Optimization and Numerical Simulation of the Cycling Process and Magnetic Shielding of a Miniature Adiabatic Demagnetization Refrigerator

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ABSTRACT

There are many space and laboratory applications for which continuous cooling from a 5 Kelvin reservoir would be useful. As such, we are designing a miniature adiabatic demagnetization refrigerator (MADR) anchored at this reservoir temperature. Continuous cooling is obtained by the use of several paramagnetic pills placed in series with heat switches. We plan to use a relatively fast cycling process in order to reduce the size of the pills. Superconducting magnets are used for operating both the pills and the magnetoresistive heat-switches. These magnets are split-pair coil magnets using high current density NbTi wires. High permeability materials are used to shield these magnets, which is crucial to minimize stray magnetic fields that could interfere with other refrigerator stages or the detectors. The magnets and their shields have been modelled using a numerical simulation and direct field measurements were made to try to discover an optimal shielding design, which was indeed found. Another computer simulation was developed to explore the feasibility of the MADR design and to test the cycling method. These tests all confirm the feasibility of our MADR design.
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1 MADR Overview

1.1 Motivation and Operation

1.1.1 Why MADR Matters

There are many scientific applications which require the cooling of radiation and particle detectors to temperatures below 1 Kelvin. The number of such applications is increasing, especially in observational astronomy, where the need for very sensitive and low noise detectors (bolometers and microcalorimeters) is essential for precision measurements [1]. However, currently available refrigeration devices from 4 K to less than 200 mK are limited to dilution refrigerators and adiabatic demagnetization refrigerators (ADRs). These two options pose a number of problems. Dilution refrigerators have complex and sizable plumbing, while conventional ADRs do not provide continuous cooling since they must warm up and re-magnetize the salt pill at some reservoir (usually a helium bath) prior to being able to cool down again. Furthermore, ADRs are often high in mass, limiting their space applications.

This thesis describes simulations and experiments done in order to further refine a design for a Miniature Adiabatic Demagnetization Refrigerator (MADR) which solves these problems and provides reliable continuous cooling to 50 mK. The MADR works on the same principle that the conventional ADR does, but by putting several small ADR “stages” in a series it is then possible to cycle a number of pills sequentially and thus achieve continuous cooling.
1.1.2 How MADR Works

ADRs in general function because of the behavior of some paramagnetic materials when placed in a magnetic field. More specifically, it is because the entropy $S$ of such materials depends on the ratio of the magnetic field applied $B$ to the temperature $T$. Increasing field strength decreases the amount of "magnetic" entropy in the system, and vice versa. Hence, for any adiabatic process performed on the salt (where there is no heat flow and therefore $S$ remains constant), if $B$ decreases, then $T$ must also decrease to keep $S$ at its initial value. Likewise, if the applied field on the salt is increased adiabatically, the temperature will increase as well. What this means in a practical sense is that if you have a detector thermally connected to a salt pill (a solid crystal of paramagnetic salt), which in turn is connected to a thermal reservoir with a heat switch, you can “charge” the pill by applying a magnetic field, then disconnect the pill from the reservoir and ramp down the field, cooling the pill and the detector with it. This is the process used in all conventional ADRs. This same idea can be extended to a series of ADRs. In a single-stage ADR as described above, the heat is pumped out of the detector and into the reservoir. In the two-stage series setup, we instead pump the heat out of the detector (which is connected to the cold-stage of the refrigerator) and into the second stage, which then pumps the heat out into the reservoir.

In the same way that a series of locks on a river can move a boat upstream, the ADRs in series effectively move entropy against the temperature gradient. The more stages added to the ADR, the lower its cold-stage temperature can be. Furthermore, the faster you cycle the pills from magnetizing to cooling and back, the smaller your pills need to be, since quicker cycling reduces the amount of heat placed on the cold stage during a single cycle. Finally, it is possible to create a PID (Proportional/Integral/Differential) controller that will keep the temperature of the cold-stage
Figure 3: Diagram of Four-stage MADR.
constant by varying the applied field up and down [2]. The magnetization of the cold-stage’s salt pill is achieved at constant temperature by cooling the stage just before the cold-stage to a slightly lower temperature and holding it there while keeping the cold-stage pill’s temperature constant. Thus the field being applied to the cold-stage pill will have to increase to keep the cold-stage from cooling down further. During this phase of the cycling the heat from the detector (or whatever is attached to the cold-stage) would be transferred to the previous stage.

As is likely obvious from the above description, a key component to the MADR design is the heat-switch. Since the entirety of the refrigerator is dependent on being able to thermally isolate each stage from the next as well as having the stages well connected thermally, we need to have a heat-switch with a high conductivity ratio between the “on position” (when it is conducting) and the “off position” (when it is insulating). To achieve this, many different types of heat-switches exist. We use a magneto resistive heat-switch in the form of a single crystal of tungsten. Such a crystal has the property that when a strong magnetic field (several Tesla) is applied to it, the crystal’s thermal conductivity decreases dramatically. With a set of magnets to control each of the heat-switches and salt pills, the MADR can be cycled for continuous cooling.

### 1.1.3 Design Specifications

In order to examine whether our MADR design will satisfy the operational requirements that we have for it, we must first establish what those requirements are and what limitations they place on our design. The most obvious requirement is some base level of cooling power, since if MADR doesn’t really cool, then it isn’t an effective cryocooler. In order to answer this question of cooling power as well as to better define what the cycling process will be for the MADR, I’ve written a simulation program that models the behavior of either a two-stage or a four-stage MADR. This program can set limits on the capabilities of our design in terms of cooling power and can help explore the effect of certain losses on the system. It is also useful to use to try a variety of cycling conditions in order to find an optimal cycling protocol.

