vectors we have so far give us three different baselines. The pattern of phase shifts associated with each of those baselines is just the difference between the two input phases. For example, the baseline associated with horns 1 and 2 has a phase shift pattern

$$\Delta \vec{\phi}_{21} = \vec{\phi}_2 - \vec{\phi}_1 = \langle 1, 0 \rangle - \langle 0, 1 \rangle = \langle 1, -1 \rangle = \langle 1, 3 \rangle. \quad (5.13)$$
The baselines that we have so far, then, are

\[
\Delta \tilde{\phi}_{10} = \langle 0, 1 \rangle, \quad (5.14)
\]
\[
\Delta \tilde{\phi}_{20} = \langle 1, 0 \rangle, \quad (5.15)
\]
\[
\Delta \tilde{\phi}_{21} = \langle 1, 3 \rangle, \quad (5.16)
\]

If we want to add a fourth input, we need to make sure that the four additional interference modulations that result are distinct from these. We take the above three baseline phase shifts \( \Delta \tilde{\phi} \), and add them to each of the three phase vectors \( \tilde{\phi} \). Each of the resulting nine vectors will be ruled out as a potential \( \tilde{\phi}_4 \), since it would give a new baseline that was redundant with one we already have. Any vector that’s not ruled out by this process can be added to the list we’ve already got, and we can repeat the procedure again.

If you go through that exercise, you find that \( \langle 1, 2 \rangle, \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle \) are all candidates.

### 5.4 Two State Modulation

The MBI uses a two state phase shifter \( (\pm 90^\circ) \). The procedure for determining modulations with two states is the same as for four states. Having only two states simplifies some of our modulo math since \( \langle 0, 1 \rangle + \langle 0, 1 \rangle = \langle 0, 0 \rangle \). We still build a basis of modulations. In this case we will need a three digit, base two number in order to have unique modulation interferences for each baseline. Like before we start with

\[
\tilde{\phi}_1 = \langle 0, 0, 1 \rangle \quad (5.17)
\]
\[
\tilde{\phi}_2 = \langle 0, 1, 0 \rangle \quad (5.18)
\]
\[
\tilde{\phi}_3 = \langle 1, 0, 0 \rangle. \quad (5.19)
\]
For MBI we wish to have the inputs modulated at all times so we are not using \( \langle 0, 0, 0 \rangle \) as a modulation in this case. Subtracting the modulations in binary is the same as the XOR operation. We now construct the modulation interferences.

\[
\begin{align*}
\Delta \vec{\phi}_{12} &= \langle 0, 1, 1 \rangle, \\
\Delta \vec{\phi}_{23} &= \langle 1, 1, 0 \rangle, \\
\Delta \vec{\phi}_{13} &= \langle 1, 0, 1 \rangle,
\end{align*}
\]

XORing these with our input modulations produces \( \langle 0, 0, 1 \rangle, \langle 0, 1, 0 \rangle, \langle 1, 0, 0 \rangle, \) and \( \langle 1, 1, 1 \rangle \). The MBI’s fourth horn modulation can be chosen from \( \langle 0, 1, 1 \rangle, \langle 1, 0, 1 \rangle, \) or \( \langle 1, 1, 0 \rangle \). Let’s select
\[\vec{\phi}_4 = \langle 1, 1, 0 \rangle \]
(5.23)
as our fourth input modulation. The resulting modulation interferences are unique and therefore, by locking into these \( \Delta \phi \) patterns we can recover the signal from any of MBI’s six base lines we wish.

\[
\begin{align*}
\Delta \vec{\phi}_{14} &= \langle 0, 1, 0 \rangle, \\
\Delta \vec{\phi}_{24} &= \langle 0, 0, 1 \rangle, \\
\Delta \vec{\phi}_{34} &= \langle 1, 1, 1 \rangle,
\end{align*}
\]

Figure 5.1 shows these input modulations and interference modulations.
Figure 5.1: On the left are the phase modulation pattern for MBI’s inputs. On the right are the resulting phase modulations after interference.
5.5 The Grid