The second restriction is the overall mass and size of the final MADR. Since one of the main goals of the MADR project is to be able to use our design in space, minimizing the mass of our design will help make such applications more feasible. To this end, I have simulated a number of different magnet-shield designs to find an optimization between minimal mass and maximal shielding of the magnet. Good magnetic shielding is vital because the cycling of the MADR requires stray fields to be at a minimum, eliminating interference between one magnet and another. Such interference would prevent accurate PID control over the pills and keep the heat-switches from operating independently, adding unnecessary heat losses through the heat-switches. Furthermore, many of the detectors that would be cooled with a MADR are also sensitive to magnetic fields.

The investigation into each of these restrictions will follow in the C++ simulation section and the FEMM simulation/stray field measurement section, and each will go into a greater depth of detail on that aspect of MADR. For now, it suffices to say that our goal for cooling power is in the range of 5 to 10 microwatts while keeping the field on each pill at the end of a cycle equal to that at the beginning, and a shielding design that is comparable or smaller than our current, one-layer design in size but with a greater shielding capability. We test the feasibility of other shielding designs by doing direct measurements of the stray fields to both calibrate the FEMM simulation and examine
the possibility of multi-layered shielding.

2 C++ MADR Simulation

2.1 Simulation Goals

In order to more appropriately determine the limitations and feasibility of our MADR design, I created a simulation program in the C++ computer language to model the behavior of such a refrigerator (the code for which is provided in Appendix B). This program was designed to answer several questions, namely:

1. Is our MADR design feasible in terms of temperature goals and cooling power?
2. What sort of cycling time can we expect and how can we optimize our cycling steps to maximize the effectiveness of the refrigerator?
3. How are our expected losses going to affect the performance of the MADR, or to what extent do the losses need to be eliminated for MADR to be practical?

Before we can consider these questions, we must first set out the structure and ideology behind the simulation so that it is clear how the simulation works and the strengths and limitations it possesses.

2.2 Methodology

The simulation program is built with a number of layers, taking advantage of the object oriented programming capabilities of the C++ language. The MADR design is broken into simple ‘building-block’ objects, which are coded, grouped into larger structures, and then finally into the full MADR setup. Within the program is a control function which imposes external conditions, namely the time dependent magnetic fields that are applied to the heat-switches and salt pills, and records the local conditions that we’re interested in, such as temperature and heat-flow. The control function moves the MADR through a series of time steps, changing the external conditions. As a foundation for the simulation here are five basic building-blocks that are used: conductors, heatswitches, reservoirs, heatloads and pills. Italics will be used throughout to distinguish the C++ program classes from the actual physical MADR components that these classes model. Each of the five will be described below.

With the building-blocks in place, simulating the overall behavior of the MADR simulation is based on connecting the blocks into grouped structures which then form each of the stages of the MADR. The structure immediately above these basic components is the link, which consists of five conductors and a heatswitch. The link behaves like a thermal conductor, though it has more complex properties than the simple conductor object. Above this there is a superlink, which consists of a reservoir, two pills and two links, forming a two-stage MADR. Above this is the vectorlink class, which contains a superlink, a heatload and the control structure, allowing us to define the steps in a MADR cycle, and also to cycle the two-stage MADR under a number of initial conditions. For the
four-stage MADR simulation there is the fourpill class, which contains four each of links and pills, plus a reservoir and a heatload. Furthermore, the fourpill class has a more complex control structure which is analogous to the two-pill vectorlink.

In each MADR type, whether two-stage or four-stage, the calculations of quantities such as magnetic field, temperature, and entropy are done using an iterative method. Initially, reasonable “first guesses” are determined for the values of quantities that are to be calculated. The program then improves on those guesses by calculating whether the initial value in question needs to be increased or decreased. This is cycled until a the current “guess” is determined to be within an acceptable error margin of the actual best solution. The initial value is chosen in such a way that the simulation will definitely find a solution and, if possible, so that the simulation takes less time than other initial values.

2.3 MADR Structures

2.3.1 Conductor

The conductor is just what its label implies, a thermal conductor between two temperatures. The conductor object stores as internal data values its own physical dimensions of length, width and height, plus the temperatures of each of the conductor’s two ends. In this construction, the length is the distance between the two temperatures and the width and height define a cross-section at the two ends. We assume that the conductor has this uniform cross-section throughout its length. Information as to the conducting material is also contained here. In our simulation copper, gold, manganin and kevlar are the four materials used.

The conductor’s most notable feature is the function for calculating and returning the amount of heat flowing through it, given the material and the two temperatures. We assume that locally within the conductor we have thermal conductivity $k$ (in units of $Wcm^{-1}K^{-1}$) in the form $k = aT^b$ for any temperature $T$. By doing an integrated average over the conductor, we see that the heat-flow $\phi$ (in W) is given by

$$\phi = \frac{a \times S \times (T_{\text{max}}^{b+1} - T_{\text{min}}^{b+1})}{(b + 1) \times L} \quad (1)$$

where $S$ is the cross-sectional area, $L$ is the length, and $T_{\text{max}}$ and $T_{\text{min}}$ are the two temperatures. A sign is applied to $\phi$ depending on the direction of the temperature gradient. Heat-flow away from the reservoir is denoted by a positive $\phi$ and flow towards the reservoir is negative, i.e. everything is signed in terms of the amount of heat flowing towards the cold stage of the MADR.