We have not described a modulation scheme which allows us identify any baseline we wish. For interferometer systems with densely packed input horns this leaves something to be desired. In a densely packed system there are likely to be redundant baselines. If readout two of these baselines individually each measurement has the same noise. If these redundant baselines could be co-added at the time of detection then the signal will still be twice as large, but the noise will not increase from the single detection because we only read the detectors once. For a 4x4 array of horns there can be a factor of twelve increase in the SNR for the shortest baselines. This assumes that we already have a stellar Michelson interferometer with a focal plane of bolometers. Compared to a system with one bolometer per baseline, this arrangement produces a \( \sqrt{N} \) increase in the signal to noise where \( N \) is the number of antennas co-added. This is because the signal strength scales up as \( N \), while the noise on a bolometer scales as \( \sqrt{N} \).

Let’s now discuss how this notation can be applied to a physical arrangement which increases signal to noise. We will return to using 4 phase states with \( 4^2 \) modulation steps in our modulation. Again in our notation these modulations will be denoted by two digit numbers in base 4 which are written \( \langle 0,0 \rangle \), \( \langle 2,2 \rangle \) etc. The modulation pattern resulting from interfering two horns is the sum of the horn modulation numbers modulo 4. For example, say horn 1 has modulation \( \vec{\phi}_1 = \langle 2,3 \rangle \) and horn 2 has modulation \( \vec{\phi}_2 = \langle 1,3 \rangle \). The resulting modulation pattern when \( \vec{\phi}_1 \) and \( \vec{\phi}_2 \) are interfered is \( \vec{\phi}_1 - \vec{\phi}_2 = \Delta \vec{\phi}_{12} = \langle 1,3 \rangle - \langle 2,3 \rangle = \langle -1,0 \rangle = \langle 3,0 \rangle \).

If we arrange horns in a 4x4 grid and modulate them in such a way that the
first digit of the modulation is the column designation and the second digit is the row designation we get something like this:

\[
\begin{array}{cccc}
\langle 0,0 \rangle & \langle 1,0 \rangle & \langle 2,0 \rangle & \langle 3,0 \rangle \\
\langle 0,1 \rangle & \langle 1,1 \rangle & \langle 2,1 \rangle & \langle 3,1 \rangle \\
\langle 0,2 \rangle & \langle 1,2 \rangle & \langle 2,2 \rangle & \langle 3,2 \rangle \\
\langle 0,3 \rangle & \langle 1,3 \rangle & \langle 2,3 \rangle & \langle 3,3 \rangle \\
\end{array}
\]

This matrix assumes a rectangular array with uniform spacing \( D \) in \( \hat{x} \) and \( \hat{y} \) and the modulation pattern for each horn in the arrangement (produced by adding \( \langle 1,0 \rangle \) and \( \langle 0,1 \rangle \) in modulo 4 the appropriate number of times). We notice that the difference between any two horns that are adjacent vertically is \( \langle 0,1 \rangle \) and any two horns that are adjacent horizontally is \( \langle 1,0 \rangle \). This means that using lock-in detection to pick out a modulation interference of \( \langle 1,0 \rangle \) will pick out all signals from baselines of separation \( D \) in the \( \hat{x} \) direction.

This arrangement leads to all baselines of the same length and orientation sharing a modulation interference pattern. The signal that comes from locking-in to, say, the \( \langle 3,0 \rangle \) modulation interference pattern will have contributions from the pairs \( \langle 3,0 \rangle \langle 0,0 \rangle; \langle 3,1 \rangle \langle 0,1 \rangle; \langle 3,2 \rangle \langle 0,2 \rangle \); and \( \langle 3,3 \rangle \langle 0,3 \rangle \). The problem is that the adding or subtracting in modulo 4 allows you to “wrap around” the edges of the array. Let’s take the \( \langle 3,0 \rangle \) interference modulation again. \( \langle 1,0 \rangle - \langle 2,0 \rangle = \langle -1,0 \rangle = \langle 3,0 \rangle \). So the \( \langle 3,0 \rangle \) modulation will pick out all horizontal baselines with length \( 3D \) and \( 1D \). Perhaps this shouldn’t come as a surprise since modulo 4 means that \( -1 = 3 \) and the baseline from \( \langle 1,0 \rangle - \langle 2,0 \rangle \) can be thought of as having length \( -1D \) while \( \langle 2,0 \rangle - \langle 1,0 \rangle \) has length \( 1D \).
5.6 A Solution to the Wrapping Problem

We shall employ a simple, if perhaps inelegant, solution. Because of our modulo-4 adding and subtracting, if we use all possible arrangements of our modulation we get “cross talk”, recovering signals from baselines we are not interested in. The solution is to not use all possible arrangements. If we are using 4 phase states but only implement a 3x3 horn grid we no longer have cross talk.

\[
\langle 0, 0 \rangle \; \langle 1, 0 \rangle \; \langle 2, 0 \rangle \\
\langle 0, 1 \rangle \; \langle 1, 1 \rangle \; \langle 2, 1 \rangle \\
\langle 0, 2 \rangle \; \langle 1, 2 \rangle \; \langle 2, 2 \rangle
\]

In general, for a grid system like this a modulation scheme with \( N \) phase states available will have \( \lfloor \frac{N}{2} + 1 \rfloor \) useable phase states. Here \( \lfloor \rfloor \) denotes truncating to the nearest integer. One way to think of this is if we have a circle with \( N \) points spaced evenly around it, and only take the first \( \lfloor \frac{N}{2} + 1 \rfloor \) of those points what we would be left with are the points at angles less than or equal to 180°. The result is that if you subtract two randomly selected angles the result will always be between +180° and -180°. This avoids the above problem because we are no longer left with two equivalent angles (e.g. 270 and -90). This angle addition is exactly what’s happening when two modulations are interfered.

Another way to look at the useable phase states is to ask how many total phase states are needed to have \( U \) useable phase states. Inverting the equation yields: \( N = 2(U + 1) \) or \( N = 2(U + 1) + 1 \). There are two equations because of \( \lfloor \rfloor \). Using \( N = 2(U + 1) + 1 \) means that the results cover less than 360°, which removes the last ambiguity between +180 and -180.

This scheme allows one to achieve a much higher signal to noise ratio, especially
for shorter baselines which have the most redundancies, than individually tagging every baseline and reading them out separately. Also, length of the baseline is proportional to digits of the modulation added in quadrature. For example an interference modulations of $\langle 2, 1 \rangle$ and $\langle 1, 2 \rangle$ would add

$$\sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5}$$

and both select baselines of length $(5 \cdot D)^{1/2}$. Therefore, the $l$-mode of interest is encoded in the lock-in modulation notation.

The size of the horn grid can be increased to 5x5 if phase shifts are evenly spaced at 45° instead of 90°. e.g. 8 phase steps to move around the unit circle of the complex plane. With arbitrary precision in phase steps a $UxU$ grid can be created with $2(U + 1) + 1$ evenly spaced phase steps.

### 5.7 Expanding without more phase steps

If it is possible to add another digit to the modulation (for 4 modulation states this means using a modulation scheme with four times as many steps in it, for 8 modulation states it would mean a scheme that is eight times as long) and extract the baselines of interest through careful book keeping, then the size of the grid can be doubled from 3x3 to 6x6. If we label the original modulation square $\langle M \rangle$ then you could describe a 6x6 block as

$$\langle M0 \rangle \langle M1 \rangle \langle M2 \rangle \langle M3 \rangle$$

Locking into $\langle 0, 1, 0 \rangle$ will select most baselines of length $D$ and oriented in $\hat{y}$. You will miss the ones that are between $\langle M0 \rangle$ and $\langle M1 \rangle$ as well as those between $\langle M2 \rangle$ and $\langle M3 \rangle$. For those you would need to lock-in to $\langle 0, 0, 1 \rangle - \langle 2, 0, 0 \rangle = \langle 2, 0, 1 \rangle$. 