The two parameters $a$ and $b$ are from literature and their values are listed in the table below.

<table>
<thead>
<tr>
<th>Material</th>
<th>$a$ [$Wcm^{-1}K^{1+b}$]</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>0.53</td>
<td>1.12</td>
</tr>
<tr>
<td>Gold</td>
<td>5</td>
<td>0.9</td>
</tr>
<tr>
<td>Kevlar</td>
<td>$3.9 \times 10^{-3}$</td>
<td>1.17</td>
</tr>
<tr>
<td>Manganin</td>
<td>$9.4 \times 10^{-4}$</td>
<td>1.2</td>
</tr>
</tbody>
</table>
2.3.2 Heat-Switch

The heatswitch class is a modified version of the conductor that contains the same type of physical dimension and temperature extrema variables. We also use the same basic formulation for determining $\phi$ as in Equation 1. However, since the thermal conductivity of tungsten is dependent on the applied magnetic field $B$, we now let $a$ and $b$ be functions of $B$. The field dependence for $a$ is measured experimentally and a regression curve is fit to $a(B)$. The dependence of $\phi$ on $b$ is relatively weak and it suffices to let $b$ vary linearly with $B$ between its extremes. See Figure 4.

2.3.3 Reservoirs & Heat Loads

The reservoir object class is the simplest of the classes created for the MADR simulation. The reservoir is a constant temperature point made to simulate the connection between MADR and the
helium bath. For the purposes of this simulation we’ve assumed that the bath is pumped \(^4\)He at 2 Kelvin, though the program allows for any reservoir temperature desired.

The \textit{heatload} is another fairly simple class that represents a copper mass on the end of the cold stage. Though once the refrigerator begins to cycle the load has no effect on the results of the simulation, it is useful to include to see how the MADR will cool down to cycling temperature from the reservoir temperature. The \textit{heatload} consists of a mass, a temperature and a function for determining the temperature change after an input of heat-flow \(\phi\) for a time \(dt\) by means of calculating the heat capacity at the initial temperature.

2.3.4 Salt Pills

The final and perhaps most important of the building-block classes is the \textit{pill} class, which encapsulates the behavior that is most fundamental to the function of the MADR. The \textit{pill} houses a temperature and volume, and it can be initialized as one of four paramagnetic salts which vary in density, molecular mass, internal magnetization field and total spin value. These salts are CPA (Chromium Potassium Alum), FAA (Ferric Ammonium Alum), CCA (Cesium Chromium Alum) and GGG (Gadolinium Gallium Garnet).

The \textit{pill} also has three functions imbedded in it: an entropy calculator, a temperature calculator and a field calculator. The entropy calculation function, using a formula from [7], yields the entropy of the salt given its temperature and the applied field. This formula is

\[
S(T, B) = N R \left\{ \ln \left( \frac{\sinh \left( \frac{(2J+1)x}{2} \right)}{\sinh \frac{x}{2}} \right) + \frac{x}{2} \coth \frac{x}{2} - \frac{(2J+1)x}{2} \coth \frac{(2J+1)x}{2} \right\} \tag{2}
\]

where we have the following definitions:

\[
x = \frac{g \beta B_{eff}}{kT} \quad B_{eff} = \sqrt{B^2 + B_{int}^2}
\]

and the following values for the different salts:

<table>
<thead>
<tr>
<th>Salt</th>
<th>J</th>
<th>Molar Mass (g/mole)</th>
<th>Density (g/cm^3)</th>
<th>(B_{int}) (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCA</td>
<td>3/2</td>
<td>571</td>
<td>2.06</td>
<td>negligible</td>
</tr>
<tr>
<td>CPA</td>
<td>3/2</td>
<td>580</td>
<td>1.82</td>
<td>0.02</td>
</tr>
<tr>
<td>FAA</td>
<td>5/2</td>
<td>482</td>
<td>1.72</td>
<td>0.05</td>
</tr>
<tr>
<td>GGG</td>
<td>7/2</td>
<td>1012</td>
<td>7.14</td>
<td>0.01</td>
</tr>
</tbody>
</table>

In these expressions, \(J\) is the angular momentum quantum number, \(g\) is the spectroscopic splitting factor, \(\beta\) is the Bohr magneton, \(k\) is the Boltzmann constant, \(B_{int}\) is the internal field, \(N\) is the number of moles of salt and \(R\) is the ideal gas constant. Note that other than \(B\) and \(T\), all of these are either fundamental constants or constants determined by the choice of salt. This entropy function is crucial to the other two functions in the \textit{pill} class since each calls on the entropy calculator in the course of its operation.

The second \textit{pill} function is the temperature calculator, which takes the current applied field value, the next field value, the heat-flow into the \textit{pill} and the size of the time-step as inputs and returns the temperature that the \textit{pill} will be during the next time-step. The way that the function works is by
calculating the current entropy, the change in entropy and then moving through possible temperature values iteratively with the next field value for $B$ until the entropies match. In other words, we try to find the unique value of $T$ that solves

$$ S(T_{\text{now}}, B_{\text{now}}) + \frac{\phi \Delta t}{T_{\text{now}}} = S(T, B_{\text{next}}) $$

which is possible since Equation 2, for $B$ constant, $S(T, B)$ is a monotonically increasing function for increasing $T$. Thus we can find such a $T$ with a convergent series of guesses $\{T_n\}$. By default we choose our first guess ($T_0$) to be 5 Kelvin and for each successive guess we set $T_{n+1} = T_n \pm \frac{2.5}{T_{\text{now}}}$, where $\Delta T$ is 2.5 Kelvin, which keeps our final solution for $T$ between absolute zero and ten Kelvin and also assures that it will converge. Due to the monotonicity of $S$, we know to choose the plus sign if the entropy calculated from our guess is too high and the minus if it is too low. Due to numerical considerations it is impractical to try to find $T$ such that the two sides of this equation are exactly equal. Rather, we simply require that they agree to ten decimal places. The amount of error is a parameter that could be changed depending on desired calculation efficiencies, though we’ve found that this error limit prevents a divergence of subsequent temperature errors without significantly reducing computation efficiency.