This means that you have less S/N than you would if you could pick out every baseline of a given length and orientation. Here are all the quadrants:

\[
\begin{align*}
\langle 0,0,0 \rangle & \quad \langle 1,0,0 \rangle & \quad \langle 2,0,0 \rangle & \quad \langle 0,0,1 \rangle & \quad \langle 1,0,1 \rangle & \quad \langle 2,0,1 \rangle \\
\langle 0,1,0 \rangle & \quad \langle 1,1,0 \rangle & \quad \langle 2,1,0 \rangle & \quad \langle 0,1,1 \rangle & \quad \langle 1,1,1 \rangle & \quad \langle 2,1,1 \rangle \\
\langle 0,2,0 \rangle & \quad \langle 1,2,0 \rangle & \quad \langle 2,2,0 \rangle & \quad \langle 0,2,1 \rangle & \quad \langle 1,2,1 \rangle & \quad \langle 2,2,1 \rangle \\
\langle 0,0,2 \rangle & \quad \langle 1,0,2 \rangle & \quad \langle 2,0,2 \rangle & \quad \langle 0,0,3 \rangle & \quad \langle 1,0,3 \rangle & \quad \langle 2,0,3 \rangle \\
\langle 0,1,2 \rangle & \quad \langle 1,1,2 \rangle & \quad \langle 2,1,2 \rangle & \quad \langle 0,1,3 \rangle & \quad \langle 1,1,3 \rangle & \quad \langle 2,1,3 \rangle \\
\langle 0,2,2 \rangle & \quad \langle 1,2,2 \rangle & \quad \langle 2,2,2 \rangle & \quad \langle 0,2,3 \rangle & \quad \langle 1,2,3 \rangle & \quad \langle 2,2,3 \rangle
\end{align*}
\]

Let’s say I want to pick out a baseline of length \(5D\). I’ve bolded \(\langle 0,1,0 \rangle\) and \(\langle 2,1,1 \rangle\) as an example of a baseline with that length. To pick out this baseline you need to lock-in to a modulation interference pattern of \(\langle 2,1,1 \rangle - \langle 0,1,0 \rangle = \langle 2,0,1 \rangle\). You’ll notice that the \(\langle 2,0,1 \rangle\) interference modulation will work between horns in the 2\(^{nd}\) and 3\(^{rd}\) quadrants as well.

I haven’t looked at this in a while so I will need to see if there are any other overlaps or cross talk in this arrangement. Also I remember talking with Ted at one point and being unsure as to whether a phase shifting scheme can work with steps that are not multiples of 90\(^{\circ}\). Our current phase shifting scheme is \(\pm 90^{\circ}\) and Ted’s example was 0, 90, 180 and 270. If we can’t mathematically step at arbitrary phase angles then we won’t be able to achieve as high S/N as we would like.

### 5.7.1 Phase Shifter Requirements

This phase shifter grid places some requirements on the phase shifters used. The phase shifters must be able to apply the incremental phase shifts discussed in this chapter. They must be able to switch from one phase shift to any other phase shift quickly. Perhaps most important for large phase shifter grids, the amount of phase shift applied must be stable and accurate. This constraint may be the most
challenging for designer as this accuracy scales with the size of the horn array, while the others do not.
Chapter 6

Beam Mapping

6.1 Beam Mapping in Theory

Beam mapping is the process of determining how MBI processes a signal from the sky onto the bolometers. The intent is to find out as much as possible about the beam patterns of the sky horns, which we should know well in the far field limit, the internal horns, which operate in the near field, and the individual gains of the bolometers, which are unknown. We wish to know how these will affect the fringes from the 6 baselines. A feature of MBI’s corrugated sky horns is that they have very low side lobes; we would also like to verify that. These measurements are also used to characterize how well we understand MBI’s errors and distortions. As we laid out in Chapter 4, the better we understand these the smaller effect they can have on power spectra estimation.

The beam mapping is done using a Gunn oscillator from the Electron Dynamics division of Hughes Aircraft Company, Model: 45726H-1000, that is tuneable in both frequency and power output, and can be amplitude modulated at low frequency, using a Millitech Gunn Modulator Regulator and a Stanford Research Systems function
Figure 6.1: MBI’s calibration mast in its old home on top of Engineering Hall.

generator, Model DS335, to provide the modulation signal. The frequency tuning was not used during beam mapping, but could be used for measuring the frequency response of MBI. The Gunn transmits into an over-moded waveguide pipe that runs the length of a 35 ft. [10.7 m] ham radio mast. Figures 6.1 and 6.2 show the mast. At the top of the mast the over-moded waveguide transitions back to single mode waveguide and emits downwards at an angle of 45° to the vertical through a pyramidal horn with gain that is to be determined.

The cryogenic systems of MBI require that the elevation angle be > 45°. With MBI positioned 10.7 m from the mast the distance to the top of the mast is $\sqrt{2} \times 10.7 = 15.1$ m. A source is considered to be in the far-field limit if it is at a distance greater than $D^2/\lambda$ away from an interferometer, where $D$ is the length of the baseline and
\( \lambda \) is the wavelength being observed. For MBI the longest baseline is 20 cm and our observing wavelength is 3 mm. This gives a far-field distance of \( 0.2^2 / 0.003 = 13.33 \) m. This means that the source on the mast is in the far-field for the longest baseline and therefore for all of the baselines.

### 6.1.1 Raster at Different Power Levels

The first step of the beam mapping procedure is mapping the beams for each element (sky horn and internal horn) separately. Three of the horns apertures are blanked off using conductive aluminum tape. During the mapping the phase modulator is set in the open condition. In principle no light should make it through gaps in the conductive tape, but keeping the phase modulators in the three taped elements turned off will help reduce any signal that may have found its way through gaps in the tape.
This procedure only requires that the phase modulator be on and transmitting through to the internal horn. The source is amplitude-modulated so that the signal can be locked into.

Ideally the source used to map the beams should be polarized. The orientation of polarization passed by the element will be aligned parallel to the polarization of the source. MBI is then rastered across the source. This will give us a measure of the response of the sky horn to its transmitted polarization across the beam.

The intensity of the source is then adjusted and another raster taken. At least three rasters at different intensities should be taken. This is done so that we can verify the linearity of the bolometers in various portions of the beam pattern, i.e. the highest power that will probably saturate the bolometers at the center of the beam, but will allow for side lobe measurements.

When the bolometer responses at different power outputs are compared at a constant pointing the gains for the bolometers can be determined. Knowing the relative strengths of the three or more source power settings we can fit the individual bolometer gains and check the linearity. If the power, \( p \), between two rasters changed by a factor of \( m \) then we expect the bolometer response \( res \) to follow \( res = mp + o \), where \( o \) is an offset. A departure from this form means we have moved out of the linear range of the bolometers. In principle many measurements could be taken and the response as a function of power mapped out explicitly. However, this is very time consuming and we would be better served to make sure that we do not leave the linear region of the bolometer response.
6.1.2 Raster Perpendicular to Source Polarization

MBI is now rotated 90° so that transmission axis of the polarization passed by MBI’s optics is perpendicular to the polarization of the source. MBI is then rastered again to obtain the element’s coupling between \(x\) and \(y\) polarization and determine \(\varepsilon\) from Chapter 4. Estimations for \(\varepsilon\) should be made by looking at relative strengths of signal between transmitted orientation of polarization and the nominally rejected polarization at the center of the beam. We do not want to confuse coupling with cross-polarization so we should stay near the center of the beam.

After the bolometer gains have been determined we can adjust the data from the raster scans and create reliable maps of the beams for sky horns and internal horns. Plotting the response of a single bolometer as function of pointing angle produces the beam pattern for the sky horn. Optimally the bolometer used for this would be the one with the largest change in response as a function of angle, though all bolometers should have the same shape of response. Also, plotting the response for all bolometers as a function of focal plane position while the source is in the center of the beam will produce a beam pattern for the internal horn. The internal beam could be mapped with the source away from the center, but this would give us less dynamic range in the bolometer responses.

It is most convenient to roll both the sky and internal beam maps into one. With a raster taken it is just as easy to map out each individual bolometer’s response as a function of angle (Az and El). Since we are keep track of both angle (sky horn beam) and bolometer position (internal horn beam) we create a beam map that includes the effects of both horns. Separating the sky and internal beams will be valuable for
determining where problems with the beams are coming from. However, if we are confident in our understanding of them we can use the combined beam.