The final function of the *pill* is the field calculator. This works in large part like the temperature calculator in terms of its iterative process to find a solution. The purpose of this function is, given the heat-flow into the *pill*, the current magnetic field and the time-step size, to return the field that would be necessary in the next time step to keep the temperature of the *pill* constant. This function takes the place of a PID device in a real MADR. It allows us to make a control structure that takes feedback from the *pill* and adjusts the field accordingly to keep the temperature constant, which is exactly what we would like to have for our cold stage in the MADR. As before, we want a solution $B$ such that

$$ S(T_{\text{now}}, B_{\text{now}}) + \frac{\phi \Delta t}{T_{\text{now}}} = S(T_{\text{now}}, B) $$

and we choose to search for a solution with a similar sequence of field values $\{B_n\}$ where $B_0 = B_{\text{max}}/2$ and $B_{n+1} = B_n \pm \frac{B_{\text{max}}}{4}$ for $\Delta B = B_{\text{max}}/4$. We place the same error limit on the resultant entropy solutions as in the temperature calculator. Since the entropy function increases monotonically with decreasing $B$, we can assign the plus sign if our guess for $S$ is too big and the minus if our guess for $S$ is too small.

### 2.3.5 Link, Superlink, Vectorlink & Fourpill classes

As alluded to above, the *link* class is a set of five conductors and a heat-switch connected as they would be in an actual MADR, connecting each successive salt pill. Note that the manganin and Kevlar conductors represent parasitic heat loads that would occur due to conduction from the reservoir temperature.

The *link* has a direction, so there is a “reservoir end” and a "cold-stage end". When it is used in the program, the *link* is first initialized by setting the temperature extrema at the ends, and then the *link* will calculate the heat-flow (in Watts) through it. This heat-flow is dependent on the field applied to the switch, the allowed error in the calculation and the size of the time-step. The *link* object calculates this heat-flow by varying the temperature at the ends of each of its components,
Figure 5: Construction of the link object. There would be a pill attached to either end of this. Though the program can also use gold wires, we simulated with copper since that is the material we used experimentally.

except for the two set temperature extrema, until it finds a set of temperatures that yield the same amount of heat-flow throughout each portion of the link. This heat-flow is then added to the heat-flow of the parasitic loads to give the total amount of heat-flow being put into the pill that is assumed to be at the “cold-stage” end of the link. Here we make the assumption that the heat capacity ($C_p$) of the link is negligible compared to that of the pill. Since the link is low mass and made of good thermal conductors, this assumption is valid.

This gives the following scheme for moving the superlink through a series of time-steps, which is the function of the vectorlink class. The first step necessary is to establish the initial temperature of each pill, the initial magnetic fields on the pills and the heatswitches, and the fields for the next time-step. The initial heat-flow is assumed to be zero. Then the pills are called to calculate their temperatures for the next step. With these temperatures recorded, the links then have their temperature extrema updated to the new pill temperatures and are subsequently called to calculate the heat-flow for the next time-step based on the new temperatures and the current fields. During this process, the temperatures of each end of each object in the link are calculated. These end temperatures are recorded for use as initial guesses for the next time-step’s calculation. This cycle repeats itself as many times as is desired. The heat-flow is conserved, since the heat-flow into the cold-stage pill is subtracted from the total calculated heat-flow through the link connecting the reservoir to the warm-stage pill when determining how much heat is being put into the warm pill (see Figure 6). This creates the following relationship: $\phi_{PillA} = \phi_{LinkA}$ and $\phi_{PillB} = \phi_{LinkB} - \phi_{LinkA}$.

The same scheme is used in the fourpill class except that now, instead of having just two links connected with pills, there are four. Each stage of the MADR is set up to conserve the heat-flow throughout the setup as a whole, making the simulation consistent with thermodynamics. The most important aspects of the vectorlink and fourpill classes are the control structures that they contain.
which drive the cycling of the pills, which will be discussed in the next section.

2.4 Cycling MADR

The cycling of the MADR program is done using a series of “phases”, each with a specific goal in mind for an effect on the MADR setup. The phases consist of while-loops that iterate until either the goal is met or one of a number of “escape conditions” occur. The escape conditions vary between from one phase to the next and some phases, due to the conditional statements inside them, don’t need any escape conditions. The purpose of the escape conditions are to end the loop if it would otherwise continue indefinitely. However, during normal operation of the MADR program under reasonable starting conditions, each phase will consist of a finite number of time-steps without using any of the escape conditions. The temperatures of the pills, the fields on the pills and the heatswitches and the heat-flow in the links are all recorded in a set of data arrays which can then be written to files for later analysis.