6.1.3 Astronomical Source

In the interest of measuring the cross-polarization for MBI we would then make a scan of a polarized astronomical source. A strong source would be preferable because we would be concentrating on keeping the source in the edges of the beam during the scan. This data could be taken before the beams were analyzed and then corrected after the beams were determined. However, without a good idea of the size of the beams we would be trusting to luck that we would cover the angles needed in the scan.

6.1.4 Recovering Visibilities

The bolometer gains and beam patterns are used to recover visibilities from MBI’s data. By using the bolometer gains and the beam patterns for two elements we can create a gain mask, $M$, for a single baseline. This gain mask corrects for differences in the responsivities of the individual bolometers and the shape of the envelope of the fringe response. In an ideal case we would take a full raster of each baseline and apply a digital lock-in to all of the bolometers to recover the signal from the baseline of interest. We then apply $M$ to correct for the bolometer and optical effects. In a perfect case all that will be left are points sampled from a sinusoid of constant amplitude and spatial frequency, and with a phase offset. We then take the bolometer response and fit it to a 3D plot of response vs. $\hat{x}$ and $\hat{y}$. The spatial frequency and orientation of the fringes are determined by the baseline, but the amplitude and phase
offset are free parameters. We fit these two parameters to the data for a given baseline and from them we can recover the complex visibility for that time and baseline.

$$M(x, y)A \sin \left( \frac{2\pi}{\lambda} \left( \cos x(\theta) - \sin y(\theta) \right) + \phi \right)$$  \hspace{1cm} (6.1)

Here $M$ is the bolometer spatial response mask, $A$ is the amplitude of the signal, $\lambda$ the wavelength of the fringe on the focal plane, $\cos x(\theta) - \sin y(\theta)$ is a coordinate rotation and $\theta$ the rotation angle of the fringe (i.e. $\theta = 0$ means the oscillations happen in the $\hat{x}$ direction.), and $\phi$ is the phase offset of the fringe. In MBI’s optical system all but $A$ and $\phi$ are set by the geometry.

Let $z = \text{complex visibility} = x + iy$. Let $\phi_1$ and $\phi_2$ be two phase shifts that correspond to two different complex numbers as follows:

$$e^{(i\phi_1)} = a_1 + ib_1,$$  \hspace{1cm} (6.2)

$$e^{(i\phi_2)} = a_2 + ib_2.$$  \hspace{1cm} (6.3)

Then, one needs to weigh $z$ by these two different phase factors to get two outputs at two different bolometer positions. But the outputs are only real parts of the multiple:

$$O_1 = \text{Re}[(x + iy)(a_1 + ib_1)]$$  \hspace{1cm} (6.4)

$$O_2 = \text{Re}[(x + iy)(a_2 + ib_2)]$$  \hspace{1cm} (6.5)

Knowing $O_1$, $O_2$, $a_1$, $a_2$, $b_1$ and $b_2$, we can solve these two equations to get a unique value for $z$. We will not be solving these equations here.

We could fit the three-dimensional data, but plotting the data and fit on the same graph is cluttered and difficult to read. Instead we will rotate the bolometer positions by $-\theta$ to align the fringes with the $x$-axis. At this point the $y$ position data
is superfluous so we discard it, leaving us with a two-dimensional plot that is easier to fit and read. We now plot a projection of the fringes onto a single axis. We will now be fitting to:

$$V = M(x')A \sin\left(\frac{2x'\pi}{\lambda} - \phi\right) + O,$$

(6.6)

where $O$ is an offset we expect it to be small.

### 6.2 Beam Mapping in Practice

During MBI’s data run we were very limited in time and there were periods when MBI didn’t have its full functionality. This meant that the thorough procedure outlined above was not feasible. Instead of the two-dimensional procedure described above we worked with one-dimensional scans in azimuth. Time constraints also meant that we were not able to make the two scans needed to determine $\varepsilon$. Our procedure is below.