The MADR program, like the real life refrigerator it is simulating, is controlled by modulating the four (in the two-pill version) or eight (in the four-pill version) magnetic fields. Because of this, the types of phases in a MADR cycle break down along a number of dichotomies in terms of what each field is doing. In general, we hold to a two basic rules for the field functions during cycling:

1. Heatswitches are opened or closed at a ramping rate which is the same for all phases. This allows for some uniformity to the cycles and makes it easier to vary the switch-cycling times.

2. The field of the cold-stage pill is always varied to keep a constant cold-stage temperature (with the exception of when the MADR is cooling down from reservoir temperature). This is consistent with the goal of having MADR provide continuous cooling at the given temperature.

In general, the non-cold-stage pills will have their fields ramped up and down at a rate which is
Figure 7: Two-pill MADR cycle with phases. A: Magnetizing Midpill; B: Cycling switches and cooling Midpill; C: Magnetizing Coldpill with Midpill isothermal; D: Recycling switches in preparation for next cycle. ColdT is the temperature of the cold-stage pill, MidT is the intermediate pill temperature, ColdB and MidB are the fields on the cold- and intermediate-stage pills, respectively. ColdSB is the field on the heatswitch connecting the two pills and MidSB is the field on the heatswitch connecting the intermediate-stage to the reservoir. Note that ColdT remains constant throughout.
an input parameter at the beginning of the simulation. The only phases in which this is not the case is when those pills are being used as a heat-sink for the next-coldest stage’s pill. In these cases, the pill in question is cooled to below the temperature of the next coldest pill and then held constant at that temperature. The temperature and fields during a cycling of the two-stage MADR is shown in Figure 7 and the phases are labelled.

The four-stage cycle is more complex but follows the same general idea, with the addition that the first and third pill/link units (labelling the cold stage as pill 1) act together and the second and fourth pill/link units also act together. In other words, the first and third pills are both charging during the same phase, the second and fourth heatswitches open and close together, etc. This greatly simplifies the process of coding the phases and resembles the actual function of the final four-stage MADR since the switches will be linked in just that way to save on the number of magnets necessary.

2.5 Program Limitations

The main limitations of this program are in its ability to account for the losses we encounter when we are actually building and using a MADR setup in the laboratory. The program does simulate some of the losses, namely conduction from thermometry wires and kevlar support wires, both of which conduct heat from the reservoir to each of the pills. However, there are a number of other losses that are present in the laboratory system that are not represented in the program. The most notable of these losses is eddy-current heating. The reason that the program does not account for this heating effect is twofold. Firstly, the current setup does not immediately allow for eddy-current heating to be included as a loss, since the program calculates heat-flow between pills and reservoirs while eddy-current heating would most realistically occur in the links. This was not a major concern during the development of the program since eddy-current heating is negligible and controllable with alterations in field ramping rates and physical link design [8].

Other losses include heating from thermometry at each of the stages, as well as radiation heating from heat sources that are not directly connected to the MADR, but could be within a line of sight of it inside a dewar. These losses, however, are controllable, since thermistor losses can be minimized by lowering the current used in four-wire measurements and radiation can be eliminated by improving heat-sinks within the MADR chamber.

The final program limitation is a computational time issue. Since changes at each time step are calculated individually, the overall cycling of the MADR in simulation takes a noticeable amount of time. It takes roughly 10 minutes to go through two cycles of the four-stage program at 100 steps per second. While this does restrict the number of cycles one can simulate in a given amount of time, the fact that each of the steps is calculated numerically rather than using more analytical techniques makes the program versatile in other ways. For example, changing the type of heat-switch wouldn’t involve solving another analytic equation, but rather just changing the heat-flow calculation function in the heatswitch object class. This opens up the possibility of making changes to the program in the future without having to completely remake the simulation.
2.6 Simulation Results

In the beginning of the discussion on the C++ program, three main questions were presented that this simulation was designed to answer. Here I present the results of the simulation of both the two- and four-stage MADRs as well as a comparison between the simulation and the cycling of an actual two-stage MADR in the laboratory.

2.6.1 Cooling Power

The amount of cooling power the MADR has is a very important specification, since it determines how effective of a refrigerator the MADR can be. Too low of a cooling power and any detector will eventually heat up while functioning, rendering the MADR’s presence moot. So how much heat can the MADR handle? Within the vectorlink and fourpill classes there is a constant applied heat variable. This represents heat placed directly on the heatload object, such as heating from a bolometer.

The key to determining what amount of applied heat is acceptable, we look not for how much can be applied without the coldest pill increasing in temperature, but rather how much heat we can apply without causing a net decrease in the magnetic fields on the pills. If there is such a net decrease, then, although there is no change in temperature over a single cycle, over a number of cycles the field will slowly be depleted and eventually it will run out. Once the MADR has fully run out of field, it cannot cycle any more. However, even if the field does not reach its full initial value, it can still cycle for a long period of time before the field will be fully depleted.

For the two-pill simulation, the vectorlink was initialized with the cold-stage pill and the heatload both at 350 mK, the cold-stage pill’s field at 0.5 T, the warm-stage pill at the reservoir temperature of 1.9 K and the warm pill’s field at 0 T. The MADR was then run for two cycles to see what happened to each of the fields. With 8 microwatts of power applied to the cold-stage, the MADR cycled twice without any net drop in the pill fields. At 9 microwatts, the MADR simulation showed minimal loss on the cold-stage, but a 0.05 Tesla drop in the field on the midpill, and the midpill actually ran out of field at the end of the second cycle. When the power was increased to 10 microwatts, the midpill expended all the field during each of the cycles and the coldpill lost 0.01 Tesla over each of the cycles. Given the initial field on the coldpill, this shows that with 10 microwatts of power on the cold-stage the MADR will function for roughly fifty cycles. Each cycle is about 25 minutes, so the estimated usable time at this power is 20 hours, which is still a significant amount of time. So the two-stage MADR achieves our cooling power goal.