#### 6.2.1 Individual Rasters

Scans in azimuth were made of elements 1 and 2. These two elements are sensitive to different polarizations, $x$ for 1 and $y$ for 2. MBI was oriented such that the transmission axes of the two elements were rotated $45^\circ$ from the polarization axis of the source. The scans were taken with the phase modulator fixed open, but not modulating, and the source being amplitude-modulated.

Figures 6.3 and 6.4 show the response of working bolometers as a function of azimuthal angle for horns 1 and 2 respectively. This four-dimensional data (position on focal plane (2), angle (1) and response (1) ) is what we use to determine the combined beam map. We expect horns 1 and 2 to have the same beam patterns, but
Figure 6.3: Bolometer response as a function of azimuthal angle for element 1 of MBI. Bolometer response depends on location in the focal plane and bolometer responsivity.
Figure 6.4: Bolometer response as a function of azimuthal angle for element 2 of MBI.
can see that they have very different beam patterns. There are a number of possible explanations for this.

The alignment of the polarizers to the source could have been misjudged, resulting in horn 1 being aligned with the source and horn 2 at 90°. In this case horn 2 is seeing low levels of polarization mixing. At the level of ~10% in ε we see this would mean 7300% error in the B-mode power spectrum assuming that this error is dominant and unaccounted for.

It is also possible that a portion of the teflon guides used to help seat the optics in the cryostat was shaved off and lodged in the throat of sky horn 2. This could conceivably result in drastically reduced sensitivity to the sky and changed beam pattern, both of which are seen. This level of beam deformity is not addressed in chapter 4. For the sake of having a number to look at let us assume this effect could be loosely modeled as ζ = 2 or 200% error in the beam. That means if we naively analyze this data assuming a Gaussian beam shape we will end up with 8400% error in the B-mode power spectrum.

There may also be light leaking around portions of the optics. For example, stray light coming around the sky horns and into the vacuum cavity, or light leaking out of the phase modulators or light that is nominally trapped between the cryostat cans could find its way to the focal plane. This would not explain the reduction in sensitivity between element 1 and 2, but assuming the signal through element 2 is very small it may explain the pathological shape of the beam. Again this error cannot be easily expressed in the formalism of the previous chapter. However, these spurious responses can be eliminated in the future by using the phase modulator to modulate
the signal. After locking-in to the phase modulator we are only sensitive to portions of the signal which passed through the modulator, which does not include any of the problems mentioned.

### 6.2.2 Fringe Envelope Determination

To determine the envelope in which the fringes will show up we multiplied the two beam patterns together and then take the square root of the values. This is shown in Figure 6.5. This is because the beam patterns map out response in $E^2$ so the multiplication of the beam patterns is proportional to $E^4$, while the interference fringes are still just proportional to $E^2$. 
After taking scans of horns 1 and 2 separately we now take a scan in azimuth with both horns 1 and 2 open, (Figure 6.6). The interference fringes are apparent. We can see that the maximum of the fringes is bounded by an envelope which resembles a Gaussian, but is clearly of a different shape. We can now divide the fringe by the effective beam. This step is the same as applying the mask $M$. Compared to the results of not removing the beam shapes, when we remove the effective beam, Figure 6.7, or just horn 1’s beam from the fringe, Figure 6.8, or just horn 2’s, Figure 6.9, we see that we do recover sinusoids that have a roughly constant amplitude as a function of azimuth when we remove a beam shape.

We can see that the beam pattern for horn 1 alone is the best fit for the interference data. This points to us not understanding beam pattern for horn 2 very well, which seem likely. Since horn 1’s beam is close to a Gaussian and the product of two
Figure 6.7: Fringe with the effective beam from elements 1 and 2 removed.

Figure 6.8: Fringe with just the beam from element 1 removed.
Gaussians is a Gaussian it may be the case that horn 2’s beam pattern is closer to a Gaussian than we thought. Perhaps there was water or ice on the window during the beam scan which distorted the beam, but had been removed before the interference scan. We will be using horn 1’s beam as the effective beam during our analysis.

6.2.3 Recovering a Sinusoidal Fit

Now we will take one azimuthal angle, 273°, and read off the bolometer responses across the focal plane. We then arrange an $x - y$ grid of the bolometers with their response as a $z$ component, rotate the fringe back onto the $x$-axis, discard the $y$ data and try to fit the sinusoid we defined earlier to it, with the exception that we will not be applying the mask $M$ because that was taken care of when we divided out the beam. We are fitting all the variables $A$, $\phi$, $\lambda$, and $O$. It should be noted here
Figure 6.10: Overlay of the interferometric elements, mirrors and bolometer horns. Also shown are the spacings and positions in inches.

that implicit in our assumptions is that the bolometer gains are all 1. This is almost certainly not the case, but we have not analyzed the data which would provide us with gain estimations.

For the 12 baseline the fringe is rotated by 11.85° or 0.207 radians. This is found by calculating the angle the two horn positions make with the x-axis as defined in Figure 6.10. In Figure 6.10 we have set the center bolometer at (0, 0). Based on the arrangement of the elements and bolometers we expect the original center of the fringe to be at a position of (0.4585", 2.0845") or 0.9" in x after rotation.

We see in Figure 6.11 that we have locked-into the inverse of the signal we
Figure 6.11: Squares are bolometer response data for Az of 273. Red line is a non-linear fit to a sine function.

desired. We expected a peak at 0.9" as it is the center of the baseline, but we see a minimum close to that point. Also based on the beam maps 273° should be just about in the center of the beam so we expect very little phase shift in the fringes from that. This data is consistent with an inverted fringe centered where we expect and means that our assumption on page 43 was wrong.

From this data we find

\[ M(x')A \sin\left(\frac{2x'\pi}{\lambda} - \phi\right) + O \]  \hspace{1cm} (6.7)

\[-1.44 \sin\left(\frac{2x'\pi}{1.4} - 2.64\right) + 0.36 \]  \hspace{1cm} (6.8)

We have shown that we can reconstruct a fringe pattern from the raw bolometer data and, in principle, estimate the parameters needed to determine a visibility.
Chapter 7

Conclusion

The last century has seen vast changes in the field of Cosmology. We have evolved from a data starved field to a data rich one with many experiments competing to advance CMB measurements. MBI was designed to explore one avenue of measurement techniques which holds great promise for sensitivity and controlling systematic errors.

MBI was designed to take advantage of the sensitivity achievable with bolometers and the $l$-mode selection inherent to interferometers. This scheme posed two main hurdles, one conceptual, incorporating incoherent detectors in a measurement technique that relies on phase sensitivity, and the other a difficulty of implementation, operating bolometers outside of a laboratory. We have shown that both were accomplished.

In this thesis we have outlined the principles upon which MBI works and tried to provide the reader with an understanding of both the particulars involved in operating MBI and what analyzing its data entails.

MBI’s first observing run did not prove to be an unequivocal success as there are a number of procedures we would have liked to do, but were not able to. However, even the subset of data we were able to collect is rich. Interference fringes were seen, which
verifies the success of the bolometric interferometry technique, and implies successful operation of the bolometers. The degree of success we had in operating the bolometers only requires someone with the time to look at the data. While we were not able to accomplish every goal we had for MBI’s first observing run, the amount and quality of data collected under our time constraints was impressive.

MBI is a proof of concept to show that interferometry can be done using bolometers and in particular using a stellar Michelson approach. At the most basic level, the production of fringes is positive proof. The next step for the MBI is to work on understanding the system as well as possible, with an eye towards the figures of merit discussed in Chapter 4, particularly the alignment of the polarization selection. Fully characterizing MBI will teach us a great deal about operating this type of interferometer. Beyond the proof of concept stage, a version of MBI with more antennas, a number of frequency bands and operating at more optimal site, such as White Mountain California, would be able to start exploring how far we can push the system in terms of sensitivity and controlling systematics as well as digging into the CMB signal. The field of CMB research could greatly benefit from the development of phase modulators with the specifications discussed in section 5.7.1.

In light of our accomplishments with MBI and the degree to which we have advanced our understanding of the operation and analysis of interferometers, we believe it is appropriate to declare MBI a successful demonstration of bolometric interferometry.
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