The four-stage MADR presents a more complicated problem. Now there are eight magnetic fields to manipulate instead of four, which makes optimizing the fourpill class’s cycles that much more difficult. The problem is simplified somewhat by synchronizing the actions of alternating pills. In other words, the first and third pills charge together and cool together, while the second and fourth act together but out of phase with the other two: Two and four charge while one and three cool, etc. The fourpill class’s cycles use this simplification, but optimization of the many exit conditions for the different phases is a task that has not been completed as of this writing. However, the success of the two-stage MADR simulation and of the laboratory prototype built in conjunction with this work are promising results that suggest that the four-stage MADR will also be successful [6].
Figure 8: Field vs time with 10 microwatts on the cold-stage. The left vertical scale is for the midpill field and the right is for the coldpill field.
Figure 9: Laboratory cycling of a two-stage MADR. Top: Fields on heat-switches. Switch A is the switch connecting the cold-stage pill to the intermediate-stage, and Switch B connects the intermediate-stage to the reservoir. Bottom: Temperatures of the two CPA salt pills. CPA1 is the cold-stage pill and CPA2 is the intermediate-stage pill. [6]
Figure 10: Temperature vs. Time for the *fourpil* cycled without any heat-flow on the cold-stage. The cold-stage is indicated with T1, T2 is the next warmest stage, etc. This represents the best optimization of the cycling procedure at the time of this writing.
far the *fourpill* has been configured to where it can run for nearly three cycles before the cold-stage pill warms above 50 mK without any sort of constant heat-flow put on the cold-stage. Also, when the pill does finally warm, it isn’t all the way to the reservoir temperature of 5K, but rather to a slightly warmer temperature of roughly 200 mK. While this isn’t in the temperature range we’d like the operation of the four-stage MADR to be, we believe that further work optimizing the simulation will yield additional progress.

2.7 Conclusion

The C++ simulation of the two-stage MADR predicted the feasibility of the two-stage prototype which in turn was confirmed in the laboratory. The simulation additionally places a preliminary upper limit on the cooling power of the two-stage MADR at 8 microwatts for indefinite cycling, and shows that with 10 microwatts there is a potentially long operational timescale that would still be useful for observational purposes. Furthermore, these results indicate that the simulation of the four-stage MADR could be optimized to define similar limits on its operational capabilities.

3 Magnetic Shielding Simulation and Testing

3.1 Introduction

As mentioned in the overview of the MADR project, one of the most important components of a well functioning and effective MADR is high magnetic fields that are localized to distinct areas, namely the heat-switches and the salt pills. Since we wish to minimize the amount of stray fields that could cause interference between the different stages of the MADR and possibly the detectors, the most logical path of action is to design some sort of shield out of high magnetic permeability materials. The question this section of the thesis will at least begin to answer are:

1. Would using several layers with multiple materials be better than a single-layer shield?
2. If so, what sorts of materials would be effective in attaining both the low-field and low-mass goals?
3. How much is gained over the single-layer shield?

The current shield design is a single layer of vanadium permendur in the shape of a hollow cylinder with solid endcaps. The superconducting magnet coils are placed inside of the cylinder. In the center of the coils there will be a salt pill or a cylindrical holmium core, if the magnet is to be used for a heat-switch. The holmium helps to focus and concentrate the field at the center of the magnet and vanadium permendur reduces the stray fields. We are looking to augment the shielding capability of this design by adding a second high-permeability layer, this one made of Cryoperm 10 (a material produced by [9]), and a layer of superconducting material, such as niobium, on top of that to catch any additional field not attenuated away by the first two layers.

The shielding was experimentally measured and the model was fit to the data. Then the existing shield was wrapped in a lead foil tape and the field was measured again, to further test the practicality
Figure 11: Current two-coil split-magnet shield design in a cross-sectional view. Each of the green dots marks a location where the simulation measures the field 0.01 inches outside the shield.
of a superconducting layer. With the simulation in agreement with the experimental data we could then begin to simulate other possible shield designs using multiple layers. The shielding simulation was done using a numerical program called FEMM: Finite Element Method Magnetics. The FEMM program operates either through a graphical user interface or via a scripting language called Lua [10]. This language provides an easy means of automating multiple shield design simulations. A basic shield design was decided on and parameters were varied within that basic design by the Lua script.

3.2 Shielding Measurements

3.2.1 Experimental Setup

We measured the field outside the existing shield using a Hall sensor and a series of calibrations, both to insure that our field values were accurate as well as to provide a calibration for the current density in the magnet, which is a parameter used in the FEMM simulation. Since the windings on the magnet are not uniform, we cannot say for sure what the current density is. However we assume that the current density is uniform and that it scales linearly with the current put through the magnet. These assumptions seem reasonable and they allow us to model the magnet to a good degree of accuracy. We use a paired coil magnet with shield that is roughly symmetric about the coil axis and symmetric across the plane separating the two halves.

In order to test the magnet and measure the stray field, the magnet was cooled with liquid helium to 4.2 K inside of a four-liter dewar. The magnet was attached to the end of an acrylic tube that both immobilized the magnet inside the dewar and acted as a position reference for inserting the Hall sensor. This tube also had two styrofoam insulating discs covered with aluminized mylor super-insulation to prevent unnecessary helium boil-off due to 300 K radiation. The Hall sensor was attached to a G-10 tube that housed both the wires from the sensor and was marked to show how deep in the dewar the sensor was (see Figure 12).

The field outside of the shield was measured at a point 1.9 inches (4.8 cm) from the axis of the magnet along the middle of the magnet and directionally parallel to the magnet’s axis. Since there is little radial or azimuthal field along the symmetry plane of the axis, our measurements are a good measure of the total field magnitude at that point. Measurements at this test point as well as in the center of the magnet give the needed reference to make our simulation accurate.

3.2.2 Calibration

One of the challenges in making these measurements was calibrating the Hall sensor. We used two different Hall sensors in the course of this experiment, the HGA-2010 (an axial sensor) and the HGT-2010 (a transverse sensor) Hall Sensors manufactured by Lakeshore. Both sensors were calibrated for room temperature use by the manufacturer, at 19.602 mV/kG and 19.125 mV/kG respectively. However, the sensors were not calibrated for low temperature use, which we would need them to be. The following method was used to calibrate the sensors at low temperature. For the axial sensor one current coil was used and for the transverse sensor two coils were used in the Helmholtz configuration. The difference in setup was to align the axis of the coil(s) with the active portion
Figure 12: Experimental setup, which was placed in a 4 liter dewar and cooled with cryogens.
of the sensor. In both cases, the coil field was measured as a function of the current put through it using the manufacturer’s sensor calibrations. Then the sensor was cooled with liquid helium to 4.2 Kelvin and the coil(s), which now had a known field-current relationship, were ramped through the same current range (see Figure 13). This provided a new calibration for the Hall sensor at low temperature. We tested this several times to ensure that the calibration was constant and did not change between coolings.

With the low-temperature Hall sensor calibration determined, it was now possible to “calibrate” our simulation. Since the coil density was not explicitly known in our magnet, it was necessary to match the simulation to field measurements in our current design before we could begin to simulate other possible shield designs. To do this we measured the field at the center of the magnet both with and without the holmium core, and outside of the magnet as described above. The same “measurements” were made using the simulation and the two were compared to find a conversion between the current put into the magnet leads and the current density in the magnet coils. In doing this we assumed that there was a linear relationship between current and current density, which is reasonable. We also allowed for some variation between the external measurement and its simulation because the simulation assumes exact axial and planar symmetry in the shield design, which our real magnet does not have. We found that the best fit conversion between current density and current is \( 1 \text{ A} = 26.67 \text{ A/mm}^2 \).

As Figures 14 and 15 show, this current density conversion makes a good fit to the data outside the shield and to the data at the center of the magnet without holmium. However, this conversion does not make a good fit to the central field data with the holmium core in place, possibly due to irregularities in our particular holmium core. Since the purpose of this investigation is to minimize the stray fields outside of the shield, the internal field is not as much of a concern in our simulation. Since the simulation shows the correct behavior qualitatively, we won’t worry too much about the magnitude of the central field, though it would be an area for later study.

As another check to the current density conversion, we then measured the stray field as a function of current with the magnet wrapped in lead tape. By measuring when the stray field becomes non-zero, meaning that the critical field of lead has been exceeded, we can check to see that the simulation’s stray field is reaching that same value for the same current. Indeed this was the case, further supporting the validity of using this conversion value. The data from this test also showed that it is crucial to keep the stray field well below the critical field of the superconductor. Notice in Figure 15 that once the critical field is exceeded (at about 150 A/mm\(^2\)) that there is a hysteresis-like effect, likely caused by an induced current in the superconductor (see Figure 17). Thus, in order for our shield to be dependable, it must attenuate the stray field enough so that it is well below the critical field of the superconductor.

### 3.3 Optimization

With the simulation correctly calibrated to agree with the data, we can now make predictions based on these simulations. The simulation modelled a shield size comparable to the design currently in use. We used a total shield radius of 1.2 inches and a total height of 1.4 inches. The thickness of the shield used in the simulation is 0.4 inches on the top and about 0.45 radially. Note that the height listed is that of the shield for each coil, and that the total magnet height would be twice that.
Figure 13: Calibration of the current coils and the Hall sensors. Top Row, right to left: Calibration of single current coil at room temperature, calibration of axial sensor at 4.2 K. Bottom Row, right to left: Calibration of Helmholtz coils at room temperature, calibration of transverse sensor at 4.2 K.
Figure 14: Field vs. Current curves for the magnetic field at the center of the magnet. The data assumes a current density conversion of 1 A = 26.67 A/mm²

Figure 15: Field vs. Current curves for the magnetic field 1.9 inches of the axis of the magnet assuming the same current density conversion.
Figure 16: Magnetic Field B vs Magnetic Field Strength H for each of the three materials. VP data is from various sources, Ho data from [11], and Cryoperm 10 data from [12] and related company literature.
Within these constraints on the size of the magnet, we constructed the shield out of two layers, each of a different material. The first layer was vanadium permendur, and the second was Cryoperm 10. The simulation altered the ratio of the vertical and radial thicknesses of the two layers independently and for each ratio the field was measured at five separate points (see Figure 10). This process was then repeated with a 0.05 inch gap between the two material layers. The goal was to optimize the design and minimize the field as much as possible, at least so that any stray fields were less than 0.2 Tesla, the critical field of niobium [3].

We expected that there would be little difference between the designs with and without gaps. However, the two simulations yielded surprisingly different results. For each we examined the maximum of the field values obtained at the five sampling locations for each of the designs. We then searched for a lowest value among these maxima. For the designs with no gap, the radial thickness is the dominant factor in determining the shield’s effectiveness, since there is a series of designs that have varying top thicknesses with little change in the maximal stray field, though an optimal design did exist. Stray fields were minimized with equal-thickness layers of vanadium permendur and Cryoperm 10 on the top and roughly 79% of the radial thickness made up of vanadium permendur. The optimal design indicated keeps the stray fields below 0.07 Tesla, which is well below the niobium critical value. Since the actual physical magnet will not have the ideal shield depicted in this simulation, we can expect that there will be a higher amount of field that leaks out. The wide margin of error suggests that our design should still work with the less than optimal shield design necessitated by experimental considerations (see Figures 19 and 20).

The simulation suggests a much different design when the gap is included. For these designs, we applied the same method for looking for an optimal design, seeking a minimum in the stray field maxima. The lowest stray field was found with the maximum allowed thickness of vanadium permendur both vertically and radially, with only a 0.01 inch thick layer of Cryoperm 10. Furthermore, the
Figure 18: The FEMM simulation field. Azimuthal symmetry is assumed about the z-axis and planar symmetry is assumed at the radial axis. R and T are the two varied design parameters mentioned in Figures 19-22.
Figure 19: Maximum stray field with no gap between shield layers as a function of the radial thickness of the VP layer. The optimal design is indicated.

Figure 20: Maximum stray field with no gap between shield layers as a function of the vertical thickness of the VP layer. The optimal design is indicated.
optimal design allows for much lower stray fields, the best design having stray fields less than 0.05 Tesla (see Figures 21 and 22). This shows that having the gap between the layers makes a significant improvement on the overall stray fields, since if the drop in field were solely due to the increased amount of vanadium permendur then the simulation without the gap would have suggested the same sort of design, rather than finding an optimum with comparable amounts of the two materials.

These results suggest the following design for the shield to be used in the future. Since the stray field is significantly improved by the use of a small gap and a thin layer of Cryoperm 10, the best design would be a shield made predominantly with vanadium permendur. On top of this layer would be placed some sort of spacing layer that is not ferromagnetic, such as a thin plastic coating. This would be covered with a Cryoperm 10 foil layer that in turn would be covered with another gap, and finally a niobium super-conducting layer. This design is capable of completely eliminating the stray fields outside of the magnet so long as there are no openings in the shield. However, since there must be such openings for the heat-switches and link objects described in the preceding sections, this shield should still significantly reduce the stray fields. The extent to which these fields would be reduced is beyond the scope of this simulation program’s capabilities due to the lack of symmetries, and empirical investigation would thus be warranted.

3.4 Conclusion

We were successful in simulating and measuring the stray fields of our current shield design, giving us a framework to simulate other possible designs for shielding. Overall, this simulation and measurement experiment yielded a number of significant results. First and foremost among these
is that it is possible to make a compact magnetic shield that will not only reduce stray fields, but reduce them to a level where a superconductor can contain the fields without nearing the critical field. This is crucial since passing the critical field can set up and maintain a substantial stray field outside of the superconductor. We’ve also found an optimal shield design, or at least one which is closer to optimal than our current design. More specific limits need to be placed on the stray fields, perhaps by the detector sensitivities, before we can optimize further in terms of the size of the shield. However, there is a lot of room for optimization since the design suggested by this simulation does not change the size of the shield at all while still significantly reducing the stray fields and keeping them well below the critical level. Our three layer design with non-ferromagnetic gaps between each is a significant improvement over the current design and could likely be reduced in size if necessary.

4 Final Remarks

In conclusion, this thesis project succeeded in a number of ways in the two main areas of investigation, namely cycling and magnetic shielding. Firstly, the C++ simulation showed that the two-stage prototype of the MADR can be cycled indefinitely with certain restrictions on the amount of heat being placed on the cold stage, and it takes into account several of the more significant losses present in the MADR system. Building upon this two-stage version, we simulated a four-stage MADR as well, which is the design that would most likely be used for a satellite cryogenic system since it would cool from a higher reservoir temperature and would have a lower cold-stage temperature than the two-stage version is capable of. This four-stage simulation was not indefinitely sustainable, but this is not, we think, due to a fundamental inability. Rather, since the four-stage MADR is significantly more complicated than the two-stage version, our current simulation is not using the optimal cycling
protocol. The temperature ranges of each of the pills, combined with the timing of the magnetic field controllers give a large number of possible parameter combinations which need to be optimized. We are confident that with some further work that this simulation can be optimized and that by doing said optimization, one would be able to find the cycling process to use for an actual four-stage MADR in the laboratory. Overall, this simulation has shown that the our MADR design is practical and feasible and that it deserves further investigation in order to perfect it.

Secondly, the modelling and measurement of the stray fields outside the magnetic shielding have provided an optimal design. This design uses three material layers with two different ferromagnetic metals (vanadium permendur and Cryoperm 10) and a superconductor (niobium) to minimize the field outside of the shield and also ensure that the field at the inner edge of the superconducting layer does not near niobium’s critical field. While any practical shielding design will be unable to shield the magnet as well as the simulated design, the large margin between the simulated stray field and the critical field allows for this fact and supports the feasibility of this design.

Further work in this area could include several things that would be useful. As mentioned above, the cycling procedure for the four-stage MADR needs to be optimized, and a three- or four-stage laboratory prototype could be built and tested. As to the shielding, more extensive stray field measurements could be taken and compared to the modelling in order to better understand the behavior of the stray fields and to better design shielding based on that.
References


